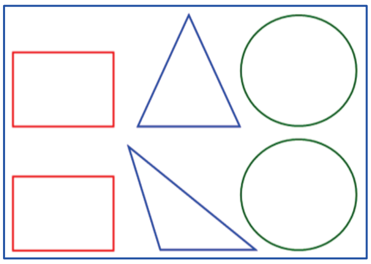
The areas of a rectangle, a triangle and a circle are equal.



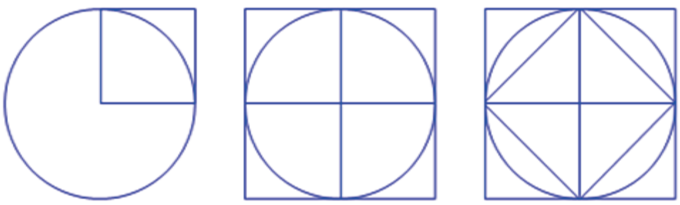
The prompt can be presented in words only, which can lead to a discussion of types of possible triangles, or combined with one of the two sets of shapes on the left.

While this seems a rather simple prompt, it can lead to a number of difficult questions for students in years 7 and 8:

- How do you work out the area of an obtuse-angled triangle?
- How do you work out the area of a circle?
- Can the length of the radius of the circle be a whole number if its area is a whole number?
- Can the dimensions of the three shapes all be whole numbers or must there be decimals?

An issue that regularly arises is the degree of accuracy appropriate for the area of the circle in comparison to the areas of the rectangle and triangle. If the areas of the rectangle and triangle are $20cm^2$, for example, is the statement proved correct if the area of the circle is within one tenth of a square centimetre? ... or one half? Can the areas ever be exactly the same? Some classes have been able to use inverse operations to deduce that the radius of the circle is, in our example, $\sqrt{(^{20}/_{\pi})}$. Is it acceptable to leave the solution in surd form and in terms of π?

On the question of the area of the circle, I always use the diagrams presented in CAME (*Thinking Maths*),



which link the area of the circle to areas of squares and triangles. Students first draw the square of the radius in the top right-hand quarter of the circle. Through the succeeding diagrams they realise that the area is $2r^2 < A < 4r^2$.

Alternative prompts

Other similar prompts that have led to sustained classroom inquiry are the following:

(1) The areas of three different types of quadrilaterals are equal.

This prompt has led to students listing types of quadrilaterals and then using their knowledge of one type to attempt to deduce the formulas for the areas of others. Another class (a high set) decided to work algebraically by making, for example, $lw = {}^1/_2(a + b)h$ in order to find values for the dimensions.

(2) The perimeters of a rectangle, a triangle and a circle are equal.

This prompt has led to students constructing particular triangles that meet the condition of the prompt with a ruler and pair of compasses. It has also generated questions about shapes with the same areas (from the main prompt):

- Is it possible to have three shapes with the same area and the same perimeter?
- Is there a relationship between the perimeters of the three shapes with equal areas?

(3) The volume of a cuboid, a triangular prism and a cylinder are equal.

The inquiry can be extended into three dimensions, giving rise to similar questions to those that arise with the main prompt.