

Simultaneous equations inquiry

$$\begin{array}{r} 5x + 7y = 9 \\ 3x + 7y = 11 \end{array}$$

This prompt can require considerable guidance at first. Students in years 10 and 11 quickly work out that $x = -1$, but they often overlook the key feature of the prompt - that is, the coefficients of x and y and the sum of the two amounts form a linear sequence. In the case of the prompt, we see 5, 7 and 9 and 3, 7 and 11. Obviously, the coefficient of y is the same in both equations to facilitate the use of the elimination method. If the teacher wishes to make the prompt more challenging, then alternatives include those on the right.

$$\begin{array}{r} 2x + 5y = 8 \\ 2x + 7y = 12 \end{array}$$

$$\begin{array}{r} 2x + 5y = 8 \\ 3x + 7y = 11 \end{array}$$

When the simultaneous equations are formed of linear sequences, then the solutions will always be $x = -1$ and $y = 2$. (A proof of this is given in the mathematical notes below.) Students can verify this common solution for simultaneous equations of the type in the prompt through inspection of a number of examples. They are also ready to acknowledge that solutions are easier to find if the coefficients of y are the same and can explain why it is permissible to multiply the terms in one equation by an integer. Many classes have requested a teacher's explanation for this procedure, but often it has turned out that questioning can elicit a correct understanding from the students themselves.

Once classes are comfortable with solving simultaneous equations, the inquiry tends to become fast-moving and multi-faceted as individuals or groups work on their own questions. Changes to the prompt have included:

- Descending linear sequences (or one ascending and one descending);
- Using sequences of other types, particularly quadratic and Fibonacci (why are the solutions for the latter $x = 1$ and $y = 1$?);
- Changing the signs, initially to subtraction (or addition and subtraction);
- Representing the equations on a graph (what is the relevance of the point of intersection?); and
- Using x^2 or y^2 or both (perhaps suggested by the teacher).