

$$24 \times 21 = 42 \times 12$$

Mathematical notes 2

Two digit numbers

Algorithm from a year 10 class:

- (1) Choose two digits *both* less than 5.
- (2) Swap/flip the digits
- (3) Double the number
- (4) Swap the digits

The algorithm gives the list below:

$$11 \times 22 = 22 \times 11$$

$$12 \times 42 = 24 \times 21$$

$$13 \times 62 = 31 \times 26$$

$$14 \times 82 = 41 \times 28$$

$$21 \times 24 = 42 \times 12$$

$$22 \times 44 = 44 \times 22$$

$$23 \times 64 = 32 \times 46$$

$$24 \times 84 = 42 \times 48$$

$$31 \times 26 = 62 \times 13$$

$$32 \times 46 = 64 \times 23$$

$$33 \times 66 = 66 \times 33$$

$$34 \times 86 = 43 \times 68$$

$$41 \times 28 = 82 \times 14$$

$$42 \times 48 = 84 \times 24$$

$$43 \times 68 = 86 \times 34$$

$$44 \times 88 = 88 \times 44$$

Key
Novel solutions
Repeats
Trivial

2 digits: start digit

12	21	31	41
13	23	32	42
14	24	34	43

Three digit numbers

Examples of algorithms:

Reverse, halve, reverse

$$226 \times 311 = 622 \times 113$$

Reverse, double, reverse

$$432 \times 468 = 234 \times 864$$

$$132 \times 462 = 231 \times 264$$

Reverse, **triple**, reverse

$$132 \times 693 = 231 \times 396$$

Reverse, **quadruple**, reverse

$$132 \times 924 \neq 231 \times 429$$

3 digits: start digits

		121	131	141
102	112	122	132	142
103	113	123	133	143
104	114	124	134	144
201	211	221	231	241
	212		232	242
203	213	223	233	243
204	214	224	234	244
301	311	321	331	341
302	312	322	332	342
	313	323		343
304	314	324	334	344
401	411	421	431	441
402	412	422	432	442
403	413	423	433	443
	414	424	434	

Algebra: two digits

$$(10a + b)(10c + d) = (10b + a)(10d + c)$$

x	10a	b	x	10b	a
10c	100ac	10bc	10d	100bd	10ad
d	10ad	bd	c	10bc	ac

$$100ac + 10ad + 10bc + bd = 100bd + 10bc + 10ad + ac$$

$$100ac + bd = 100bd + ac$$

$$99ac = 99bd$$

$$ac = bd$$

$a = \frac{bd}{c}$	$c = \frac{bd}{a}$
$b = \frac{ac}{d}$	$d = \frac{ac}{b}$

Algebra: three digits

$$(100a + 10b + c)(100d + 10e + f) = (100c + 10b + a)(100f + 10e + d)$$

x	100d	10e	f
100a	10000ad	1000ae	100af
10b	1000bd	100be	10bf
c	100cd	10ce	cf

x	100f	10e	d
100c	10000cf	1000ce	100cd
10b	1000bf	100be	10bd
a	100af	10ae	ad

$$10000ad + 1000ae + 100af + 1000bd + 100be + 10bf + 100cd + 10ce + cf = 10000cf + 1000ce + 100cd + 1000bf + 100be + 10bd + 100af + 10ae + ad$$

$$9999ad + 990ae + 990bd = 9999cf + 990ce + 990bf$$

$$101ad + 10ae + 10bd = 101cf + 10ce + 10bf$$

$$101(ad - cf) = 10(ce + bf - ae - bd)$$