24 x 21 = 42 x 12

Mathematical notes 1

A complete set of two 2-digit numbers with equal products when digits reversed (ratio of pairs of numbers on the left- and right-hand sides of the equation in bold):

1:2, 2:1 (or x2, x½)
13 x 62 = 31 x 26
14 x 82 = 41 x 28
23 x 64 = 32 x 46
24 x 84 = 42 x 48
34 x 86 = 43 x 68

1:3, 3:1 (or x3, x⅓)
12 x 63 = 21 x 36
13 x 93 = 31 x 39
23 x 96 = 32 x 69

1:4, 4:1 (or x4, x¼)
12 x 84 = 21 x 48

2:3, 3:2 (or x$^3/_2$, x⅔)
46 x 96 = 64 x 69
26 x 93 = 62 x 39
24 x 63 = 42 x 36

3:4, 4:3 (or x$^4/_3$, x¾)
36 x 84 = 63 x 48

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In looking at the general case, a, b, c and d are defined as whole numbers, where $a \neq b$ and $c \neq d$.

(1) To find the relationship between *a, b, c* and *d* for two pairs of 2-digit numbers with equal products when their digits are reversed:

First 2-digit number: $10a + b$
Second 2-digit number: $10c + d$

So,
$(10a + b)(10c + d) = (10b + a)(10d + c)$

$$100ac + 10ad + 10bc + bd =$$
$$100bd + 10bc + 10ad + ac$$

$$100ac + bd = 100bd + ac$$

$$99ac = 99bd$$

$$\boldsymbol{ac = bd}$$

(In the classroom this has led to the formula $a = {}^{bd}/_{c}$ or similar.)

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(2) To find the relationship between *a, b, c* and *d* for two pairs of 2-digit numbers with equal products when their digits are reversed and one number is doubled and the other halved:

As above, with following:
A. $2(10a + b) = 10d + c$
B. $\frac{1}{2}(10c + d) = 10b + a$

(From A) I. $20a + 2b = 10d + c$
(From B) II. $10c + d = 20b + 2a$

Therefore,
(From II.) $d = 20b + 2a - 10c$
Substitute expression for d into I.

$$20a + 2b = 10(20b + 2a - 10c) + c$$
$$20a + 2b = 200b + 20a - 100c + c$$
$$2b = 200b - 100c + c$$

$$99c = 198b$$

$$\mathbf{c = 2b}$$

Substitute expression for c into II.

$$10(2b) + d = 20b + 2a$$

$$20b + d = 20b + 2a$$

$$\mathbf{d = 2a}$$

$$\mathbf{c = 2b,\ d = 2a}$$

Similarly, for equations in which the ratio differs:

1:3, 3:1 $c = 3b,\ d = 3a$
1:4, 4:1 $c = 4b,\ d = 4a$
2:3, 3:2 $2c = 3b,\ 2d = 3a$
3:4, 4:3 $3c = 4b,\ 3d = 4a$

List of solutions using $ac = bd$

a	c	b	d
1	4	2	2
1	6	2	3
1	6	3	2
1	8	2	4
1	8	4	2
1	9	3	3
2	6	3	4
2	6	4	3
2	8	4	4
2	9	3	6
2	9	6	3
3	8	4	6
3	8	6	4
4	9	6	6