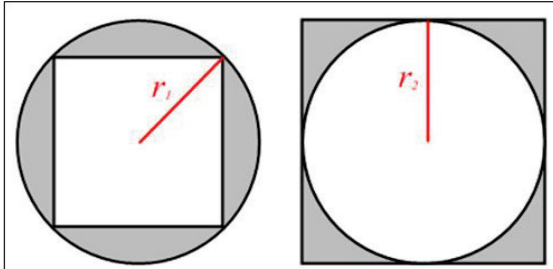
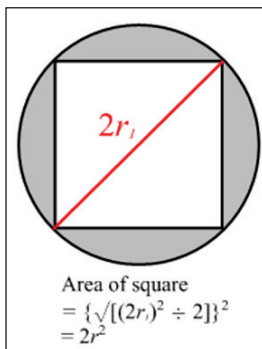


A square fits better into a circle than a circle fits into a square.

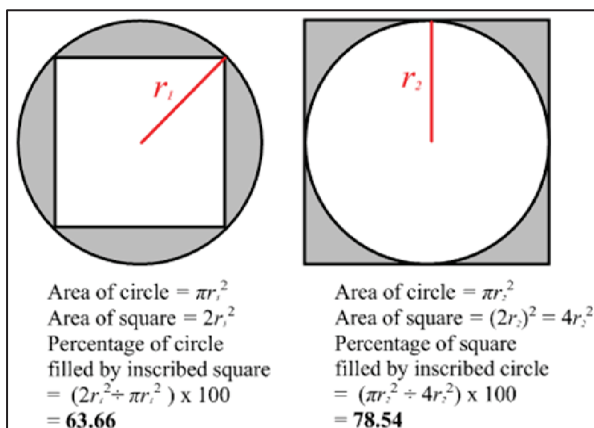
Start with two diagrams:



Use Pythagoras' Theorem to find the area of the inscribed square.



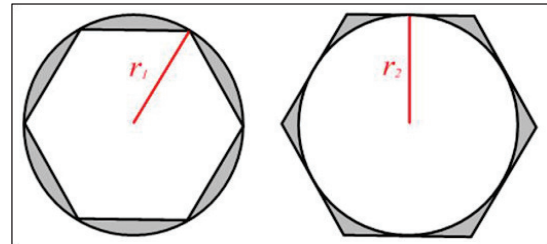
To find the percentage of the larger shape covered by the inscribed shape:



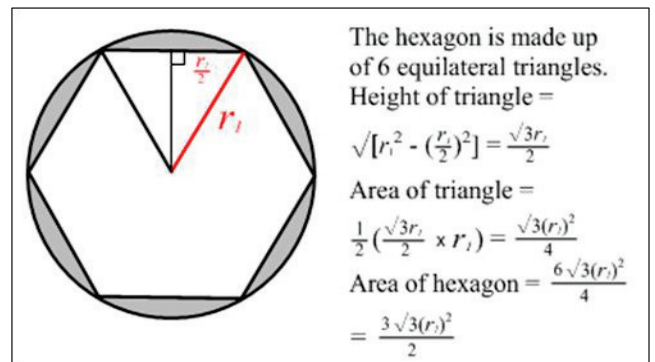
So the statement is proved false in all cases.

Is it always the case that the circle fits better into an n-sided polygon than the inscribed polygon fits in the circle?

The case of a hexagon and a circle:

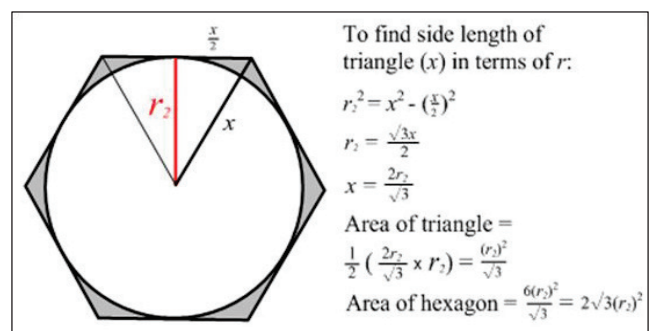


Hexagon inscribed in circle



Area of circle $= \pi r_1^2$
 Area of hexagon $= \frac{3\sqrt{3}(r_1)^2}{2}$
 Fraction of circle filled by inscribed hexagon
 $= \frac{3\sqrt{3}(r_1)^2}{2} \div \pi r_1^2$
 $= \frac{3\sqrt{3}}{2\pi}$

Circle inscribed in hexagon



Area of circle $= \pi r_2^2$
 Area of hexagon $= 2\sqrt{3}(r_2)^2$
 Fraction of hexagon filled by inscribed circle
 $= (2\sqrt{3}(r_2)^2 \div \pi r_2^2)$
 $= \frac{2\sqrt{3}}{\pi}$

As $\frac{2\sqrt{3}}{\pi} > \frac{3\sqrt{3}}{2\pi}$, the circle fits better in the hexagon than the hexagon in the circle.