Lesson Notes

$$\square x + \square = \square x + \square$$

Untypically, the inquiry starts with a teacher-directed episode in order to establish the prompt as a template. The teacher invites students to place integers in the empty squares, thereby creating an equation with the unknown on both sides. This can be followed by a question related to finding the unknown number represented by *x*. The openness of the question depends on the prior learning of the class. At this point, students suggest ways to solve the equation with the teacher guiding the discussion towards a legitimate method. If no knowledge is forthcoming, I have used a *Thinking Maths* worksheet (CAME 21) that uses an interesting method based on partitioning.

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$$5x + 3 = 2x - 7$$
$$5x + 10 - \cancel{7} = 2x - \cancel{7} \quad (3 = 10 - 7)$$
$$5x + 10 = 2x$$
$$3x + \cancel{2x} + 10 = \cancel{2x} \quad (5 = 3 + 2)$$
$$3x + 10 = 0$$
$$3x + \cancel{10} = -10 + \cancel{10}$$
$$3x = -10$$
$$x = -10 \div 3\text{, etc.}$$

The method used in CAME 21

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For the first part of the lesson the teacher might explicitly regulate the activity of the class by selecting appropriate regulatory cards.

The teacher can also ask if every equation created by randomly selecting integers has a solution. Students quickly come up with a conjecture that they do, although their solutions may need correction. In particular, if $a > c$ and $b > d$ in the equation $ax + b = cx + d$, students will often write $(a-c)x = (b-d)$, whereas $(a-c)x + (b-d) = 0$.

Once the class has gained confidence in solving equations, there are two ways to proceed. The teacher might judge that the time is right to introduce a general solution for the case in the prompt, where a > c and d > b (or for another defined relationship). The solution below is based on the 'partition method':

$$ax + b = cx + d$$
$$ax + \cancel{b} = cx + \cancel{b} + e \quad (d = b + e)$$
$$ax = cx + e$$
$$\cancel{cx} + fx + = \cancel{cx} + e \quad (a = c + f)$$
$$fx = e$$
$$fx \div f = e \div f$$
$$x = e \div f$$

Alternatively, the class could proceed directly to suggesting changes to the prompt. This has given rise to these comments in key stage 3 classes:

- Change the operation(s) to subtraction.
- Use fractions instead of integers.
- Re-design the equation with the unknown on one side only.

  $$\square x + \square = \square$$

  (In all-ability classes, this might constitute a 'support' inquiry from the start of the lesson.)
- Include a second variable on both sides of the equation.
- Re-design the equation to include an algebraic fraction.

  $$\frac{\square x + \square}{\square} = \frac{\square x + \square}{\square}$$
- Add a third expression equal to the other two.

Students might be invited to 'decide what the problem is', with the teacher again using the cards to model regulation of the inquiry. The inquiry can then go off in different directions, with students grouped in the classroom on the basis of the level of challenge they set themselves.