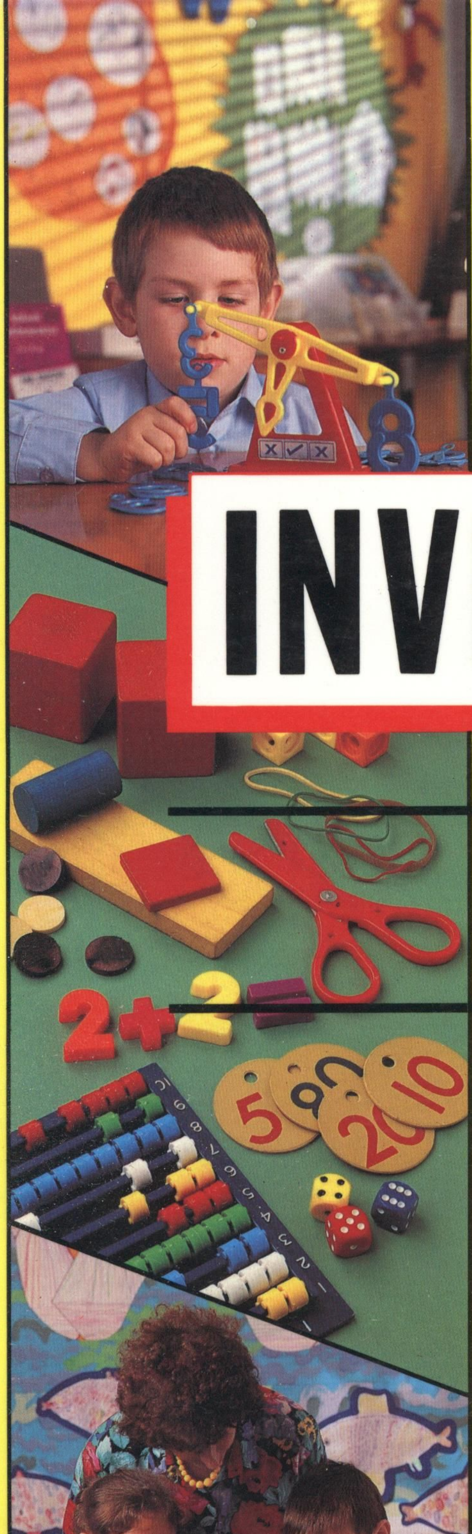
1995 CURRICULUM



starting INVESTIGATIONS

For National Curriculum
levels 1-3

SPECTRUM MATHS

Dave Kirkby

starting

INVESTIGATIONS

For National Curriculum levels 1-3

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Using the Teacher's Notes

CONTENT

This heading states the focus of the investigation in terms of a particular mathematical topic, e.g. Triangles, Addition, Number patterns.
A more detailed description of the potential content is outlined here on each of the teacher's pages.

Apparatus

This section indicates to the teacher what apparatus is likely to be required for the investigation, e.g. Cubes or Number cards. Where appropriate, the teacher is alerted to the availability, at the back of the book, of a 'special paper' which the pupils can use to record their work.

LEVEL	UA	N	SSM	HD
1				
2				
3				
4				
5				
6				

KEY UA Using and Applying Mathematics
N Number
SSM Shape, Space and Measures
HD Handling Data

The teacher's notes for each investigation contain the above table. This table refers to the Programmes of Study and levels 1–6 of the National Curriculum. An attempt has been made to locate, by means of dots in the table, the approximate content level for each investigation, but it must be appreciated that many activities can be performed at a variety of different levels.

This symbol appears on every page and highlights the advice to teachers on recording.

This section contains the teaching notes but not necessarily the answers. The notes are intended as a guide to the possible directions the investigation may take. They contain background mathematics for the teacher, but should not be seen as an indication of what can be expected from all pupils. The pupils should feel free to follow their own lines of enquiry, which may very well not coincide with these notes.

This section may also include suggestions about recording, points for discussion, warnings, etcetera.

QUESTIONS

These suggested follow-up questions may lead to further investigations. They may also provide teachers with some ideas for potential areas for development. Many of the investigations are rich in opportunities for introducing a variety of mathematical ideas. Questions can help to link different ideas and concepts. Although the questions are written simply, teachers may need to adapt the phrasing and language to suit their pupils.

EXTENSIONS

- It is hoped that pupils will develop sufficient interest and confidence to extend their work in their own way.
- This section contains suggestions that teachers may wish to use with particular pupils, whilst encouraging them to develop their own ideas.

Using the Pupils' Sheets

You will need

The basic information about apparatus also appears on the pupils' sheets, so that the children have some idea of what materials they need.

The pupils' sheets are written using as few words as possible. However, pupils may still need some help in getting started.

Find or **Investigate** precedes an indication of where to start. Sometimes the indication is deliberately vague.

Encourage the pupils to become responsible for their own lines of enquiry, and to extend them in some way.

INTRODUCTION

Most schools use a mathematics scheme. Teachers using these require a range of support material to supplement the scheme. Such material is provided by **Spectrum Maths.**

INVESTIGATIONS
SPECTRUM
MATHS

This is a series of three books of investigations primarily for the primary years, although secondary school teachers with low attaining pupils will also find these books useful.

They are defined in terms of three ability levels. Broadly defined, these levels are:

Starting Investigations (Infants)
More Investigations (Lower Juniors)
Go Further With Investigations (Upper Juniors)

Each book contains:

40 pupil investigations in the form of photocopymasters.
Detailed teacher notes accompanying each investigation.
Special papers in the form of photocopymasters to aid pupils to record their work.

HOW CAN IT BE USED?

Spectrum Maths investigations can be used in a variety of ways:

(a) to consolidate other work in the school mathematics scheme
(b) as a completely separate supplement to the scheme
(c) as a means of introducing a new topic within the scheme
(d) to provide enrichment material at appropriate times.

The 40 pupil investigations in each book are non-sequential.
Investigations can be selected by the teacher to suit individual needs.
The teacher's notes contain clear indications of both the **content** area and the required **apparatus** for each investigation. This will aid the teacher who wishes to be selective. Some teachers may wish to select a group of investigations based on a particular mathematical theme e.g. **multiplication**. Others may choose investigations requiring the use of a particular piece of apparatus e.g. **cubes**.

The material is flexible in terms of organisation.
Some examples include:

Individual investigations: pupils working individually on their own particular investigation.

Small group investigations: the class divided into groups, each group working on a different investigation.

Class investigation: the whole class working on the same investigation. This may be the easiest form of organisation for teachers who are starting on this type of work.

School investigation: several classes working on the same investigation. This enables teachers to discuss and compare experiences amongst each other. It can also lead to combined work displays.

WHAT IS AN INVESTIGATION?

An **investigation** presents pupils with an open mathematical situation and invites them to explore it.

In most mathematical activities, a goal is specified and an answer is sought. There is no 'answer' to an investigation. It is the 'journey', not the 'destination' which is the goal.

The **Spectrum investigations** pupil material provides guidelines and suggestions of ways in which the pupil explorations may lead, and ideas for helping pupils continue their 'journey'.

As pupils become practised in making 'journeys' they will need to experience some of the following:

- understanding the starting point
- trying some examples
- recording results (diagrams, tables, drawings, lists etc.)
- devising methods of recording
- spotting patterns
- describing patterns
- checking results
- generalising results
- systematically organising the 'journey'
- devising strategies
- writing an account of the 'journey'
- extending the 'journey'

The 'journey' is often referred to as 'mathematical process' and lists like those above as 'process objectives'.

CALCULATORS

Spectrum Mathematics: Investigations does not contain many activities which focus on the use of a calculator. Nevertheless, pupils will often find a calculator a valuable aid, particularly when extending an investigation. The provision of calculators is left to the discretion of the teacher.

The activities which a calculator may be required are:

Starting Investigations 2, 7, 8, 10, 15, 17, 19, 25, 33, 35, 38.

More Investigations 2, 7, 11, 12, 17, 21, 25, 29, 32, 33, 37.

Go Further With Investigations 1, 4, 5, 7, 9, 12, 13, 16, 17, 19, 21, 25, 29, 33, 36, 40.

1 Odds and evens

ODDS AND EVENS

Sorting dominoes into sets: both sides even numbers; both sides odd numbers; and one side an even number, the other an odd number.

Apparatus

Use a set of dominoes and start by removing those with a blank.
Domino paper (special paper 3).

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●	●		
4				
5				
6				

- Odds and evens.
- Number patterns.
- Recording observations.

On domino paper. The dominoes can be recorded using either dots or numbers.

The dominoes in each set are:

Both numbers odd

[1 | 1] [1 | 3] [1 | 5]

[3 | 3] [3 | 5]

[5 | 5]

Both numbers even

[2 | 2] [2 | 4] [2 | 6]

[4 | 4] [4 | 6]

[6 | 6]

One odd and one even number

[1 | 2] [1 | 4] [1 | 6] [2 | 3] [2 | 5]

[3 | 4] [3 | 6] [4 | 5] [5 | 6]

QUESTIONS

? Which dominoes have 10 dots altogether, with both sides odd?

? Which dominoes have 8 dots altogether, with both sides even?

? Which dominoes have one side odd, one side even, and a difference of 3?

EXTENSIONS

→ Try sorting the dominoes according to the dot total.

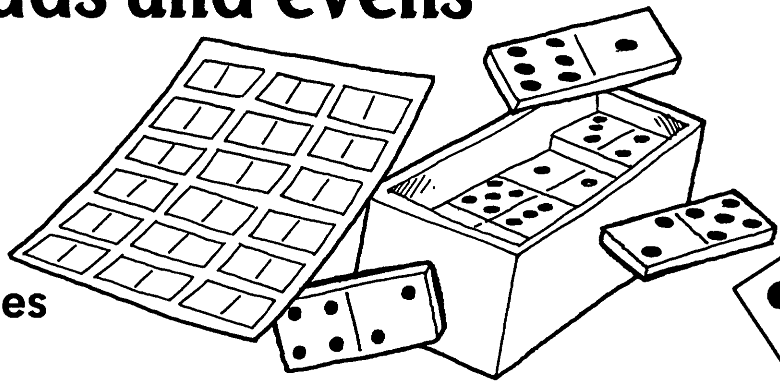
→ Try sorting them according to the dot difference.

→ Try sorting them into pairs, doubles, trebles, and so on.

Odds and evens

You will need

domino paper
a set of **dominoes**



Take out all the dominoes with blanks.
Sort the others into 3 sets like these:

Both numbers **odd**.

Both numbers **even**.

One **odd** and
one **even** number.

Find how many dominoes there are
in each set.

2 Pairs

ADDITION

Addition bonds. Searching for different pairs of numbers with the same total.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●	●		
4				
5				
6				

- Addition facts up to 20.
- Addition patterns.
- Recording observations.

Apparatus

Use cards numbered 1 to 20

On paper or in a book. Two possibilities are:

Total 12	or	
1, 11		1 + 11 = 12
2, 10		2 + 10 = 12
. .		. .
. .		. .
. .		. .
. .		

There are five different pairs that total 12.
These can be arranged in order.

1	2	3	4	5
11	10	9	8	7

The number of possible pairs for other totals are:

Total	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number of pairs	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9

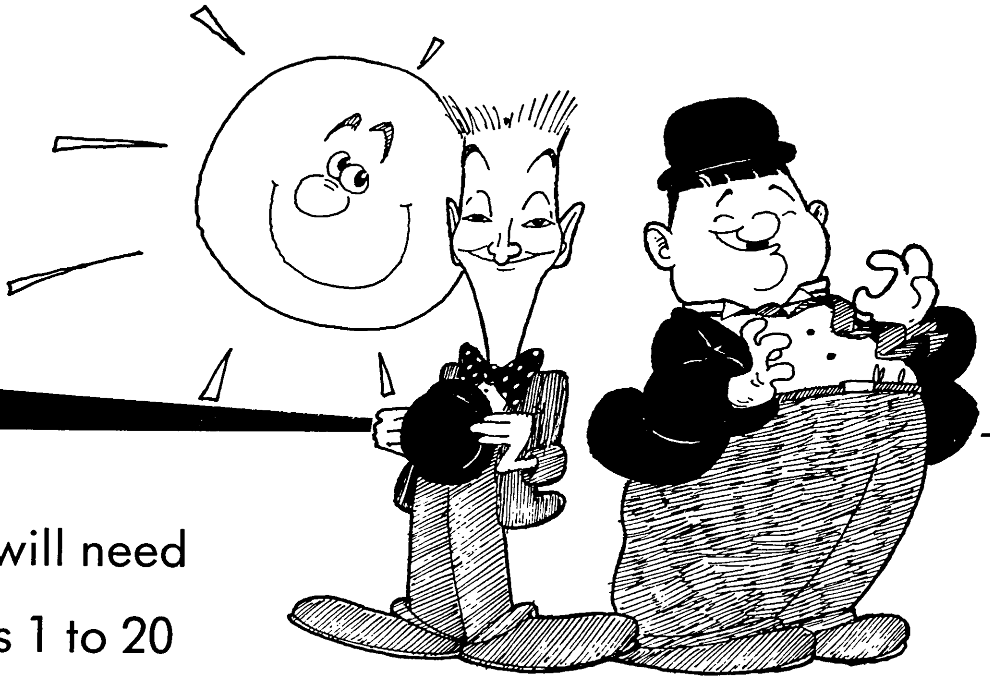
QUESTIONS

- (?) What number is paired with 7?
- (?) How many pairs have both numbers odd/even?
- (?) Which pair has the smallest difference?

EXTENSIONS

- Try different totals up to 20.
- Try to guess how many pairs you will find if the total is 16.
- Try with totals greater than 20.
- Try with a different set of cards, e.g. 1 to 15.

Pairs



You will need

cards 1 to 20

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Make **pairs** that total **12**.

Here are two pairs:

Find some more.

starting
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3 Dot tot

ADDITION

Addition bonds based on the sum of the dots on dominoes.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

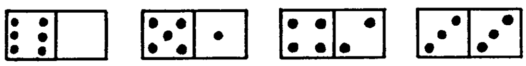
- ● Addition bonds.
- ● Patterns in addition.
- ● Recording results.

Apparatus

A full set of 28 dominoes is required.
These can be made from card if necessary.
Domino paper (special paper 3).

On domino paper. The dominoes can be recorded using either dots or numbers.

There are four dominoes with a dot total of 6.



The other totals are:

0	[|]			
1	[| 1]			
2	[| 2]	[1 | 1]		
3	[| 3]	[1 | 2]		
4	[| 4]	[1 | 3]	[2 | 2]	
5	[| 5]	[1 | 4]	[2 | 3]	
6	[| 6]	[1 | 5]	[2 | 4]	[3 | 3]
7	[1 | 6]	[2 | 5]	[3 | 4]	
8	[2 | 6]	[3 | 5]	[4 | 4]	
9	[3 | 6]	[4 | 5]		
10	[4 | 6]	[5 | 5]		
11	[5 | 6]			
12	[6 | 6]			

QUESTIONS

? Can you find three different dominoes with 5 dots each?

? What is the smallest/largest possible dot total?

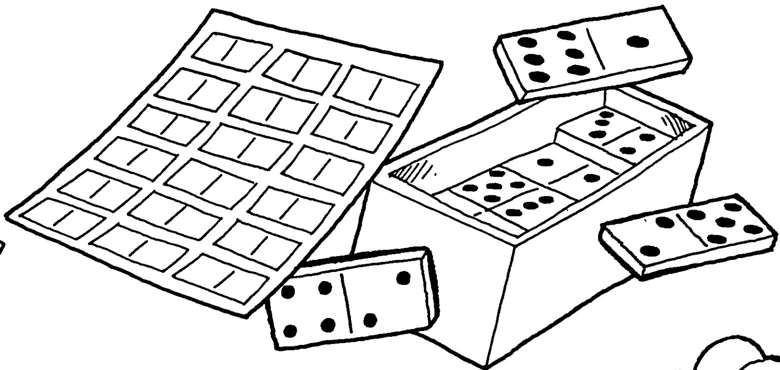
EXTENSIONS

→ Try different totals.

→ Try to find out what dot total you will find most often.

→ Try domino differences.

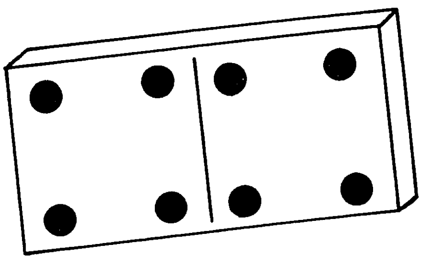
Dot tot



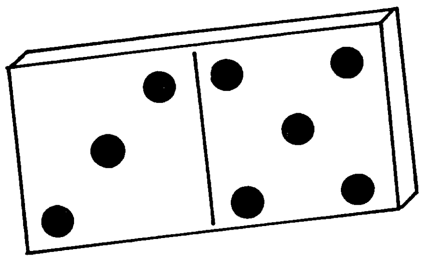
You will need

dominoes
domino paper

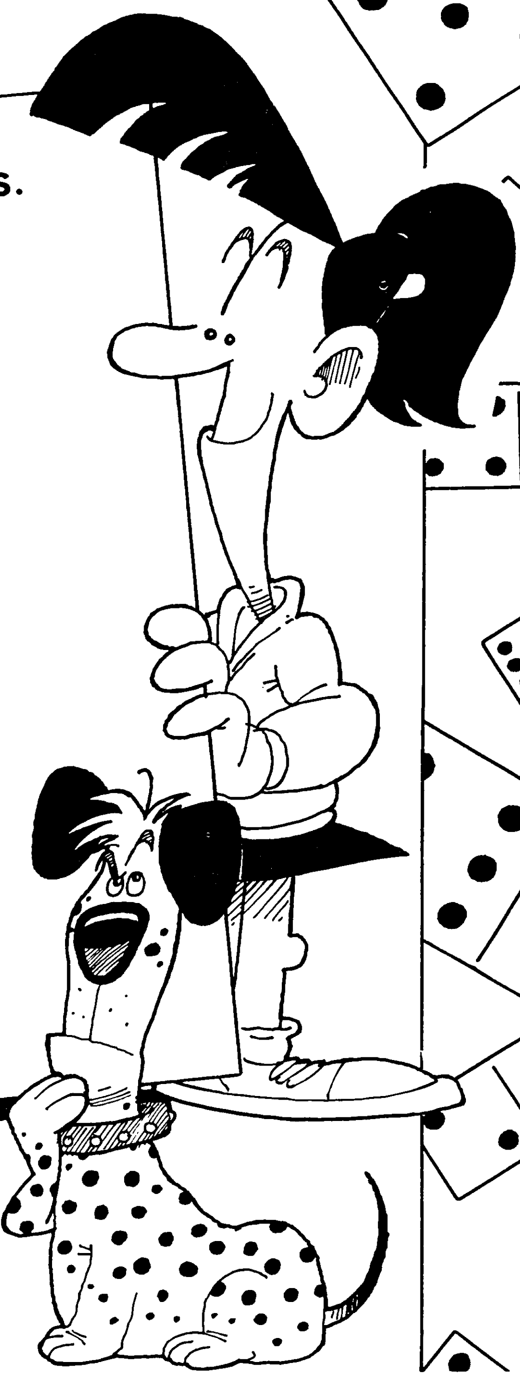
These **dominoes** each have **8 dots**.



8 dots



8 dots



Find dominoes with **6 dots.**

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Rockets!

COLOUR PATTERNS

SHAPE PATTERNS

Different arrangements of three rectangles within a larger rectangle. Also, different arrangements according to colour.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●	●	
3	●			
4				
5				
6				

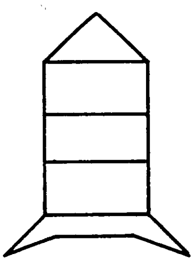
- Patterns using 2D shapes.
- Rectangles.
- Recording results.

Apparatus

You could use Logibloc rectangles or make 2 x 1 rectangles from coloured card.
Rocket paper (special paper 4).

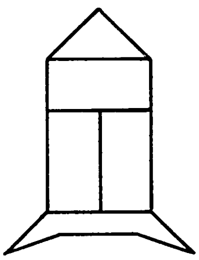
On rocket paper. The rockets can be coloured.

There are six different rockets with this arrangement:



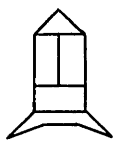
R	R	Y	Y	B	B
Y	B	R	B	R	Y
B	Y	B	R	Y	R

There are six different rockets with this arrangement



R	R	Y	Y	B	B
Y B	B Y	R B	B R	R Y	Y R

and, as variations of the above, there are six like this.



QUESTIONS

? How many different rockets can you make with the red here R or here R ?

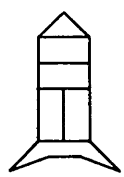
? How many different rockets have red next to blue, like this R B ?

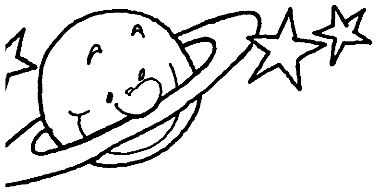
EXTENSIONS

→ Try with 2 red rectangles and 1 yellow rectangle.

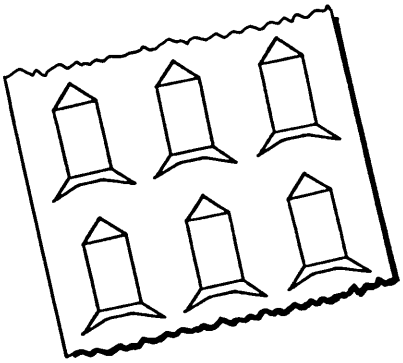
→ Try choosing from a collection of 3 red, 3 yellow and 3 blue rectangles. (You could choose 3 rectangles all the same colour.)

→ Try with a different rocket and rectangles of 2 colours.



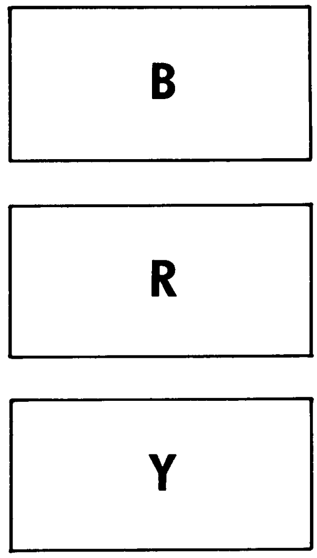


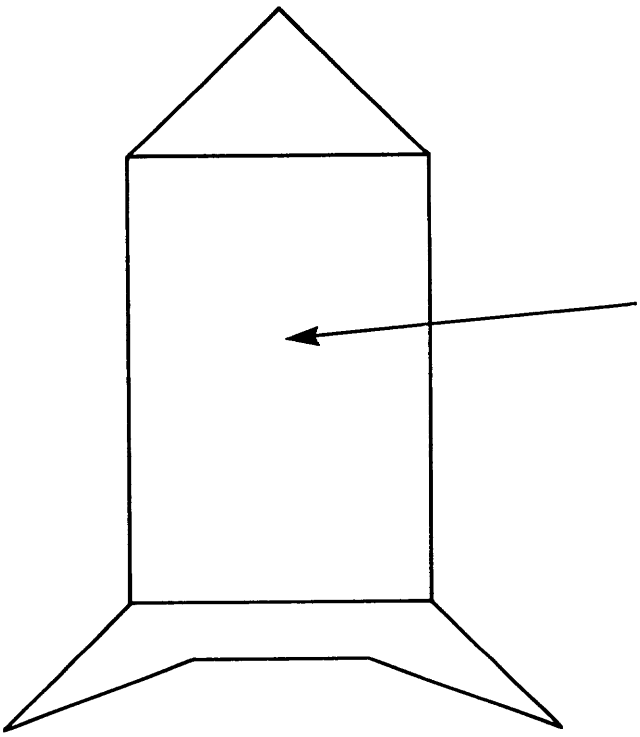
Rockets!



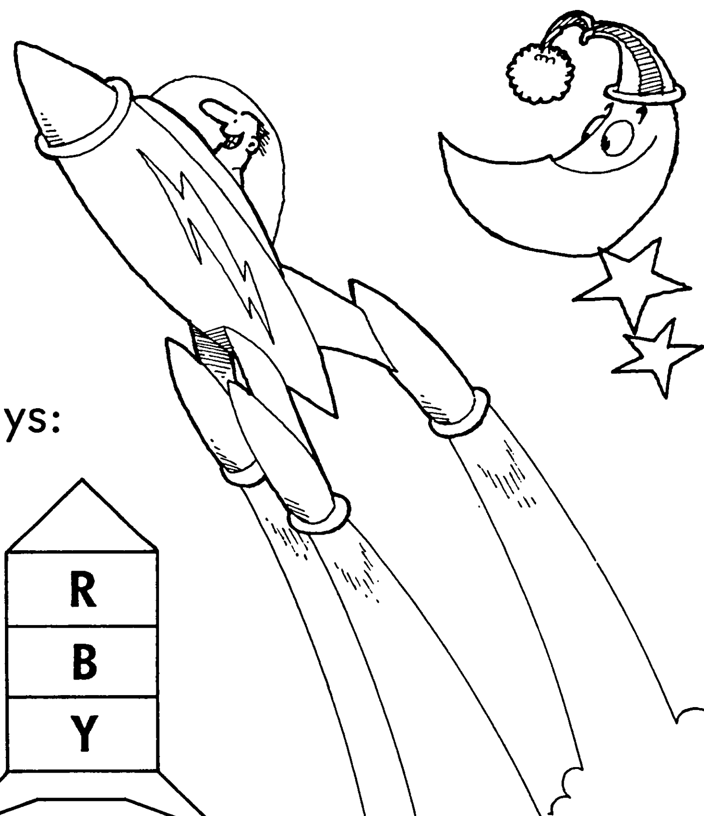
You will need

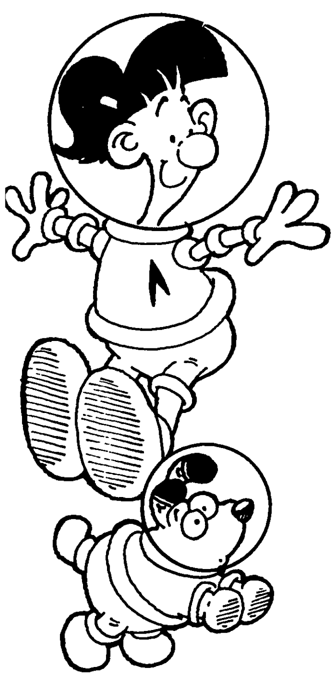
rocket paper
3 **rectangles**,
one **blue**,
one **red**,
and one **yellow**



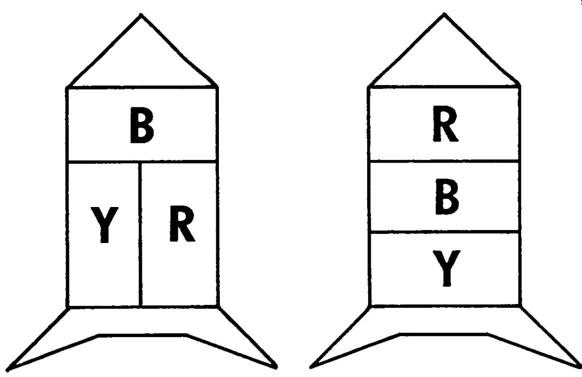


Fit them in here to make a **coloured rocket**.

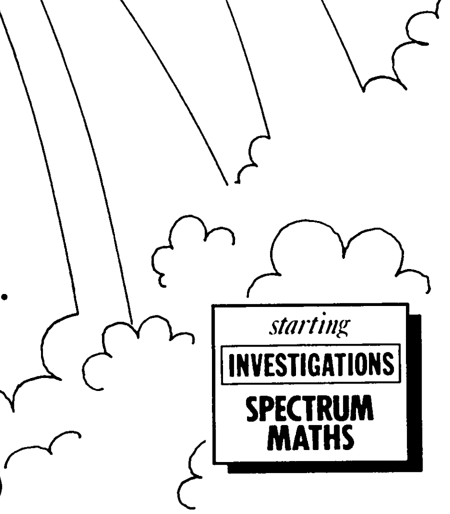


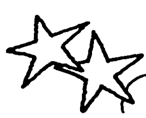


Here are two ways:



Find different coloured **rockets**.





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5 Snip a shape

SHAPE

Making different shapes by cutting out shapes from the folded edges of pieces of paper.

Apparatus

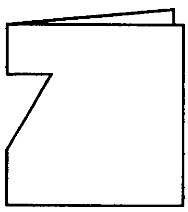
Rectangular pieces of paper, not too small.
Scissors are also needed.

LEVEL	UA	N	SSM	HD
1	●			
2	●		●	
3	●			
4				
5				
6				

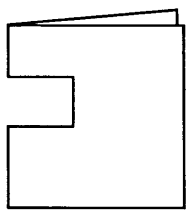
● Creating pictures and patterns in shape.

On paper or in a book. The cut-outs can be mounted on coloured paper and labelled.

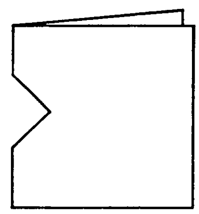
Pupils can experiment with different cut-outs.
Some recognisable shapes arise from the following cuts.



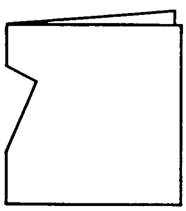
Triangle



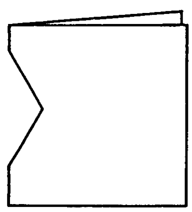
Rectangle



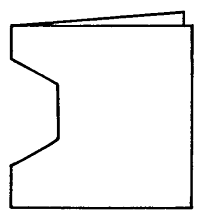
Square



Kite



Diamond



Hexagon

QUESTIONS

? Can you make a square?

? Can you make a circle?

? Can you make an eight-sided shape?

EXTENSIONS

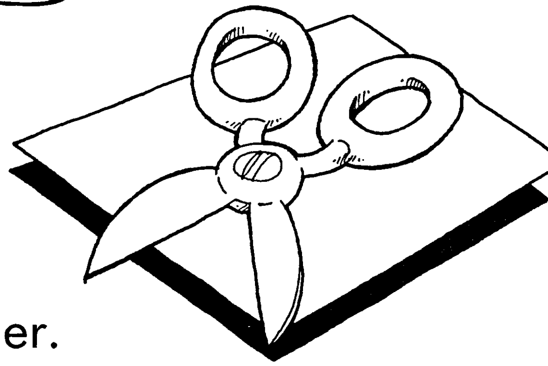
→ Try cuts using two straight lines only.

→ Try cuts using curved lines.

→ Try folding the paper twice and cutting through four sheets.

Snip a shape

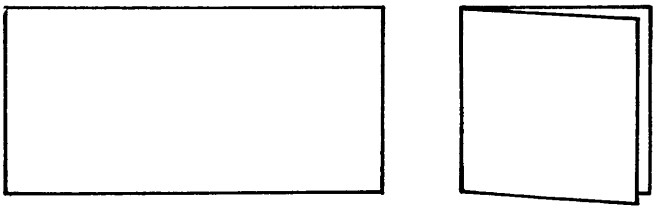
5



You will need

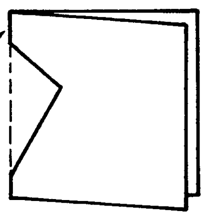
scrap paper
scissors

Fold some paper.

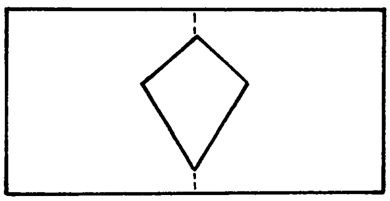


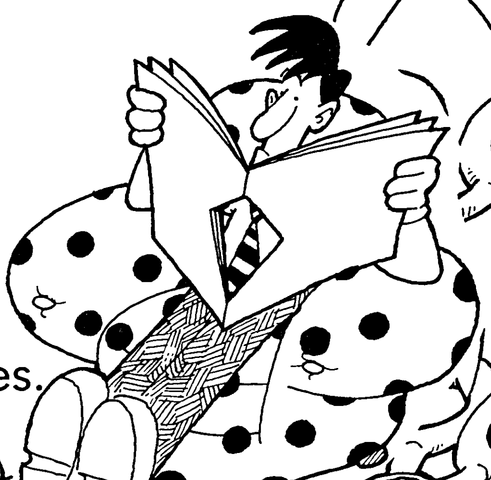
Cut a piece out of the folded edge.

fold



Open out the paper.
See what shape you have made.





Fold some more paper.
Make some other shapes.

6 Letters

AREA

Designs of letters of the alphabet on squared paper.
Measurement, in squares, of the areas of the letters.

Apparatus

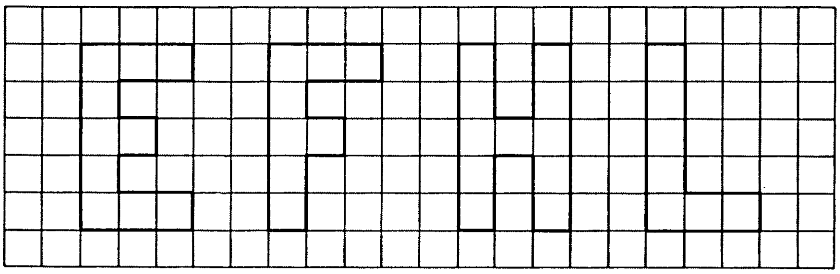
The letters are drawn on squared paper.

LEVEL	UA	N	SSM	HD
1	●			
2	●		●	
3	●			
4			●	
5				
6				

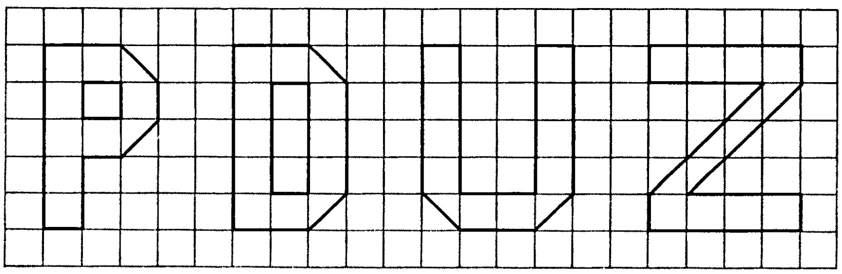
● Area by counting squares.
● Creating pictures using 2D shapes.

On squared paper (2 cm).

Pupils can start by drawing letters using whole squares only, e.g.:



They might then progress to using half-squares as well, e.g.:



QUESTIONS

Can you draw an E that covers 10 squares (has an area of 10 square units)?

Can you draw different letters that cover the same number of squares (have the same area)?

Which letters have symmetry? (Discuss examples of letters with horizontal and vertical axes of symmetry, M ~~E.~~ Pupils can fold their letters.)

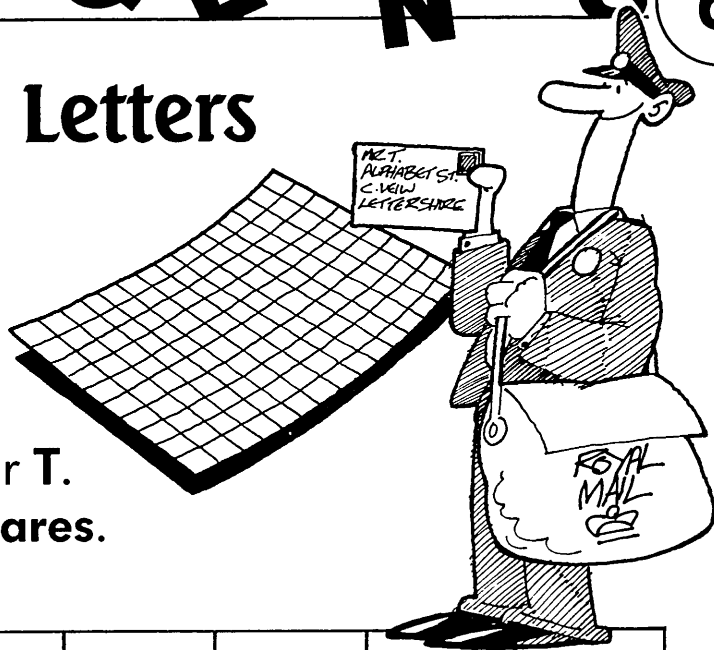
EXTENSIONS

Try drawing Ts that cover different numbers of squares (with different areas).

Try drawing numbers instead of letters.

Try drawing pictures.

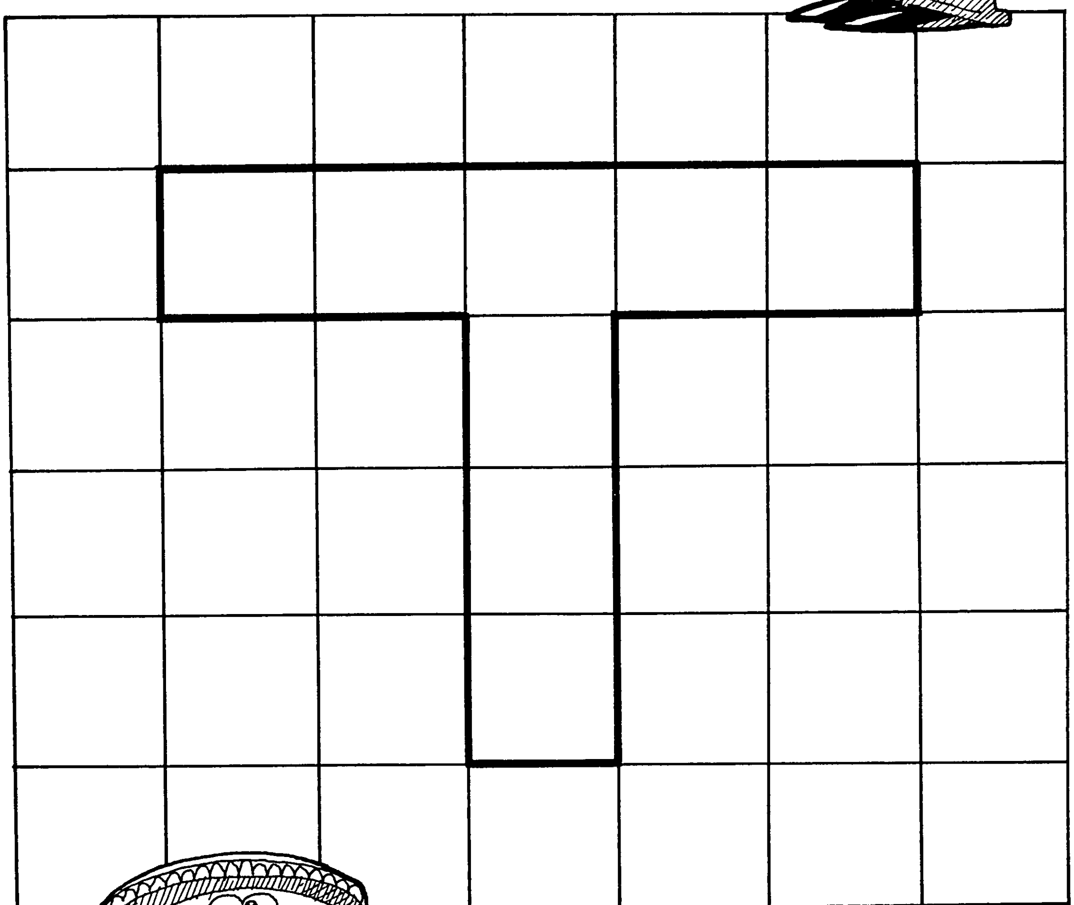
Letters

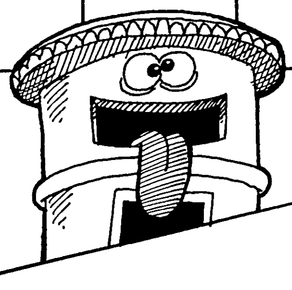


You will need

squared paper

Draw this letter **T**.
It covers **8 squares**.





Design some more letters.
Find how many **squares** they cover.

starting
INVESTIGATIONS
SPECTRUM MATHS

7 Coin count

MONEY

ADDITION

Different ways of making given sums of money using 1p, 2p, 5p and 10p coins.

Apparatus

Use 1p, 2p, 5p and 10p coins.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●	●		
4				
5				
6				

- Addition of money.
- Patterns in addition.
- Recording results.

On paper or in a book. Draw round coins, use sticky circles, or use rubber stamps.

There are 15 different ways of making 12p.

10p 2p

10p 1p 1p

5p 5p 2p

5p 5p 1p 1p

5p 2p 2p 2p 1p

5p 2p 2p 1p 1p 1p

5p 2p 1p 1p 1p 1p 1p

5p and 1p x7

2p x6

2p x5 and 1p x2

2p x4 and 1p x4

2p x3 and 1p x6

2p x2 and 1p x8

2p and 1p x10

1p x12

QUESTIONS

? How many ways can you find using 3 coins?

? Can you find three ways without using 1p coins?

? Can you find three ways without using 5p coins?

EXTENSIONS

→ Try making other totals.

→ Try finding six ways of making each total.

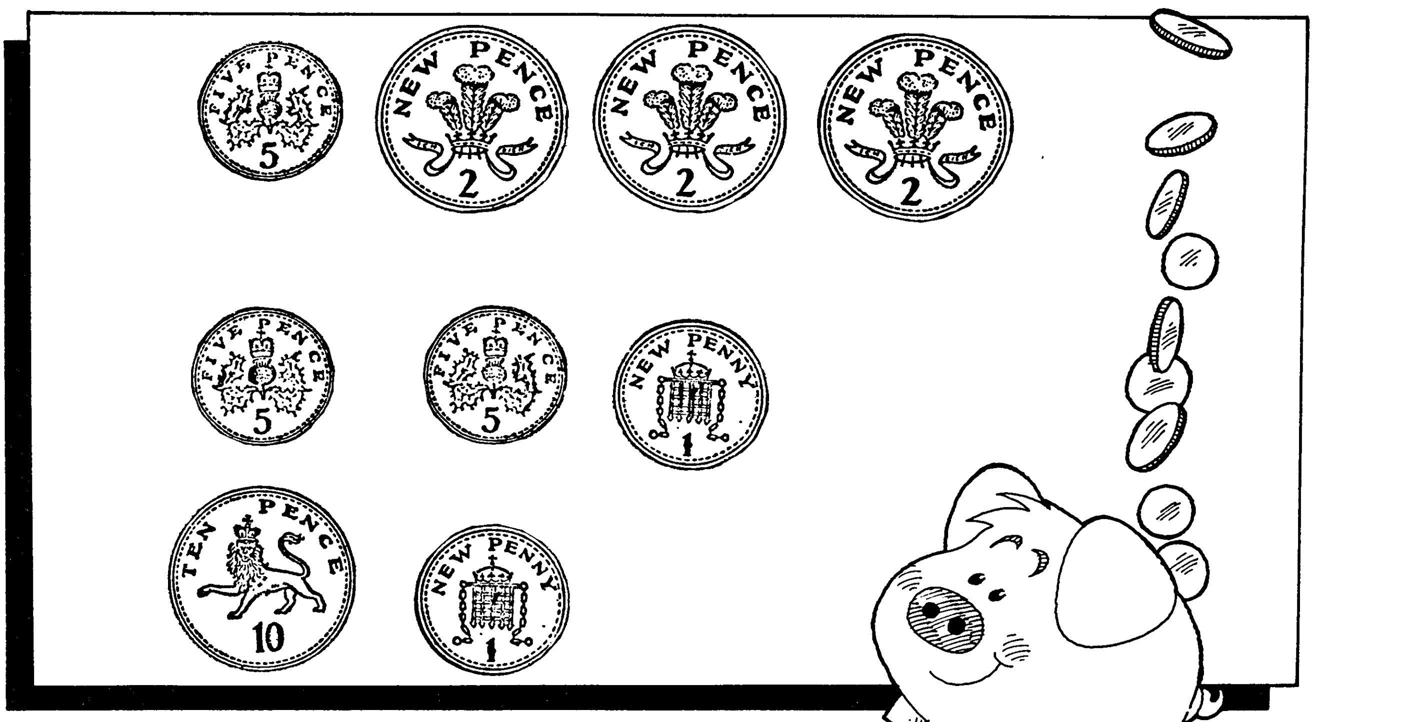
Coin count

You will need

these coins

1p 2p 5p 10p

Here are **three ways** of making **11p**.



Find three ways of making **12p**.
Find three ways of making **16p**.

starting
INVESTIGATIONS
SPECTRUM MATHS

8 Staircases

NUMBER PATTERNS

SHAPE PATTERNS

Using cubes to build staircase models. Counting the number of steps and the number of cubes.

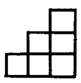
LEVEL	UA	N	SSM	HD
1	●			
2	●		●	
3	●	●		
4				
5				
6				

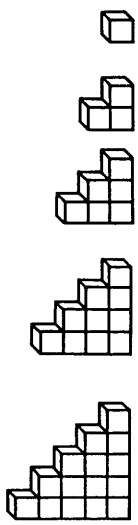
● Number patterns.
● Patterns in shape.

Apparatus

Interlocking cubes are ideal (e.g. Multilink).

On paper or in a book. Draw a table as suggested here. On squared paper. The staircases can be drawn side-on:





steps	cubes
1	1
	+2
2	3
	+3
3	6
	+4
4	10
	+5
5	15
.	.
.	.
.	.
10	55

QUESTIONS

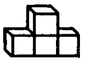
? How many cubes for a 4-step staircase?

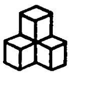
? If you use six cubes, how many steps are there?

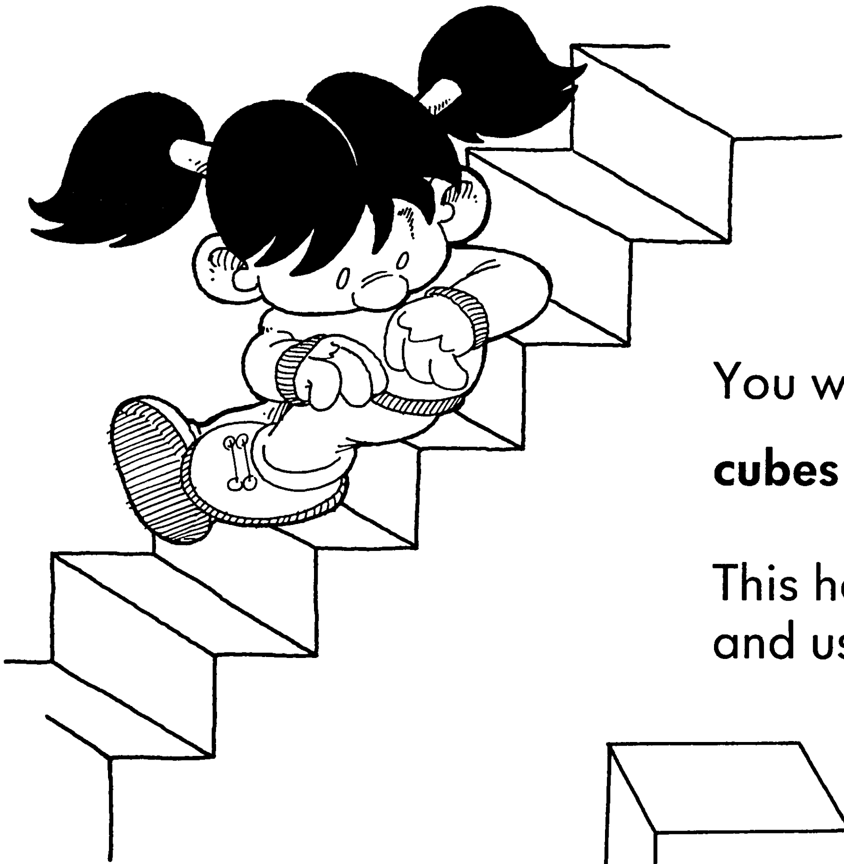
? How many cubes for a 10-step staircase?

EXTENSIONS

→ Try double staircases. 

→ Try staircases like this. 

→ Try staircases like this. 

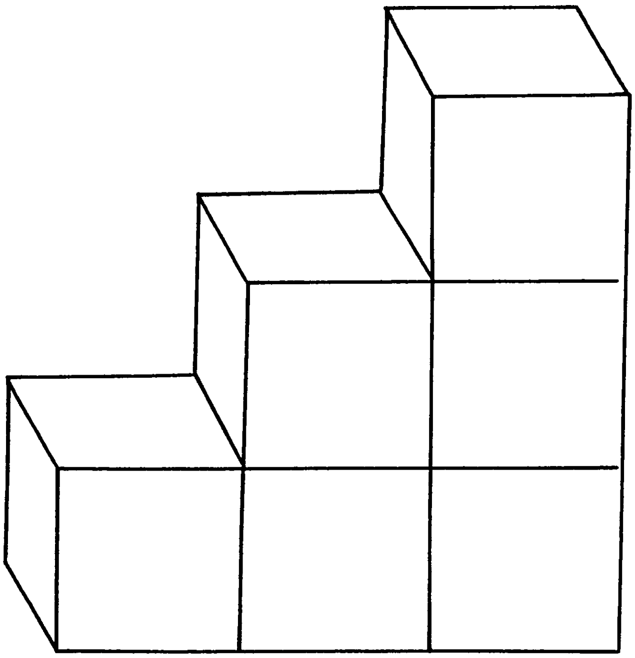


8 Staircases

You will need

cubes to build **staircases**

This has **3 steps**
and uses **6 cubes**.



Build some more **staircases**.

starting
INVESTIGATIONS
SPECTRUM MATHS

9 Sticky stamps

ADDITION

Finding different arrangements of 2p and 3p stamps, given an overall total cost of postage.

Apparatus

Paper or card rectangles can be used for the 'letters'.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●	●		
4				
5				
6				

- Addition of money.
- Number patterns.

On paper. The stamps can be drawn on paper or card rectangles to represent letters. The stamps can then be coloured. Pupils could also use sticky-paper squares, different colours for 2p and 3p, and write 2p and 3p on them.

The possibilities are:

9p	3p	3p	3p	
	3p	2p	2p	2p
	2p	3p	2p	2p
	2p	2p	3p	2p
	2p	2p	2p	3p

10p	2p	2p	2p	2p	2p
	2p	2p	3p	3p	
	2p	3p	2p	3p	
	2p	3p	3p	2p	
	3p	2p	2p	3p	
	3p	2p	3p	2p	
	3p	3p	2p	2p	

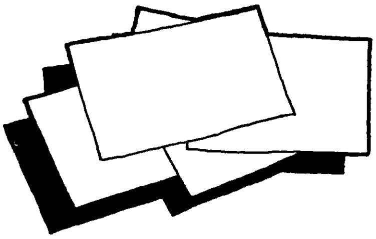
QUESTIONS

- What stamps would you put on a letter that costs 11p?
- What is the greatest/least number of stamps needed for a letter that costs 18p?
- What is the price of five 2p stamps and three 3p stamps?

EXTENSIONS

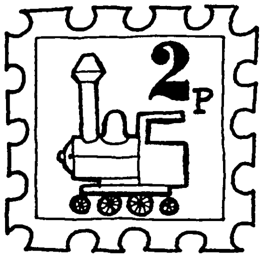
- Try with letters for other costs.
- Try changing the prices of the two stamps.
- Try with three different stamps.

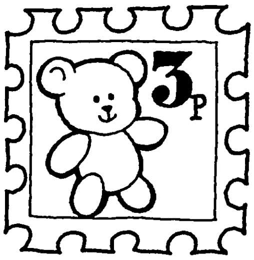
Sticky stamps

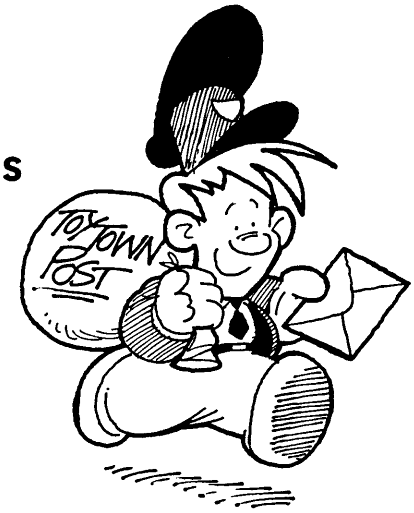


You will need

paper rectangles

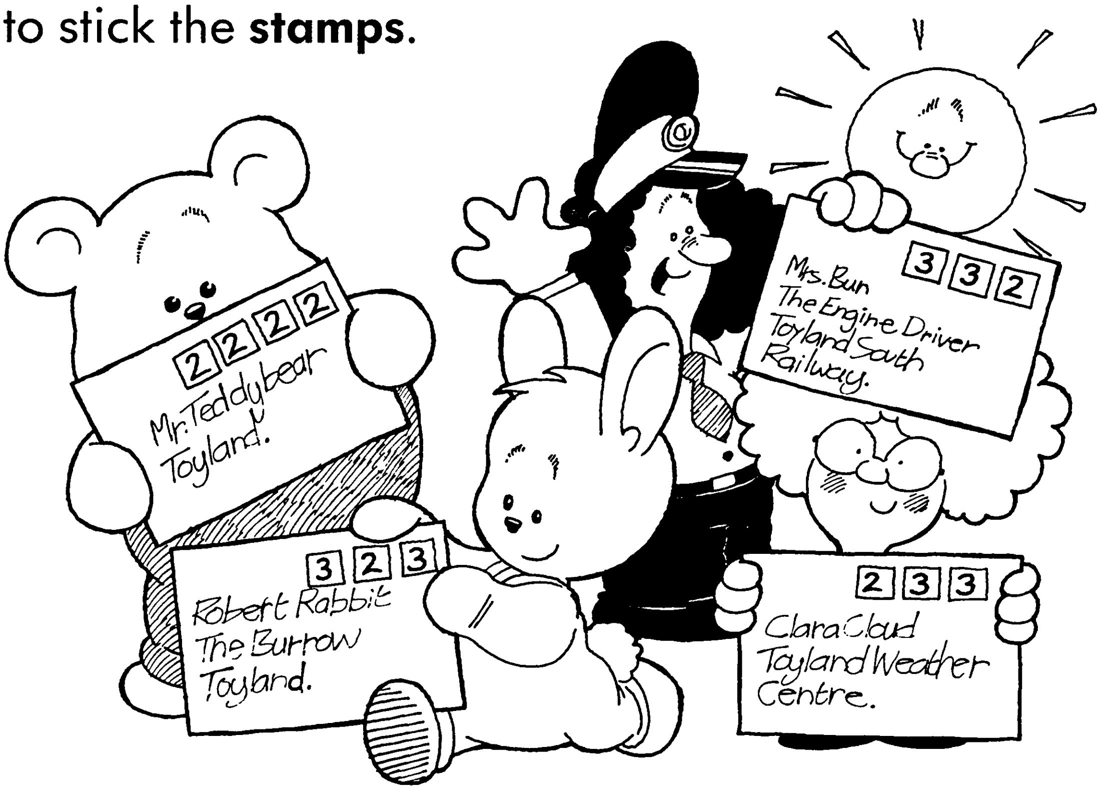






In Toyland, only **2p** and **3p stamps** can be used.
A letter costs 8p to send.

These envelopes show different ways to stick the **stamps**.



Find different ways to stick the stamps if a letter costs **9p**.

10 Trios

ADDITION

Finding sets of three different numbers which, when added, total 12.

Apparatus

Numbercards can be used if required.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4		●		
5				
6				

- Summing three single-digit numbers.
- Recording results.

On paper or in a book. Two possibilities are:

Total 12	or	
1, 2, 9		$1 + 2 + 9 = 12$
1, 3, 8		$1 + 3 + 8 = 12$
.		. .
.		. .
.		. .

The possible different trios are:

1	2	9	2	3	7	3	4	5
1	3	8	2	4	6			
1	4	7						
1	5	6						

If repetitions are allowed, the other possible trios are:

1	1	10	3	3	6	5	5	2
2	2	8	4	4	4			

QUESTIONS

? How many trios can you find using a 2?

? Which trio has numbers in counting order (consecutive)?

EXTENSIONS

→ Try allowing numbers to be repeated.

→ Try with a different target total.

→ Try with four numbers, two numbers, and so on.

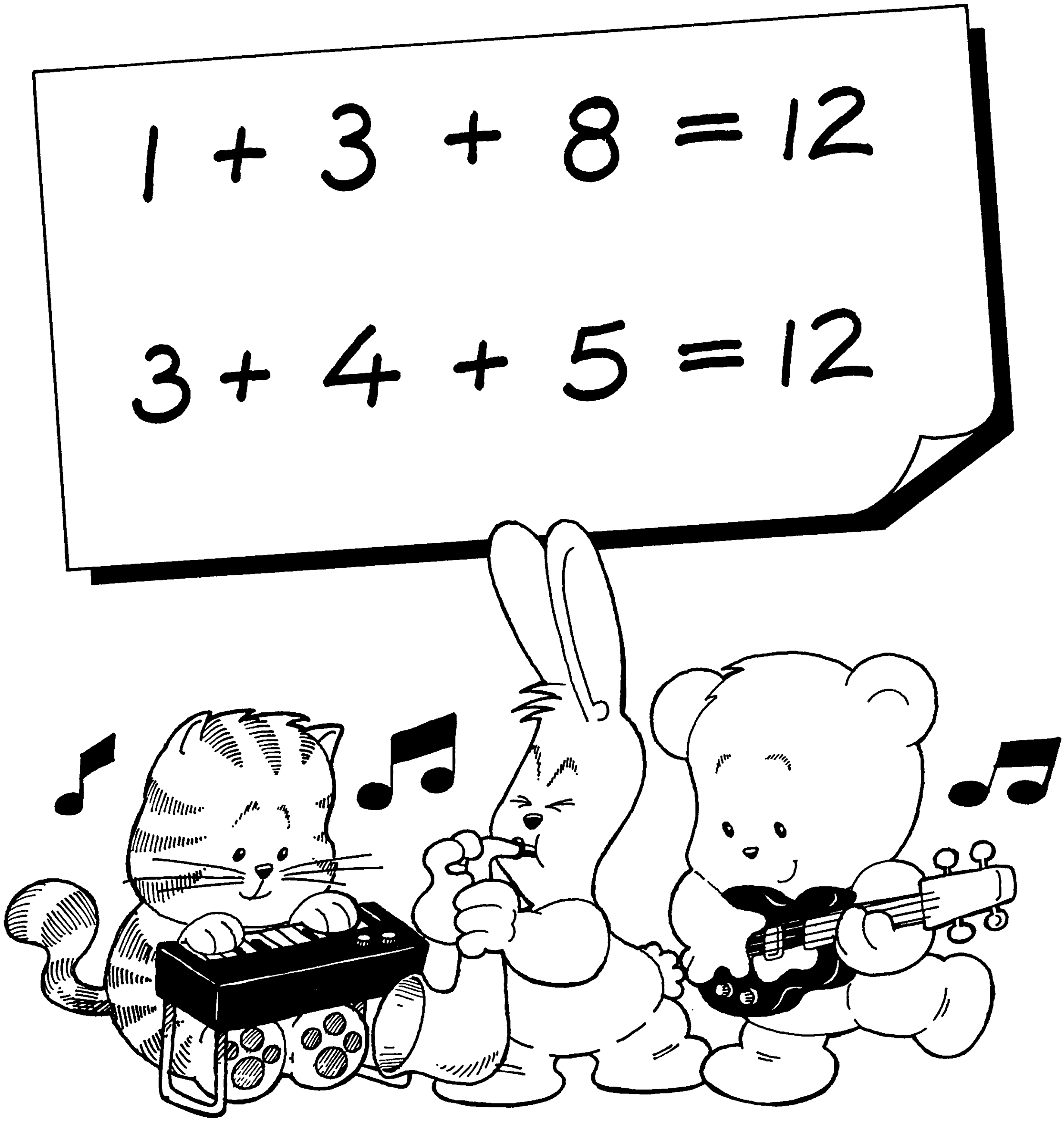
→ Try with $\square + \square - \square = 12$.

Trios

10

Add **three numbers** to make **12**.

The **three numbers** must be different.



Find some more.

starting
INVESTIGATIONS
SPECTRUM MATHS

11 Make a mat

SHAPE PATTERNS

Different patterns formed by arranging five 2 x 1 rectangles inside a larger (2 x 5) rectangle.

Apparatus

Use Logibloc rectangles or make 2 x 1 rectangles from card.
Mat paper (special paper 1).

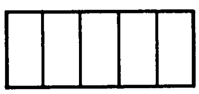
LEVEL	UA	N	SSM	HD
1	●		●	
2	●	●	●	
3	●			
4				
5				
6				

- Appreciate spatial properties through moving shapes around.
- Everyday shapes – rectangles.
- Classifying and recording.

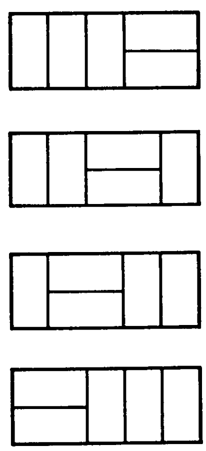
On mat paper. The arrangements can then be outlined and coloured.

The different arrangements are:

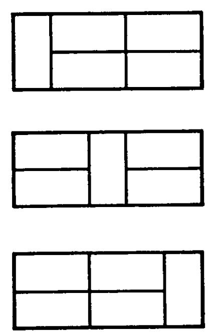
5 'uprights'



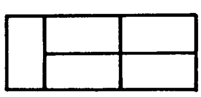
3 'uprights', 2 'flats'



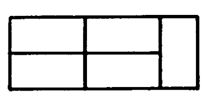
1 'upright', 4 'flats'



These assume that, for example,



and



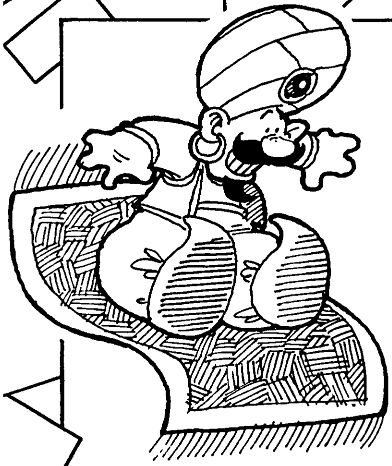
are different.

QUESTIONS

- Which pattern has most 'upright' mats? (Explain upright in this context.)
- Can you make a pattern with three 'flat' mats? (Explain flat in this context.)
- Which patterns look exactly the same upside down?

EXTENSIONS

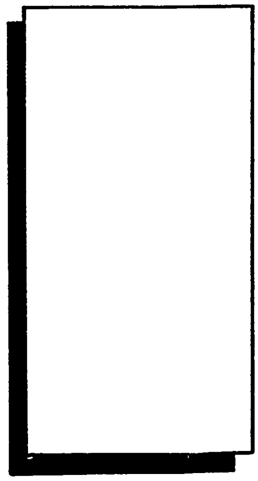
- Try fitting 6 mats into a 2 x 6 rectangle.
- Try fitting 8 mats into a 4 x 4 square.
- Try making patterns using mats in two separate colours.



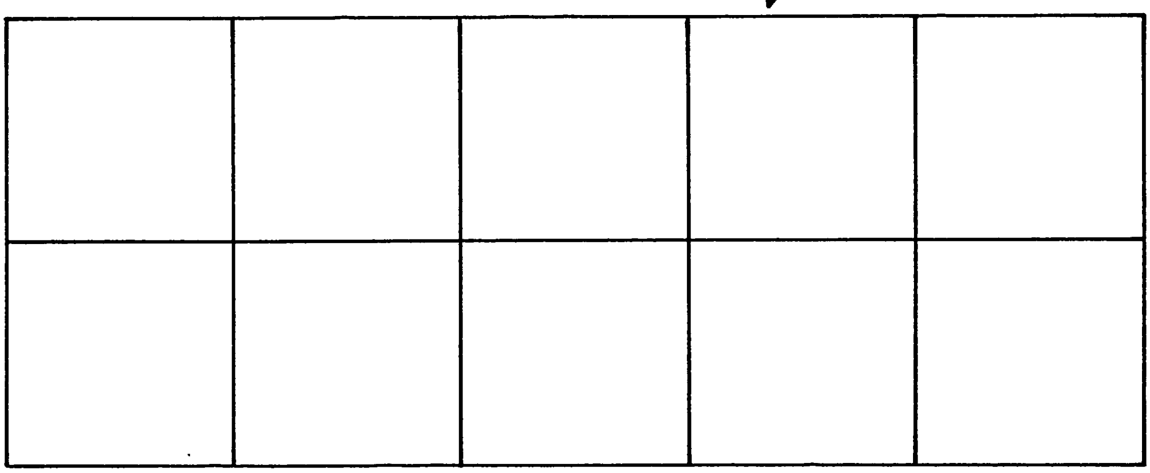
Make a mat

You will need

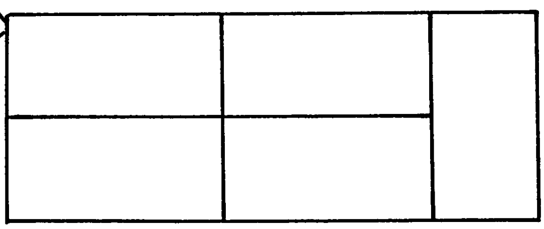
5 rectangles for **mats**
mat paper

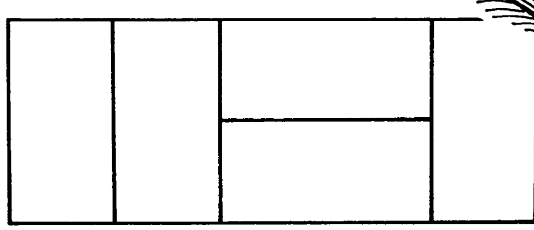


Fit the **mats** inside here
to make a **big mat pattern.**



Here are two ways:





Find some more **big mat patterns**.

starting
INVESTIGATIONS
SPECTRUM MATHS

12 Keep your balance

ADDITION

Exploration of different possible positions of 3 weights on a number balance.
Addition bonds for numbers up to 10.

Apparatus

Use a number balance and weights.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- ● Addition facts.
- ● Patterns in addition.
- ● Recording results.

On paper or in a book. Two possibilities are:

Balance 7	or
1 and 6	1 + 6 = 7
2 and 5	2 + 5 = 7
.	. .
.	. .
.	. .

One systematic approach is to place one weight on 2, 3, 4, 5, . . . in turn on one side of the balance, and then find different positions for the other weights on the other side.

These are the different possibilities:

2	(1, 1)				
3	(1, 2)				
4	(1, 3)	(2, 2)			
5	(1, 4)	(2, 3)			
6	(1, 5)	(2, 4)	(3, 3)		
7	(1, 6)	(2, 5)	(3, 4)		
8	(1, 7)	(2, 6)	(3, 5)	(4, 4)	
9	(1, 8)	(2, 7)	(3, 6)	(4, 5)	
10	(1, 9)	(2, 8)	(3, 7)	(4, 6)	(5, 5)

QUESTIONS

? How many ways are there of balancing a 6 on one side?

? If one weight is on 7 and another on 2, where will the third weight have to be? (Two possible answers.)

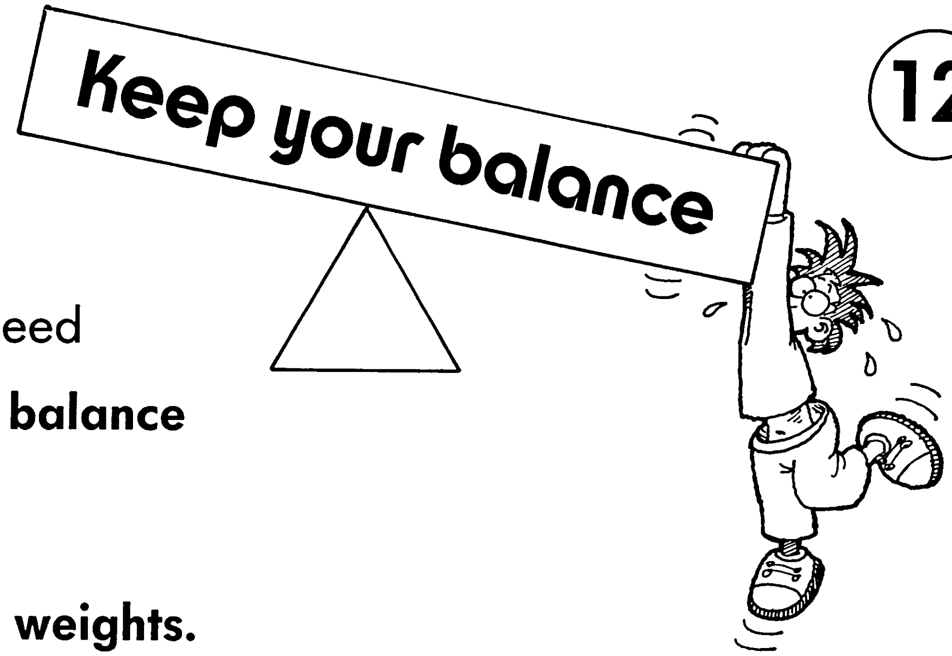
? If one weight is on 6, where could the others be? (Seven possible answers.)

EXTENSIONS

→ Try with four weights, two on each side.

→ Try with four weights, one on one side, three on the other.

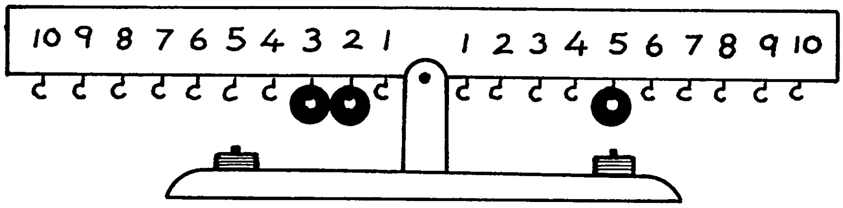
Keep your balance



You will need

a **number balance**
weights

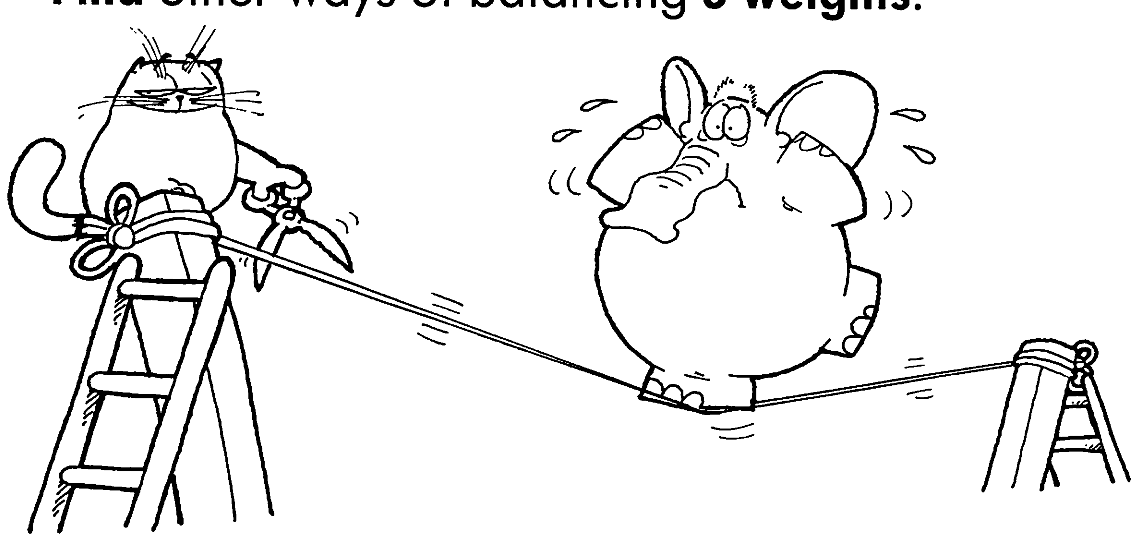
Balance **3 weights.**



Write down the numbers.

$$3+2=5$$

Find other ways of balancing **3 weights**.



starting
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SPECTRUM MATHS

13 Split the strip

NUMBER PATTERNS

Investigating different ways of splitting a given number into three smaller numbers.

Apparatus

Interlocking cubes are ideal (e.g. Multilink).

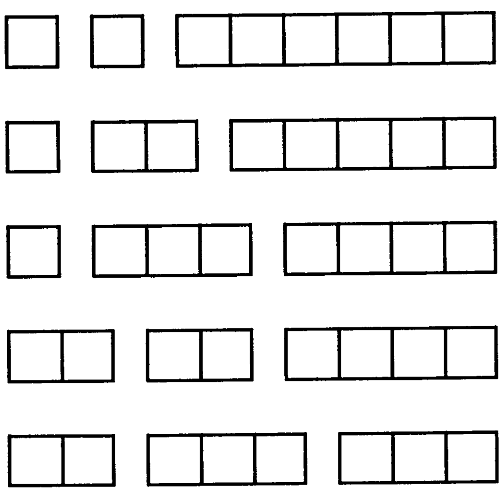
LEVEL	UA	N	SSM	HD
1	●	●		
2	●	●		
3	●			
4				
5				
6				

- Addition using objects.
- Addition facts.
- Patterns in addition.
- Sorting and recording.

On paper or in a book.

$1 + 1 + 6 = 8$
$1 + 2 + 5 = 8$
. .
. .
. .

The strip of 8 cubes can be split in the following different ways:



(1, 1, 6)
(1, 2, 5)
(1, 3, 4)
(2, 2, 4)
(2, 3, 3)

QUESTIONS

? How many different ways can you find?

? How many ways have two parts the same, for example, (2, 2, 4)?

? If a split has three parts (3, 4, 7), how many cubes were in the strip to start with?

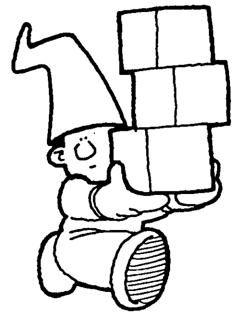
EXTENSIONS

→ Try with strips of different number of cubes.

→ Try splitting strips in two.

→ Try splitting strips in four.

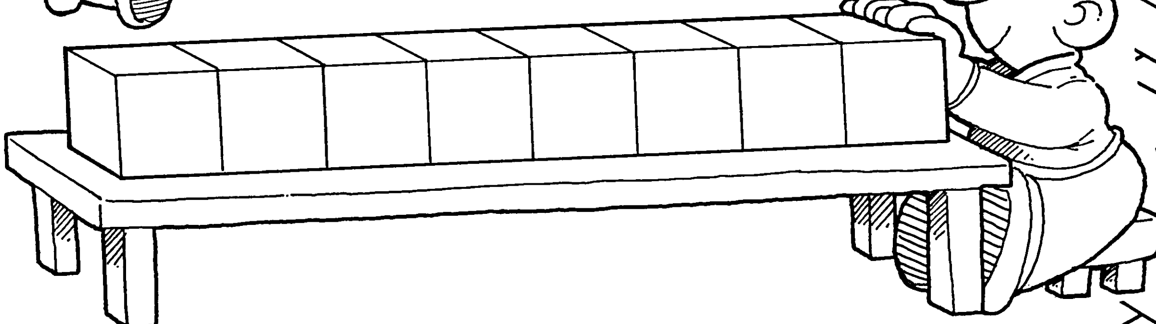
Split the strip



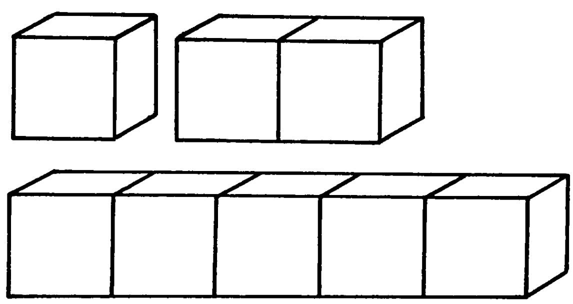
You will need

cubes

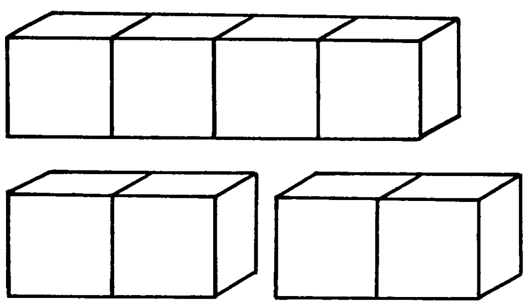
Join **8 cubes** like this.



Then **split** the strip into three.
Here are two ways:



1 + 2 + 5 = 8



4 + 2 + 2 = 8

Find other ways of **splitting** it into three.

14 Card tricks

SUBTRACTION

Subtraction bonds involving the numbers 1 to 6.

Apparatus

Use cards numbered 1 to 6.

LEVEL	UA	N	SSM	HD
1	●	●		
2	●	●		
3	●			
4				
5				
6				

- Subtraction with numbers up to 10.
- Number patterns.
- Recording results.

On paper or in a book.

$4 - 3 = 1$
$5 - 3 = 2$
.
.
.

A systematic approach could start by 'fixing' the 6 and finding all the solutions to $\boxed{6} - \boxed{} = \boxed{}$; then 'fixing' 5, and so on.

Altogether there are 12 ways of placing the cards:

$\boxed{6} - \boxed{1} = \boxed{5}$	$\boxed{5} - \boxed{3} = \boxed{2}$
$\boxed{6} - \boxed{2} = \boxed{4}$	$\boxed{5} - \boxed{4} = \boxed{1}$
$\boxed{6} - \boxed{4} = \boxed{2}$	$\boxed{4} - \boxed{1} = \boxed{3}$
$\boxed{6} - \boxed{5} = \boxed{1}$	$\boxed{4} - \boxed{3} = \boxed{1}$
$\boxed{5} - \boxed{1} = \boxed{4}$	$\boxed{3} - \boxed{1} = \boxed{2}$
$\boxed{5} - \boxed{2} = \boxed{3}$	$\boxed{3} - \boxed{2} = \boxed{1}$

QUESTIONS

- ? Can you find two ways of using $\boxed{4}$ $\boxed{3}$ and $\boxed{1}$?
- ? How many different ways can you find using $\boxed{5}$?
- ? How many different ways give the answer $\boxed{1}$?
- ? Which card have you used most often?

EXTENSIONS

- Try using cards numbered 1 to 9.
- Try using cards numbered 3 to 9.
- Try using the cards like this: $\boxed{} + \boxed{} = \boxed{}$.

Card tricks

You will need **cards 1 to 6**

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ $\boxed{5}$ $\boxed{6}$

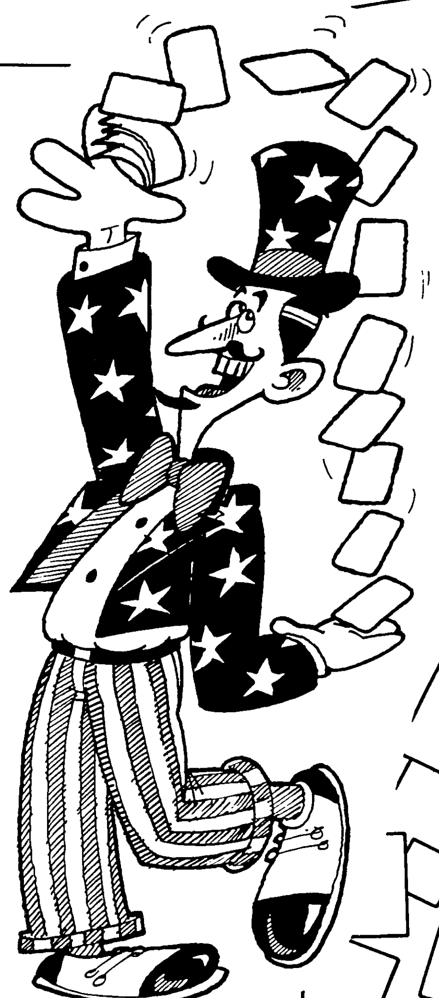
$\boxed{} - \boxed{} = \boxed{}$

Place cards to show

$\boxed{6} - \boxed{2} = \boxed{4}$

Now show

$\boxed{4} - \boxed{3} = \boxed{1}$



Find some more ways of using the cards like this.

15 Pick of the pence

MONEY

ADDITION

Making given amounts of money using different numbers of coins from a bank of 5p, 10p and 20p coins.

Apparatus

Use 5p, 10p and 20p coins.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- Addition of money.
- Patterns in addition.
- Recording results.

On paper or in a book. Draw round coins, use sticky circles, or use rubber stamps.

Here are some ways of making 30p with different numbers of coins:

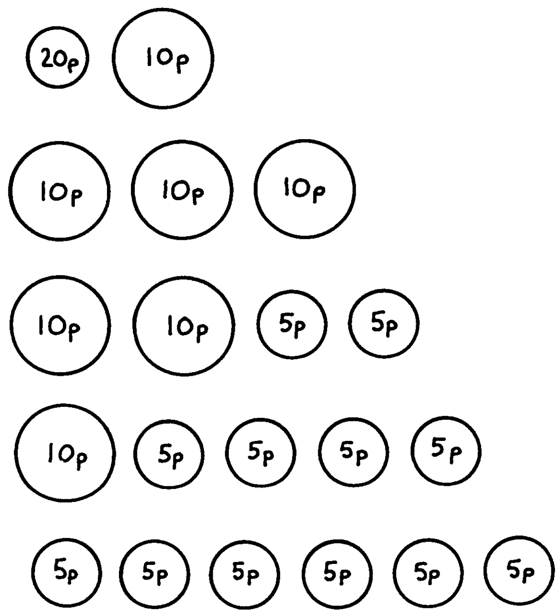
2 coins

3 coins

4 coins

5 coins

6 coins

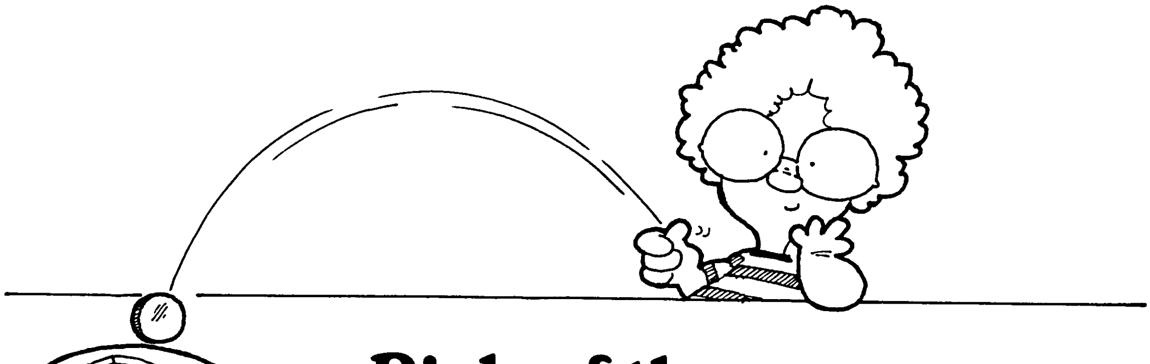


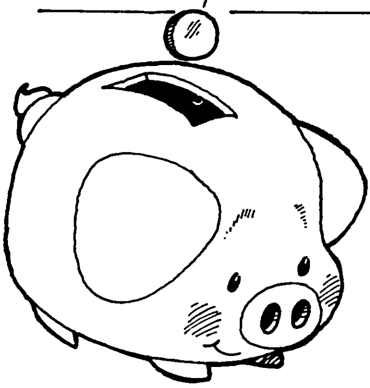
QUESTIONS

- What amounts can be made with one coin?
- Can you find two different ways of making 30p with three coins?
- How can you make 50p with four coins?

EXTENSIONS

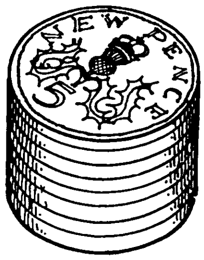
- Try to make different amounts of money.
- Try with different coins, e.g. 2p, 5p, 10p.
- Try using no more than two of each coin.





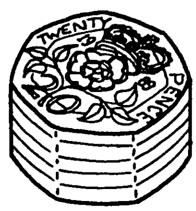
Pick of the pence

You will need these coins



5p

10p

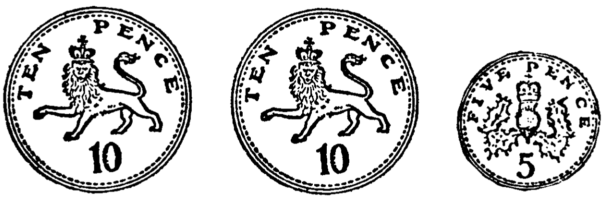


20p

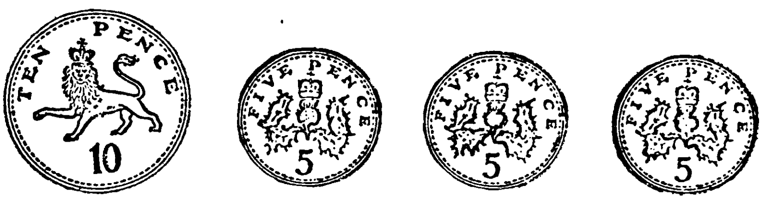
Make **25p** with different numbers of coins.

2 coins

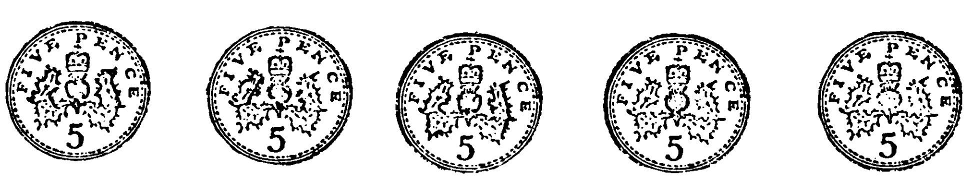
3 coins



4 coins



5 coins



Make 30p with different numbers of coins.

16 Shapes from squares

AREA

Drawing different shapes on squared paper, using given numbers of squares per shape.

Apparatus

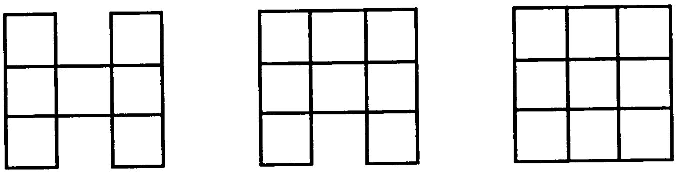
Squared paper.

LEVEL	UA	N	SSM	HD
1	●	●	●	
2	●	●	●	
3	●			
4			●	
5				
6				

- Counting.
- Area by counting squares.
- Drawing 2D shapes.
- Recognition of squares, rectangles.
- Recording results.

On squared paper. The shapes can be coloured, cut out and then stuck on a backing sheet. For example, one sheet for all the different 5-square shapes, one for all the 8-square shapes, and so on.

One way of doing this is to draw the shape for, say, 6 squares and then add one square. Thus the children could build up series of shapes, for example:



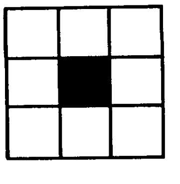
7 squares 8 squares 9 squares

QUESTIONS

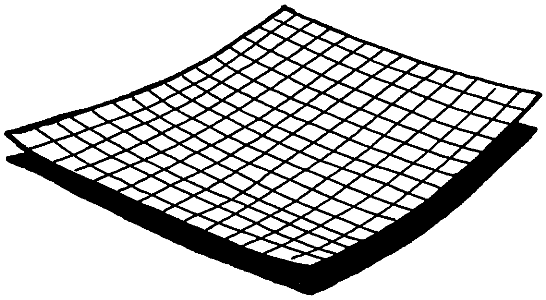
- Which shape is the smallest/largest?
- What is the distance around each shape?
- Is the distance around the 4-square shapes the same?

EXTENSIONS

- Try drawing different shapes all made from 6 squares.
- Try drawing shapes with holes, like this.



Shapes from squares

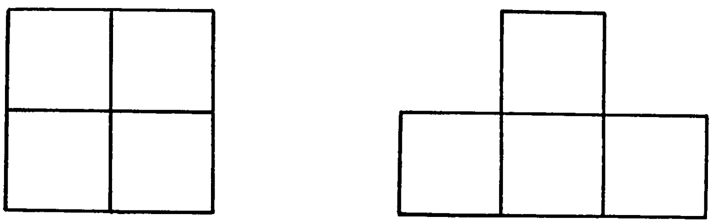


You will need

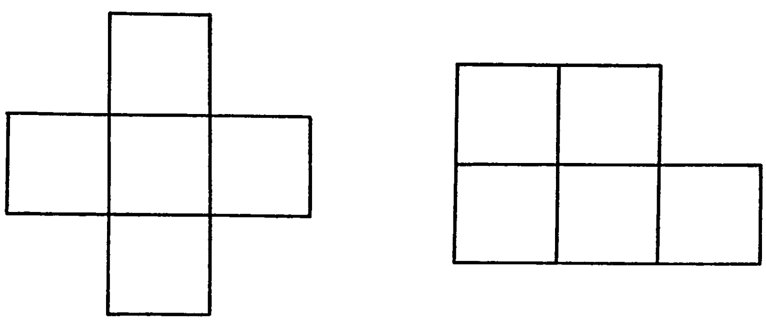
squared paper

Draw some shapes on squared paper.

Here are two made from **4 squares** each.



Here are two made from **5 squares** each.



Draw two shapes made from **6 squares** each.

Do the same for **7, 8, 9** and **10 squares.**

17 Totals

ADDITION

Addition of a single-digit number to a two-digit number.
Possible totals for different arrangements of three digits.

Apparatus

Use cards numbered 1 to 9 and select three with which to find the different arrangements.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●	●		
4		●		
5				
6				

- Ordering numbers.
- Addition of single and two-digit numbers.
- Odds and evens.
- Recording results.

On paper or in a book. Write sums as suggested here.

There are three different possible totals, each of which can be obtained in two ways.

$$\begin{array}{r} 24 \\ +\ \ 5 \\ \hline 29 \end{array} \qquad \begin{array}{r} 25 \\ +\ \ 4 \\ \hline 29 \end{array}$$

$$\begin{array}{r} 42 \\ +\ \ 5 \\ \hline 47 \end{array} \qquad \begin{array}{r} 45 \\ +\ \ 2 \\ \hline 47 \end{array}$$

$$\begin{array}{r} 52 \\ +\ \ 4 \\ \hline 56 \end{array} \qquad \begin{array}{r} 54 \\ +\ \ 2 \\ \hline 56 \end{array}$$

QUESTIONS

- ? What is the smallest/largest possible total?
- ? Which total is nearest to 40?, to 35?
- ? Which totals are odd/even?

EXTENSIONS

- → Try with a different set of 3 cards.
- → Try choosing three cards from a set of four.
- → Try a different arrangement, for example:

$$\begin{array}{r} \square\square\square \\ +\ \ \square\square \\ \hline \end{array}$$

Totals

You will need these cards **4** **5** **2**

Place them in these boxes to make an **addition sum**.

Then find the **total**.

	☐	☐
+		☐

Here is one way:

	2	4
+		5
Total	2	9

Find some more **totals**.

starting INVESTIGATIONS SPECTRUM MATHS

Square count

AREA

On squared paper, drawing different shapes made up from squares and half-squares.

Apparatus

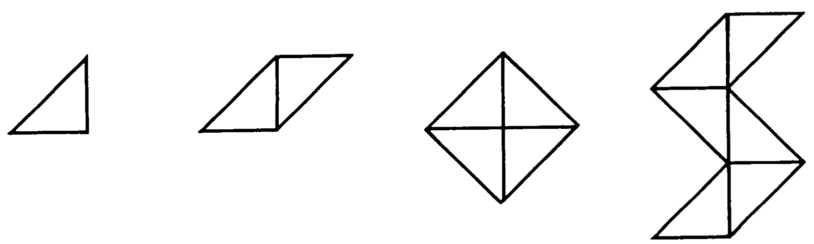
Squared paper.

LEVEL	UA	N	SSM	HD
1	●	●	●	
2	●		●	
3	●			
4			●	
5				
6				

- Counting.
- Area by counting squares.
- Drawing 2D shapes.
- Recognition of squares, triangles.

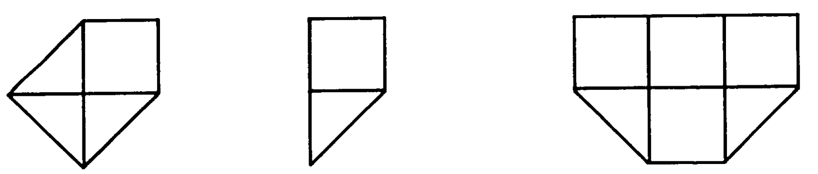
On squared paper. Pupils could colour the whole squares in one colour and the half-squares in another.

These shapes are made from half-squares only:



1 half-square   2 half-squares   4 half-squares   6 half-squares

These shapes are made from half-squares and whole squares:



QUESTIONS

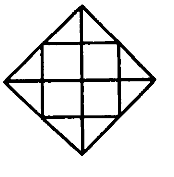
? What is the total number of squares in each shape?
(e.g.: 1 square + 6 half-squares = 4 squares)

? Which shape is the smallest/largest?

EXTENSIONS

→ Try drawing different shapes with 4 half-squares, 5 half-squares, and so on.

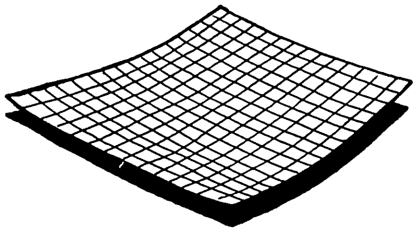
→ Try drawing different squares and rectangles.



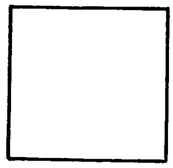
Square count

You will need

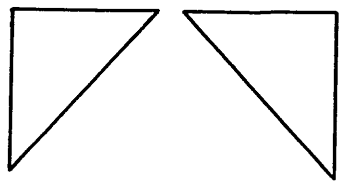
squared paper



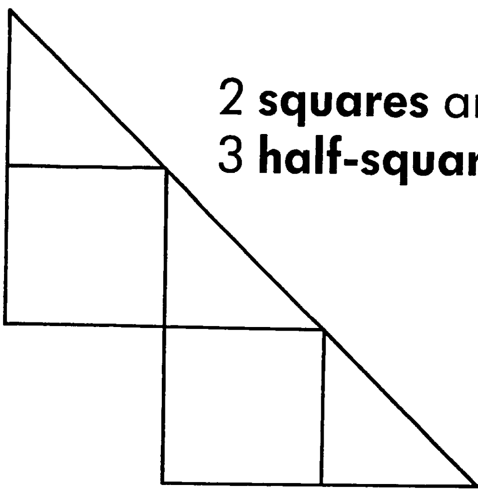
Draw some **squares**



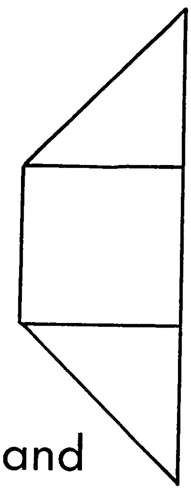
and some **half-squares.**



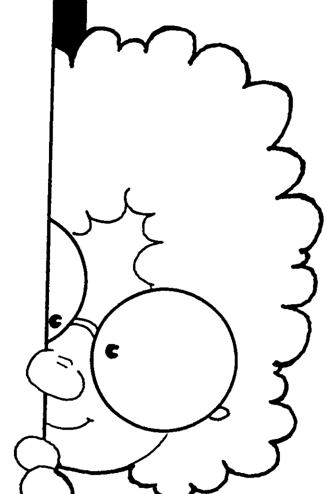
Now draw some **shapes** like these.

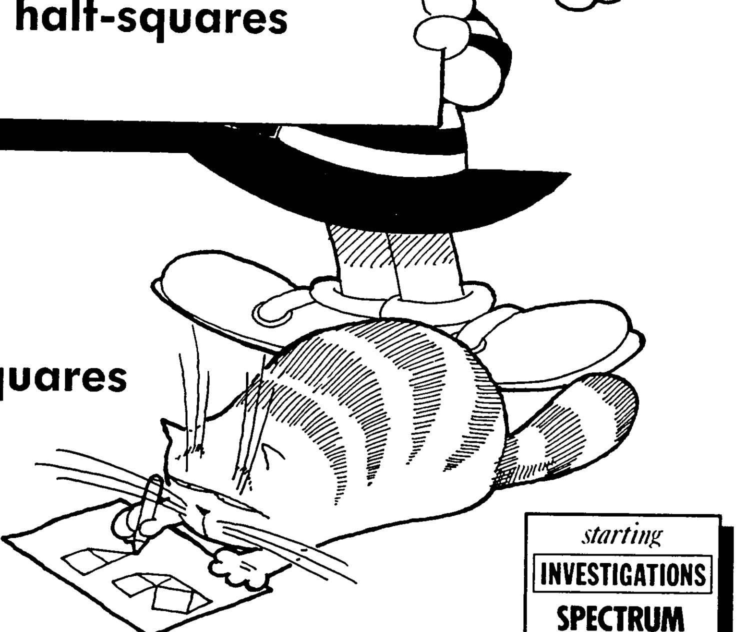


2 **squares** and
3 **half-squares**



1 **square** and
2 **half-squares**





Draw some more **shapes.**

Count the **squares** and **half-squares** in each shape.

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19 Three coins

MONEY

ADDITION

Investigating the different amounts that can be made with just three coins chosen from sets of 1p, 2p, 5p and 10p coins.

Apparatus

Use 1p, 2p, 5p and 10p coins.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- ● Addition of money.
- ● Recording results.

On paper or in a book. Draw round coins, use sticky circles, or use rubber stamps. Alternatively, pupils can write: 14p = 10p + 2p + 2p.

The different possible amounts, up to 30p, are:

1p		11p	(5, 5, 1)	21p	(10, 10, 1)
2p		12p	(5, 5, 2)	22p	(10, 10, 2)
3p	(1, 1, 1)	13p	(10, 2, 1)	23p	
4p	(1, 1, 2)	14p	(10, 2, 2)	24p	
5p	(1, 2, 2)	15p	(5, 5, 5)	25p	(10, 10, 5)
6p	(2, 2, 2)	16p	(10, 5, 1)	26p	
7p	(5, 1, 1)	17p	(10, 5, 2)	27p	
8p	(5, 2, 1)	18p		28p	
9p	(5, 2, 2)	19p		29p	
10p		20p	(10, 5, 5)	30p	(10, 10, 10)

Some amounts can be made in more than one way, for example 12p: (5, 5, 2) or (10, 1, 1).

QUESTIONS

- ? What is the smallest/largest amount possible?
- ? What amounts cannot be made?
- ? What amounts can be made with 3 coins of the same value?
- ? What amounts can be made if 10p coins are not allowed?

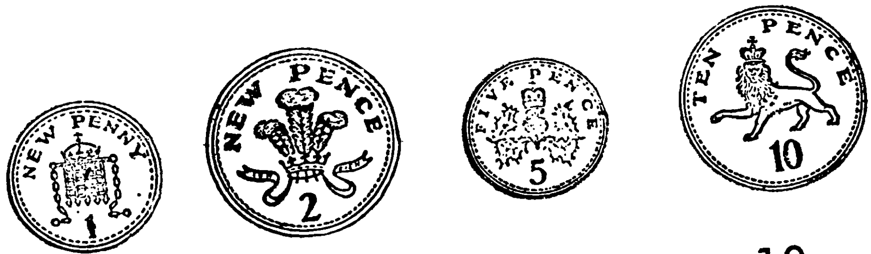
EXTENSIONS

- → Try using one 20p coin as well as the other coins.
- → Try choosing 4 coins.

Three coins

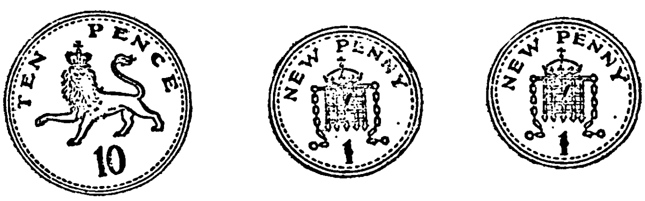
You will need

these coins



1p 2p 5p 10p

Choose **3 coins.**



makes 12p

makes 9p

Find more ways of using **3 coins.**

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SPECTRUM MATHS

Buildings

SHAPE PATTERNS

Different arrangements of six cubes joined face to face.

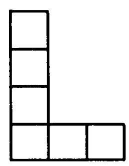
Apparatus

A large collection of interlocking cubes is needed.

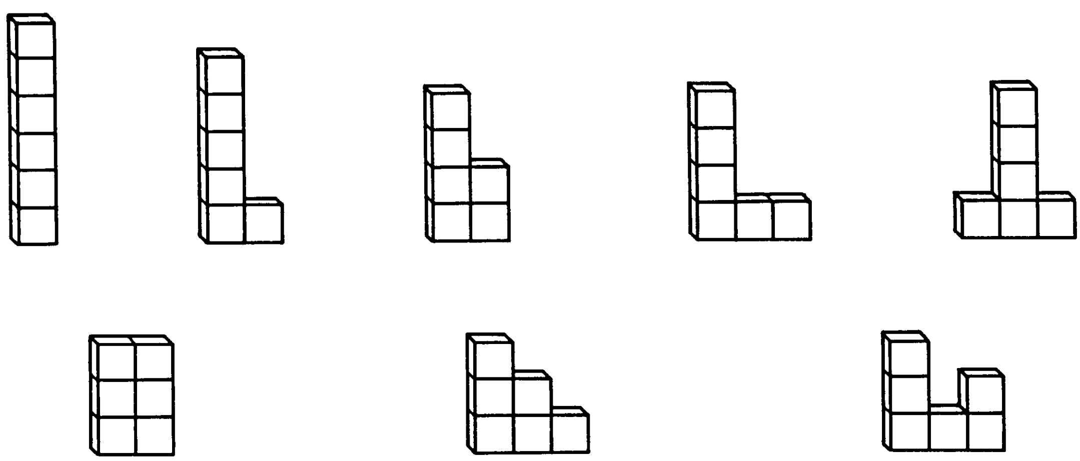
LEVEL	UA	N	SSM	HD
1	●		●	
2	●		●	
3	●			
4				
5				
6				

● Building 3D shapes. Moving shapes around.
● Recognition of cubes and other shapes.

It is not really appropriate to record these, though it is possible to show these models side-on using squared paper:



Many different buildings can be made with six cubes. These examples are all one unit in width. They can be built systematically – first those that are 6 cubes high; then those that are 5 cubes high; then 4 cubes high, and so on.



QUESTIONS

? How many cubes high is each of your buildings?

? How many different buildings can you make with a height of 4 cubes?

? Can you make buildings with just one flat roof?

EXTENSIONS

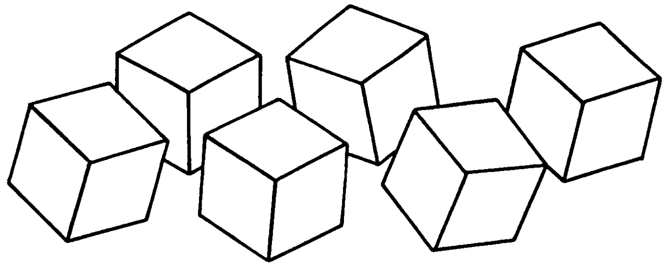
→ Try with a different number of cubes.

→ Try counting the total number of squares on the top and four sides of each building (surface area, less base, in squares).

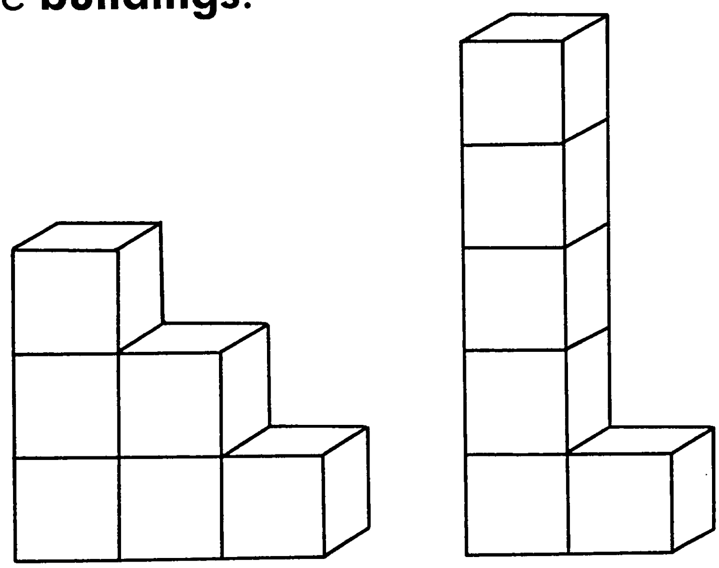
Buildings

You will need

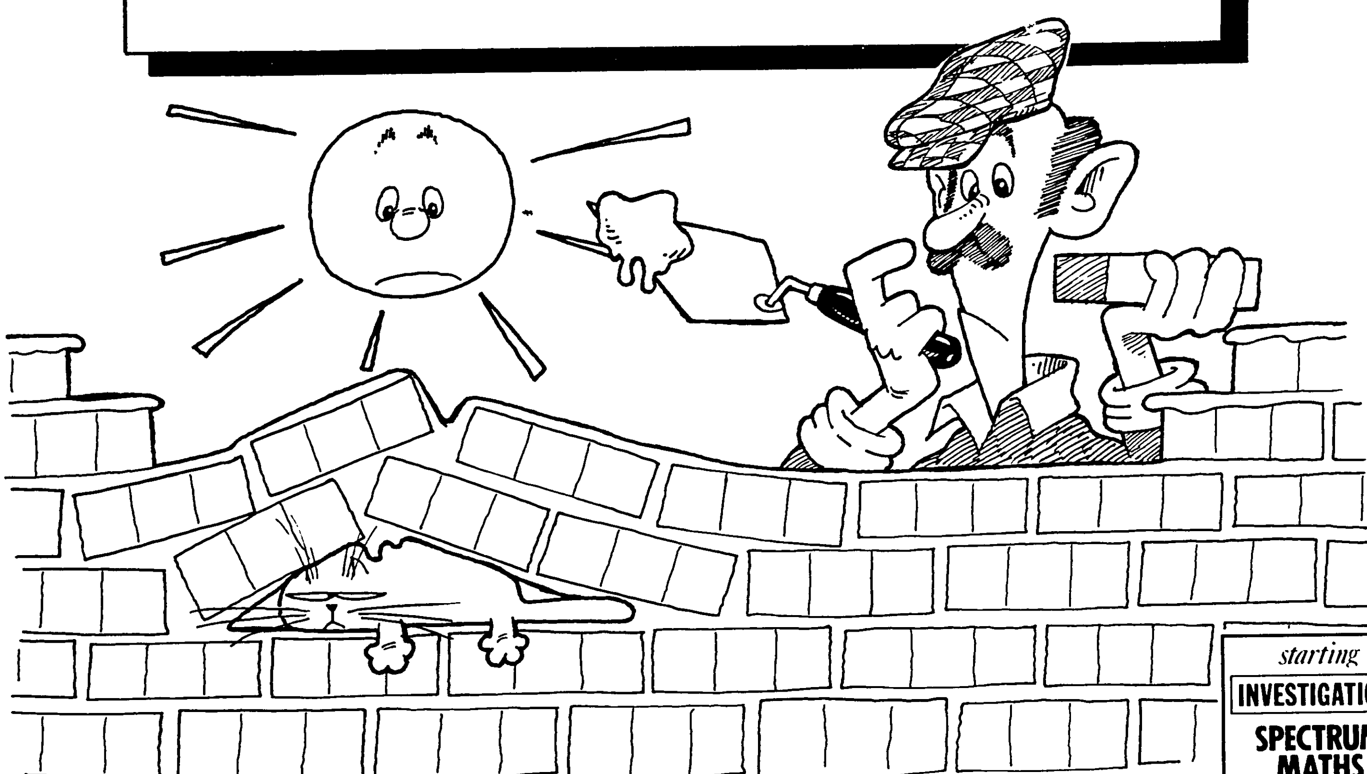
6 cubes



Make **buildings**.



Make some more **buildings**.



Rectangles

RECTANGLES

The construction of rectangles of different sizes using sets of number rods of the same length.

LEVEL	UA	N	SSM	HD
1	●		●	
2	●	●	●	
3	●	●		
4				
5				
6				

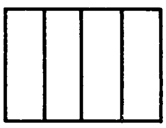
- Number patterns.
- Appreciate spatial properties through moving shapes around.
- Recognition of rectangles.
- Recording results.

Apparatus

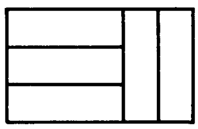
Use squared paper to draw the rectangles. A set of number rods is needed.

On squared paper (1 cm).

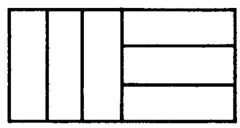
The different rectangles will have dimensions that are multiples of 3; for example,



3 x 4

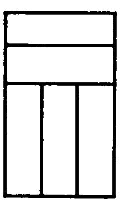


3 x 5

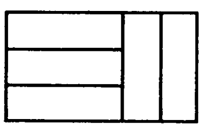


3 x 6

The activity will highlight the equivalence of 3 x 5 and 5 x 3 rectangles; for example,

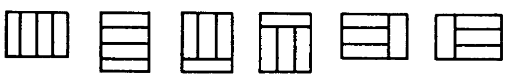


5 x 3



3 x 5

Some rectangles can be built in different ways:



QUESTIONS

? Can you make a square? How many rods do you need? How many squares are there on each side?

? How many different rectangles can you build with four 3-rods?

? Can you build a rectangle that is 5 squares long and 2 squares wide?

EXTENSIONS

→ Try finding all the different ways of arranging five 3-rods to make a 3 x 5 rectangle.

→ Try with 4-rods.

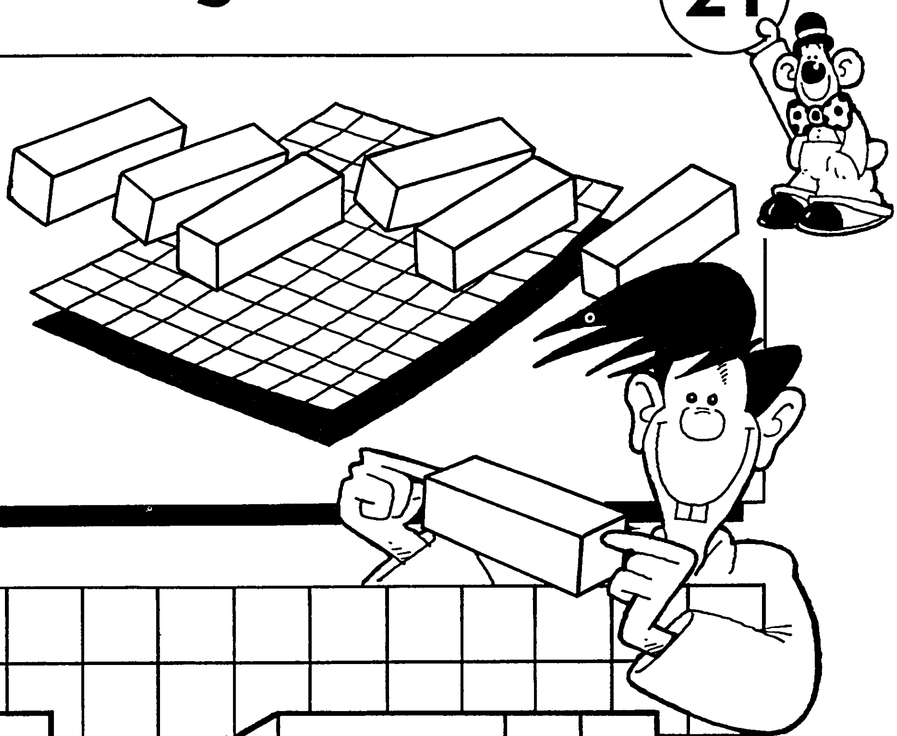
→ Try with two sets of rods, for example, 3-rods and 4-rods.

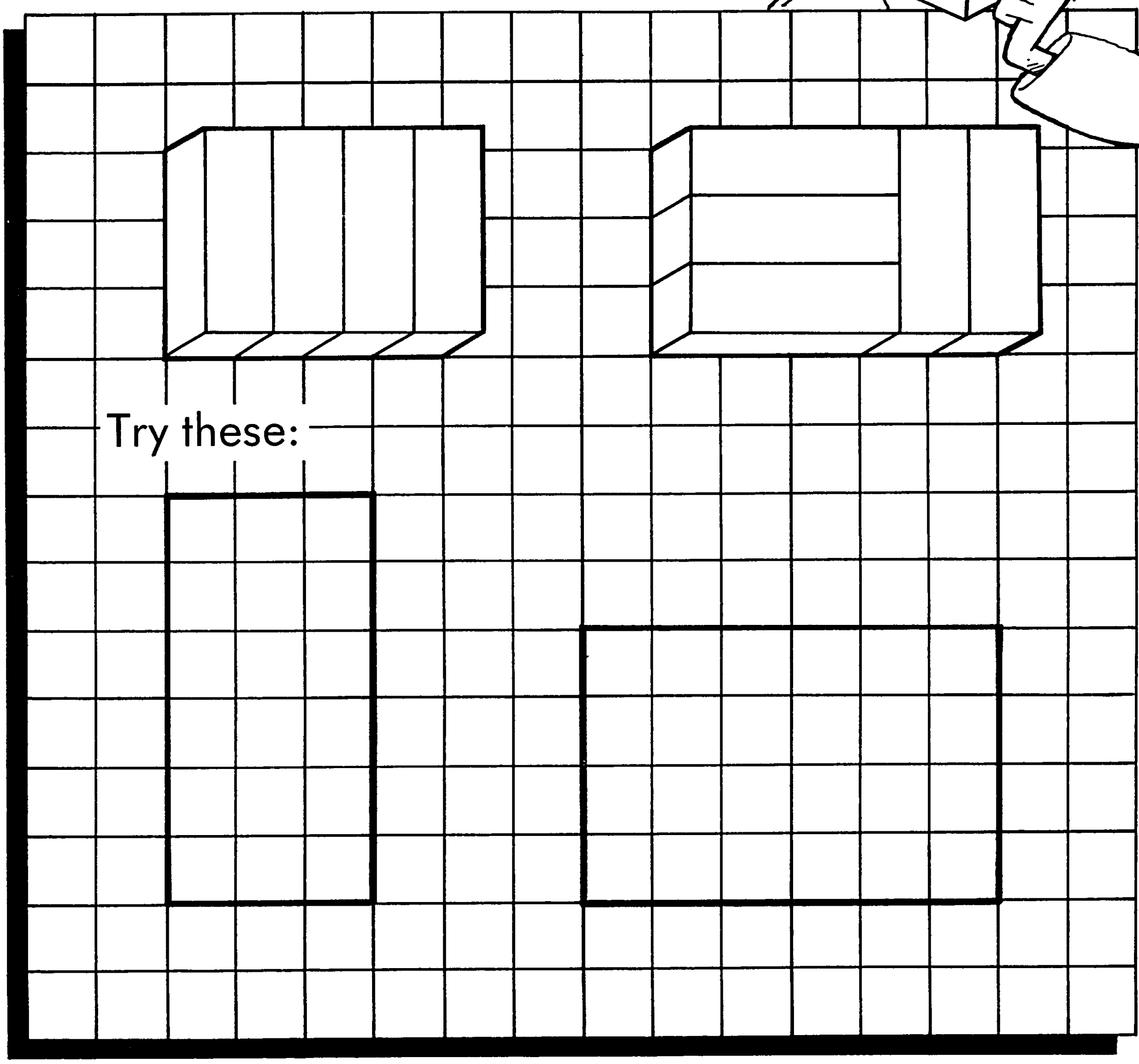
Rectangles

You will need

3-rods
squared paper

Fill **rectangles**.





Draw some different **rectangles**
and try to fill them.

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SPECTRUM MATHS

22 Strips

COLOUR PATTERNS

Arrangements of four squares of two and three different colours.

Apparatus

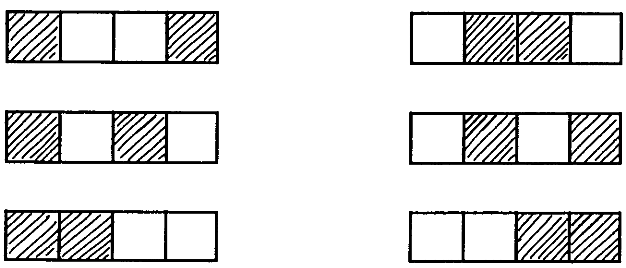
Provide the children with Logiblocs or squares of coloured card.
Squared paper.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●	●	
3	●			
4				
5				
6				

● Patterns using 2D shapes.
● Recording results.

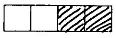
On squared paper. The strips can be coloured.

There are six different arrangements:



If three white and one red square are used, there are only four different arrangments:

QUESTIONS

? Are  and  the same?

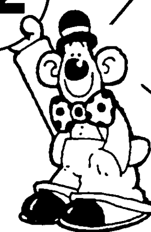
? Which arrangements look exactly the same upside down?

EXTENSIONS

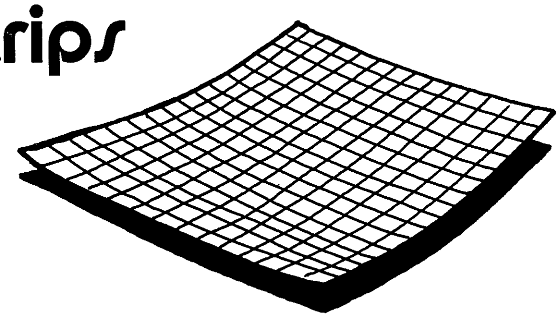
→ Try strips with five squares, two white and three red.

→ Try two red and two white squares in various patterns to make a larger square.

→ Try strips with four squares, two white, one red and one blue.



Strips

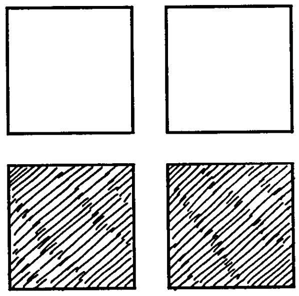


You will need

squared paper

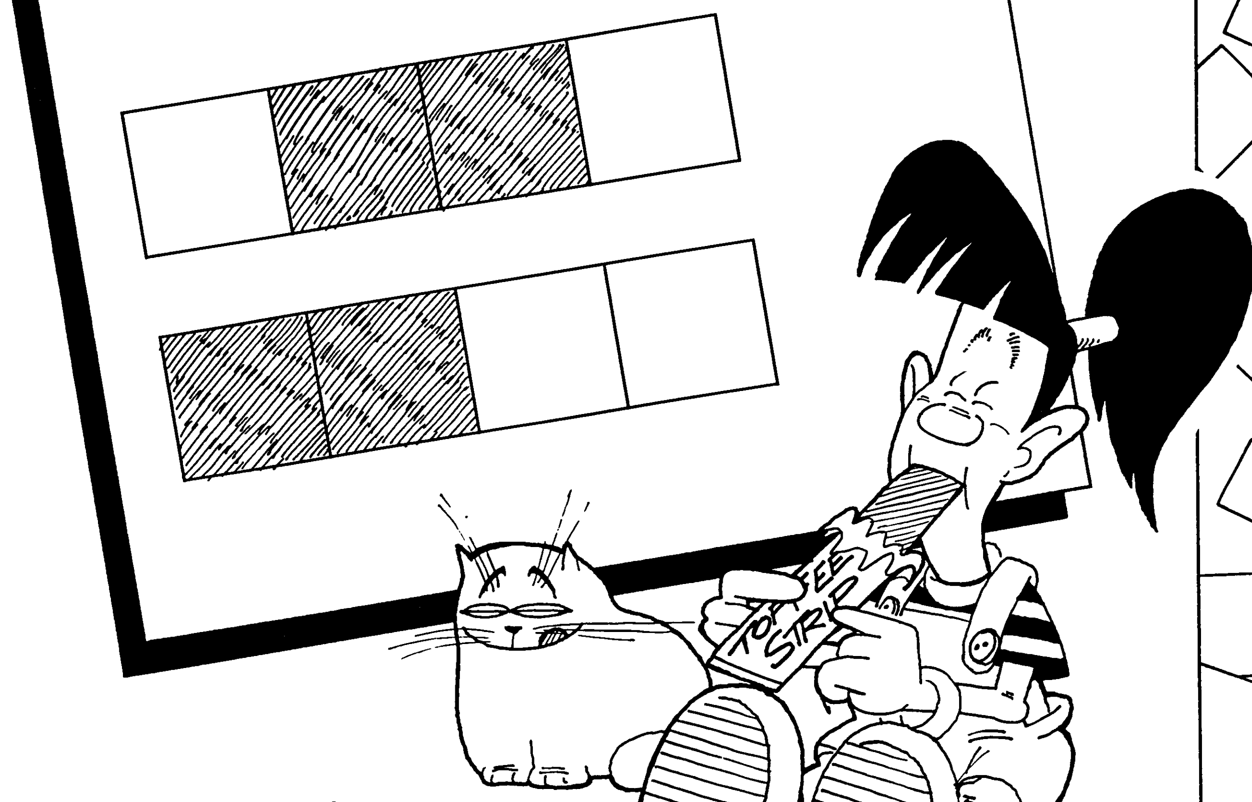
two **white squares**

and two **red squares**



Put them in a **strip**.

Here are two ways:



Find some other ways.

Dice sort

ODDS AND EVENS

Different combinations of the numbers on three dice: all numbers odd; all even; or mixtures of odd and even.

Apparatus

Use three ordinary dice or three cubes numbered 1 to 6.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- Odds and evens.
- Recording results.

On paper or in a book: 3, 3, 5
On squared paper: 3 3 5 

There are ten different 'all odd' combinations:

1 1 1	1 5 5
1 1 3	3 3 3
1 1 5	3 3 5
1 3 3	3 5 5
1 3 5	5 5 5

Some discussion will be needed about, for example, 1 3 5 and 3 5 1.

QUESTIONS

? What is the total of each set of three numbers?

? One dice shows 3, one shows 1, and the three numbers add up to 9. What number does the other dice show?

? Can you make a total of 10 with two 'odd' dice and one 'even' dice?

EXTENSIONS

→ Try showing even numbers only.

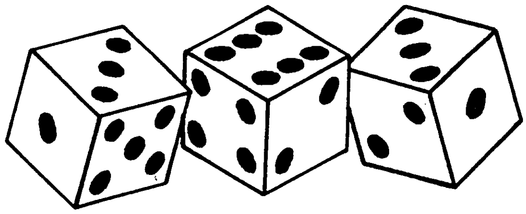
→ Try showing one odd and two even numbers.

→ Try using spinners, numbered 1 to 10.

Dice sort

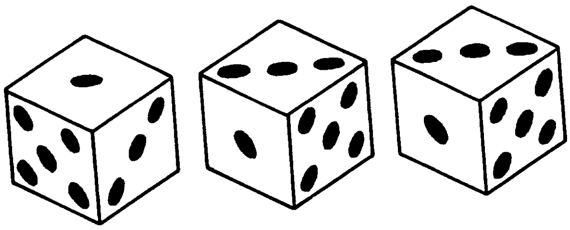
You will need

3 dice

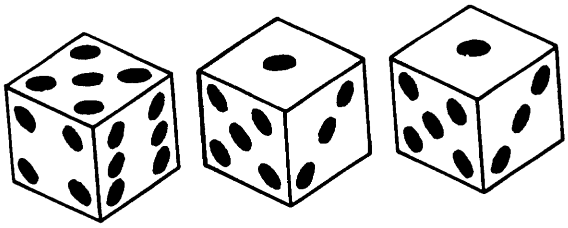


Show **odd numbers** only.

Here are two ways:



1 3 3



5 1 1

How many ways can you find?

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24 Trains

ADDITION

Combinations of different multiples of two numbers to make different totals.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●	●		
4		●		
5				
6				

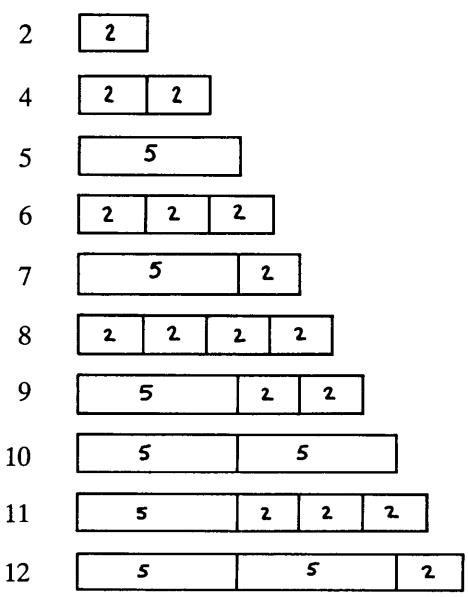
- Addition facts.
- Patterns in addition.
- Multiplication patterns.
- Recording results.

Apparatus

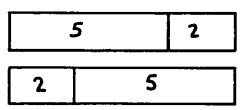
Use Cuisenaire or Colour Factor number rods.
Squared paper.

On paper. Draw around the rods or, more easily, on squared paper. The rods can then be coloured.

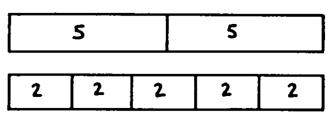
Pupils can be systematic by starting with 2 as a total, and then trying to make 3, then 4, and so on.



Make sure the children understand that these are the same:



Note that some totals can be made in different ways, e.g.:



QUESTIONS

? Can you make a train for every number up to 12?

? Can you make different trains for some of the numbers?

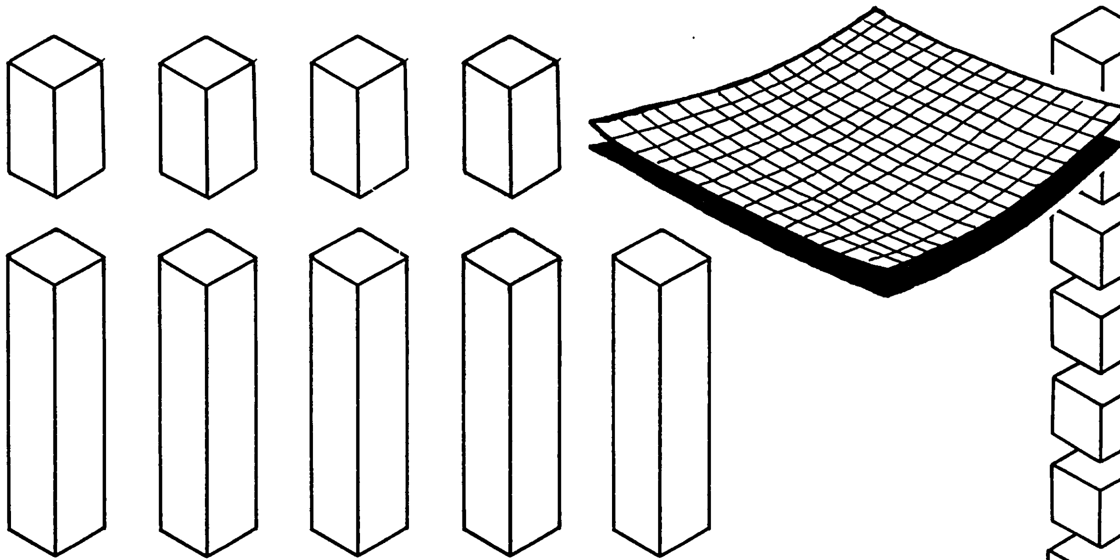
EXTENSIONS

→ Try with different rods, e.g. 3-rods and 4-rods.

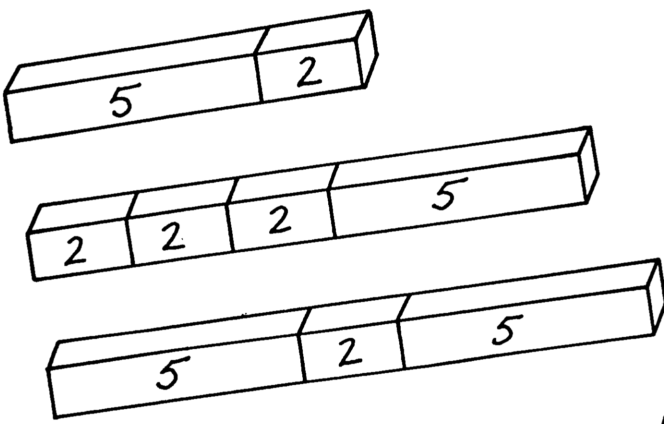
→ Try with three different sets of number rods.

Trains

You will need **squared paper**
2-rods and **5-rods**



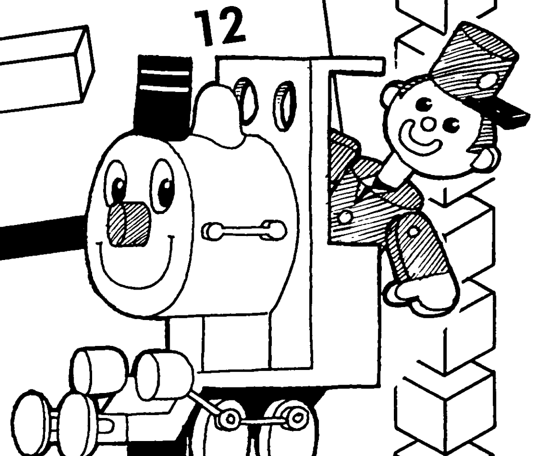
Make **trains** like these.



7

11

12



Find some more **trains**.

25 Dice sums

ADDITION

Sums of the dots on the top faces of two thrown dice.

Apparatus

Use two dice numbered 1 to 6.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

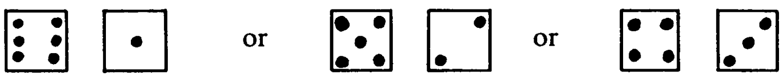
- Addition facts.
- Patterns in addition.
- Recording results.

On paper or in a book: Total 8 2+6 3+5 4+4

On squared paper: Total 8 $\boxed{2}$ $\boxed{6}$ $\boxed{3}$ $\boxed{5}$ $\boxed{4}$ $\boxed{4}$

or

There are three different ways of obtaining a total of 7:



All totals between 2 and 12 are possible:

2	$\boxed{1}$ $\boxed{1}$		
3	$\boxed{1}$ $\boxed{2}$		
4	$\boxed{1}$ $\boxed{3}$	$\boxed{2}$ $\boxed{2}$	
5	$\boxed{1}$ $\boxed{4}$	$\boxed{2}$ $\boxed{3}$	
6	$\boxed{1}$ $\boxed{5}$	$\boxed{2}$ $\boxed{4}$	$\boxed{3}$ $\boxed{3}$
7	$\boxed{1}$ $\boxed{6}$	$\boxed{2}$ $\boxed{5}$	$\boxed{3}$ $\boxed{4}$
8	$\boxed{2}$ $\boxed{6}$	$\boxed{3}$ $\boxed{5}$	$\boxed{4}$ $\boxed{4}$
9	$\boxed{3}$ $\boxed{6}$	$\boxed{4}$ $\boxed{5}$	
10	$\boxed{4}$ $\boxed{6}$	$\boxed{5}$ $\boxed{5}$	
11	$\boxed{5}$ $\boxed{6}$		
12	$\boxed{6}$ $\boxed{6}$		

QUESTIONS

- How many different ways can you make a total of 9?
- What totals can you make when both dice show the same?
- How many different totals can you make?

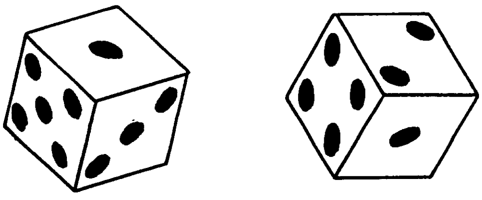
EXTENSIONS

- Try making totals with three dice.
- Try with different dice, e.g. with 1, 3, 5, 7, 8, 9 and 2, 4, 5, 6, 7, 9 written on blank faces. (Make sure that 9 and 6 cannot be confused.)

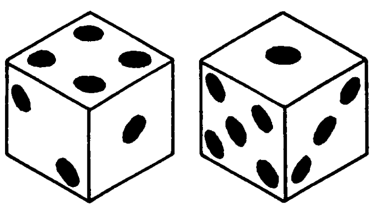
Dice sums

You will need

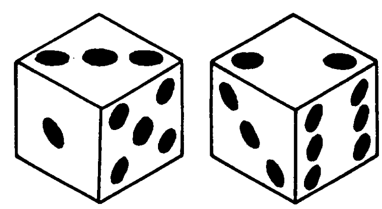
2 dice



These throws have a **total of 5**.



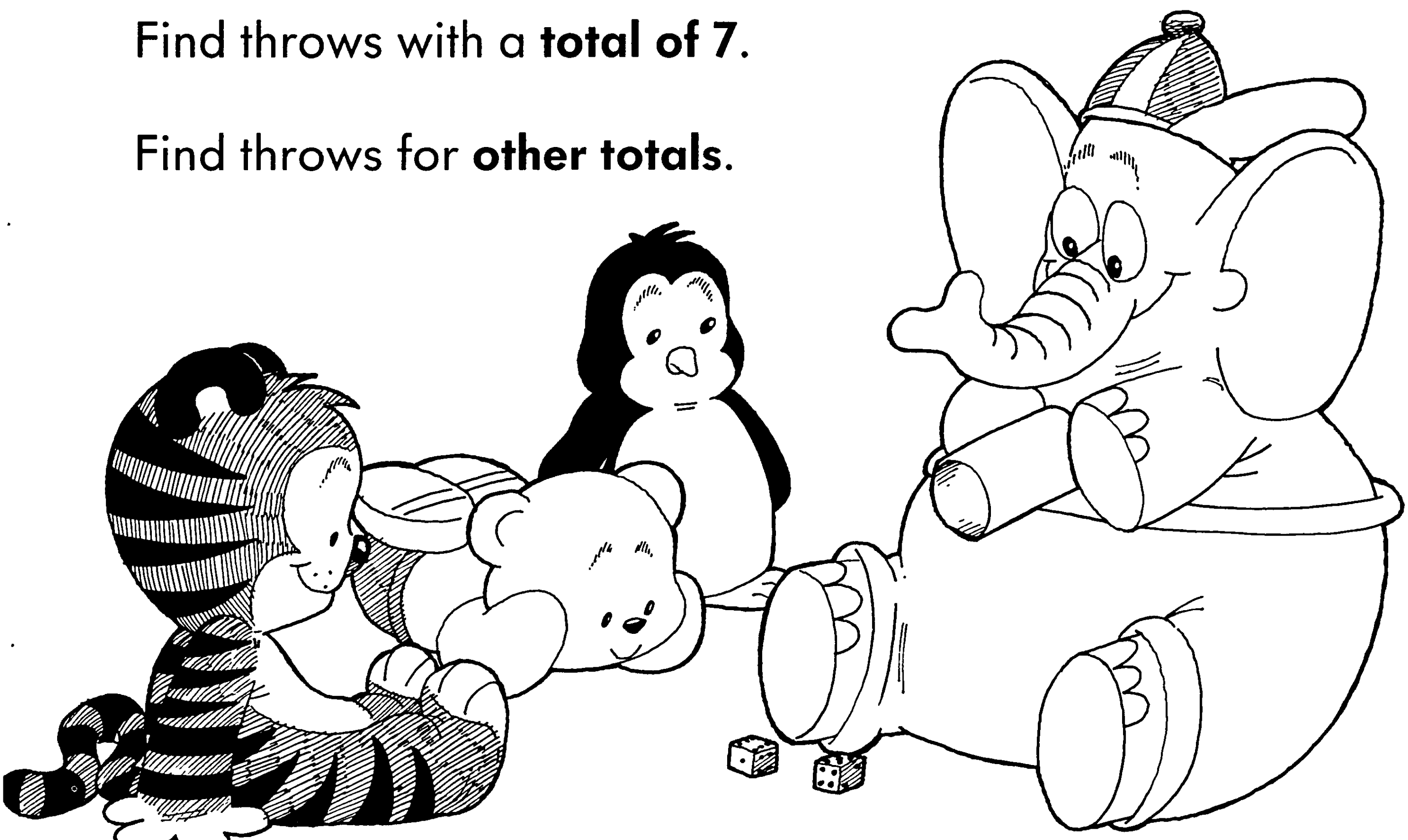
$4 + 1 = 5$



$3 + 2 = 5$

Find throws with a **total of 7**.

Find throws for **other totals**.



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26 Flags

COLOUR PATTERNS

Making different 2-colour patterns by colouring the squares of 2 by 2 flag shapes.

Apparatus

The flag paper (special paper 5) provides the flag shapes. Alternatively, squared paper can be used.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●	●	
3	●			
4				
5				
6				

● Patterns using 2D shapes.
● Choose classification criteria.

On flag paper. The flags can be coloured.

It is important to encourage a systematic approach. For example, the children could start by finding different flags with one yellow square, then flags with two yellow squares, and so on.

1 yellow square

Y R	R Y	R R	R R
R R	R R	R Y	Y R

2 yellow squares

Y Y	Y R	R R	R Y	Y R	R Y
R R	Y R	Y Y	R Y	R Y	Y R

3 yellow squares

R Y	Y R	Y Y	Y Y
Y Y	Y Y	Y R	R Y

Also, no yellow squares

R	R
R	R

and 4 yellow squares

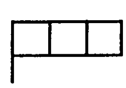
Y	Y
Y	Y

QUESTIONS

? How many different flags have 1 yellow square?

? Which flags will look the same upside down?

EXTENSIONS

→ Try different flags, e.g. 

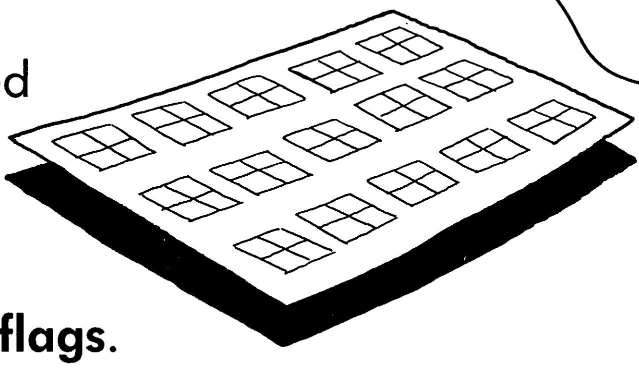
→ Try three colours.

→ Try designing flags with triangles.

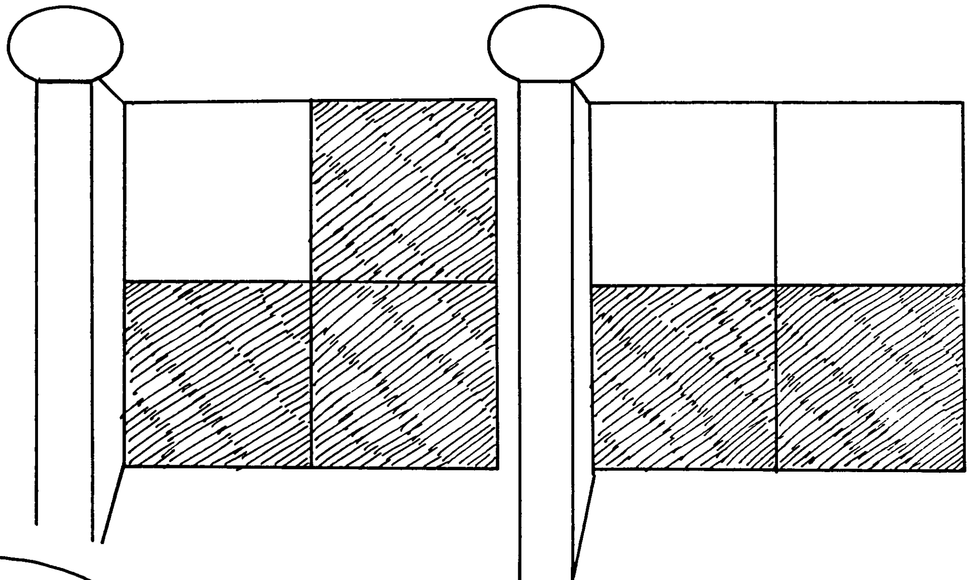
Flags

You will need

flag paper



Draw some **flags**.
Colour the squares red or yellow
to make different **flags**.



Make some more **flags**.

27 Counter colours

COLOUR PATTERNS

Arrangements of three different coloured counters in a vertical strip.

Apparatus

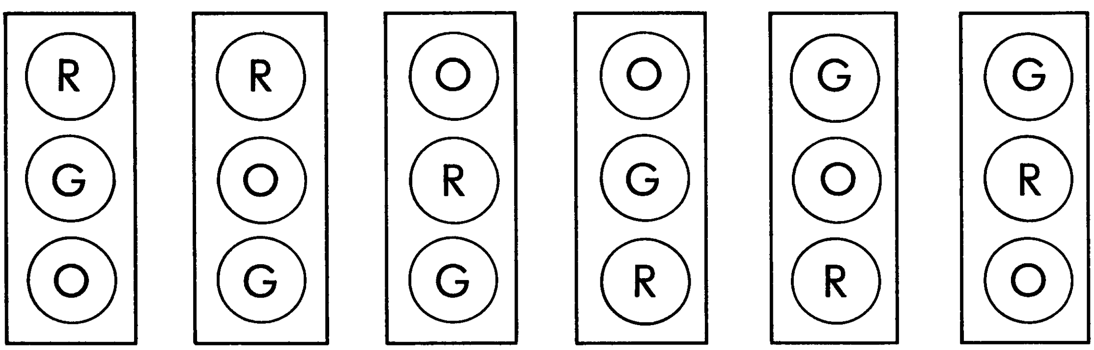
Red, orange and green counters.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●	●	
3	●			
4				
5				
6				

- Patterns using 2D shapes.
- Recording results.

On paper or in a book. Draw round coins or use sticky circles. Alternatively, draw them free-hand.

There are six different arrangements using one counter of each colour:

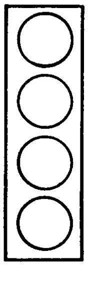


QUESTIONS

- How many different arrangements have red at the top?
- What is the correct arrangement for traffic lights?

EXTENSIONS

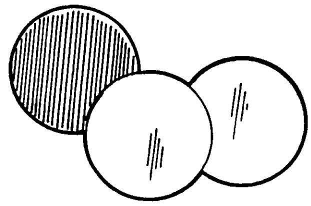
- Try choosing from four colours.
- Try four colours in a strip like this:



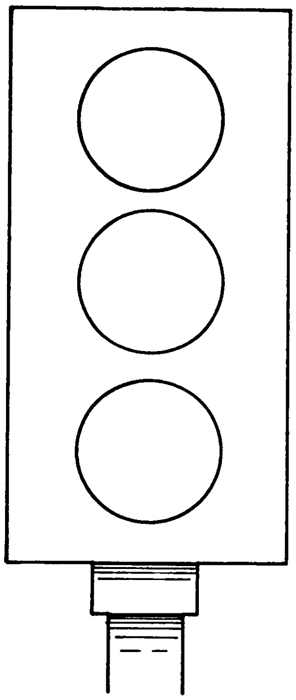
Counter colours

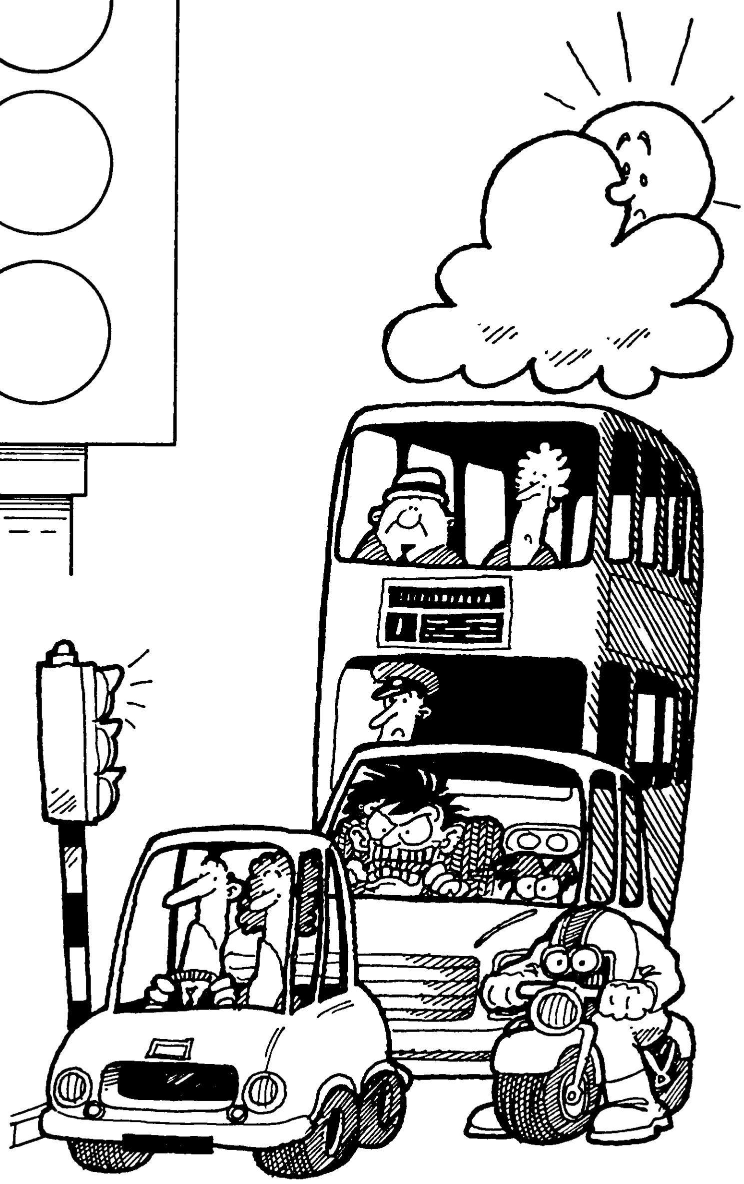
You will need

one red, one orange and
one green counter

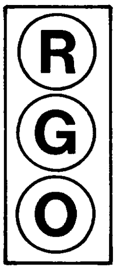


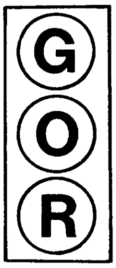
Put one counter on each circle.





Here are two ways:





How many different ways
can you find?

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Spot the domino

NUMBER PATTERNS

Different arrangements of up to six counters used as the spots on dominoes.

Apparatus

Use up to six counters intitially, then increase the number up to twelve.
Domino paper (special paper 3).

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- Addition patterns.
- Recording results.
- Choose sorting criteria.

On domino paper. The dominoes can be recorded using either dots or numbers.

The different dominoes are:

6 counters:	[| 6]	[1 | 5]	[2 | 4]	[3 | 3]
5 counters:	[| 5]	[1 | 4]	[2 | 3]	
4 counters:	[| 4]	[1 | 3]	[2 | 2]	
3 counters:	[| 3]	[1 | 2]		
2 counters:	[| 2]	[1 | 1]		
1 counter:	[| 1]			
0 counters:	[|]			

QUESTIONS

? How many different dominoes can you make with 4 counters?

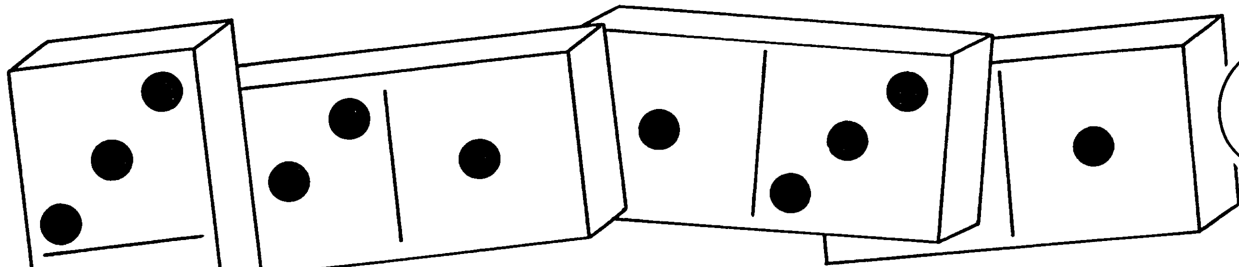
? How many dominoes have a blank?

? Which dominoes have one side with two counters more than the other side?

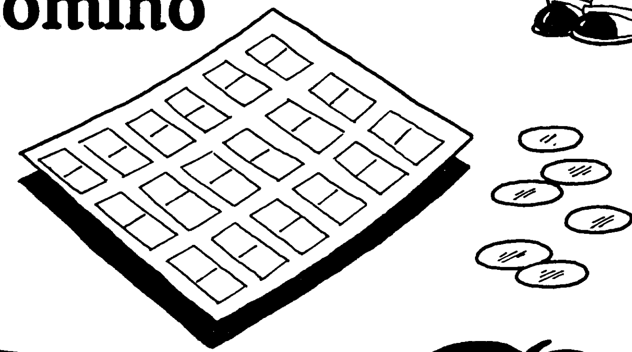
EXTENSIONS

→ Try allowing one extra counter.

→ Try using up to ten counters.

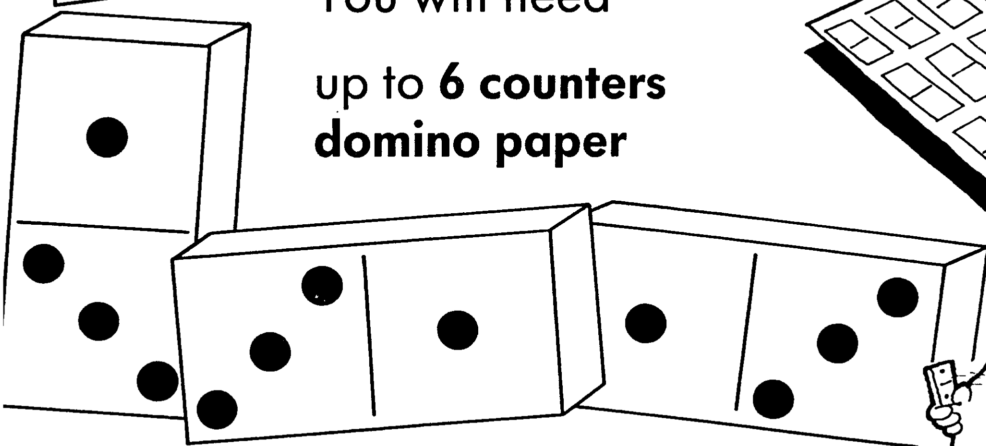


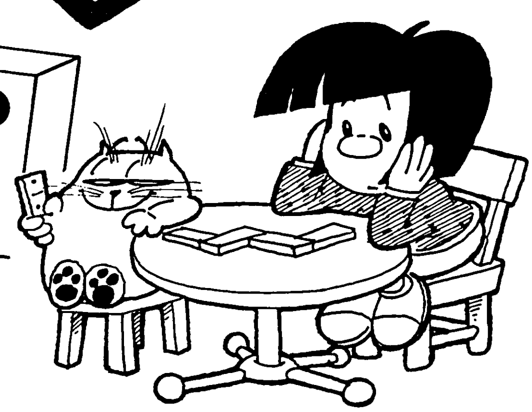
Spot the domino



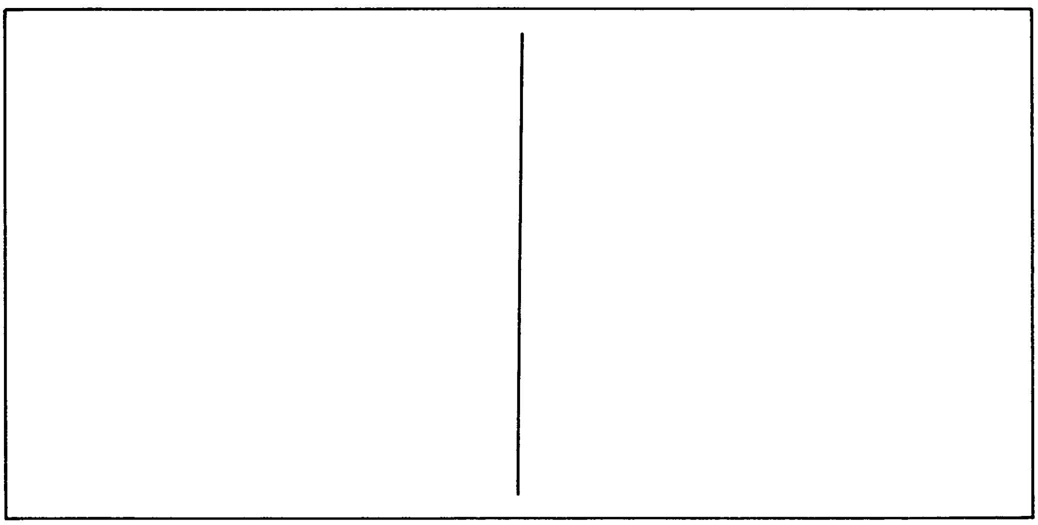
You will need

up to **6 counters**
domino paper

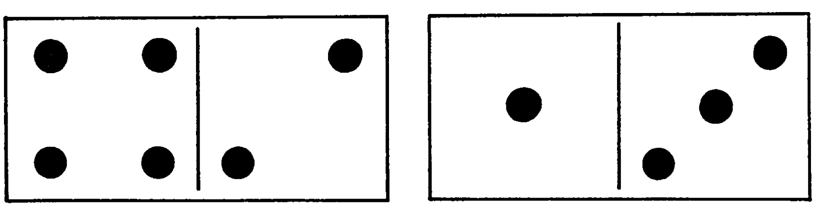


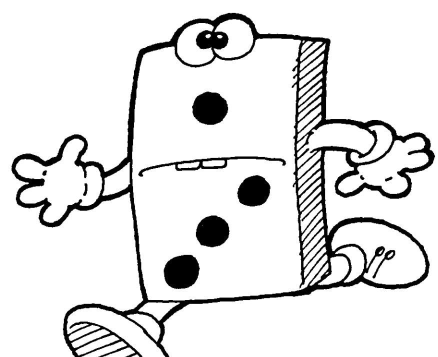


Make **dominoes** with the counters.



Here are two dominoes:





Make some more **dominoes**.

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29 Let's add

ADDITION

Addition bonds in the form of vertical sums, using the numbers 2 to 8.

Apparatus

Use cards numbered 2 to 8 initially, and then include 1 and 9 if appropriate.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- Addition facts up to 10.
- Addition patterns.
- Recording results.

On paper or in a book.

$$\begin{array}{r} 2 \\ +6 \\ \hline 8 \end{array}$$

There are twelve different possible additions:

$$\begin{array}{r} \boxed{2} \\ +\boxed{6} \\ \hline \boxed{8} \end{array} \quad \begin{array}{r} \boxed{3} \\ +\boxed{5} \\ \hline \boxed{8} \end{array} \quad \begin{array}{r} \boxed{5} \\ +\boxed{3} \\ \hline \boxed{8} \end{array} \quad \begin{array}{r} \boxed{6} \\ +\boxed{2} \\ \hline \boxed{8} \end{array} \quad \begin{array}{r} \boxed{2} \\ +\boxed{5} \\ \hline \boxed{7} \end{array} \quad \begin{array}{r} \boxed{3} \\ +\boxed{4} \\ \hline \boxed{7} \end{array}$$

$$\begin{array}{r} \boxed{4} \\ +\boxed{3} \\ \hline \boxed{7} \end{array} \quad \begin{array}{r} \boxed{5} \\ +\boxed{2} \\ \hline \boxed{7} \end{array} \quad \begin{array}{r} \boxed{2} \\ +\boxed{4} \\ \hline \boxed{6} \end{array} \quad \begin{array}{r} \boxed{4} \\ +\boxed{2} \\ \hline \boxed{6} \end{array} \quad \begin{array}{r} \boxed{2} \\ +\boxed{3} \\ \hline \boxed{5} \end{array} \quad \begin{array}{r} \boxed{3} \\ +\boxed{2} \\ \hline \boxed{5} \end{array}$$

It is important to discuss pairs, e.g.

$$\begin{array}{r} \boxed{5} \\ +\boxed{2} \\ \hline \boxed{7} \end{array} \quad \begin{array}{r} \boxed{2} \\ +\boxed{5} \\ \hline \boxed{7} \end{array}$$

QUESTIONS

- How many different addition sums can you make using $\boxed{6}$?
- How many different ways can you find the total $\boxed{7}$?
- What is the highest/lowest total you can find?

EXTENSIONS

- Try introducing the card numbered 1.
- Try using cards numbered 1 to 9.
- Try subtracting instead of adding.

Let's add

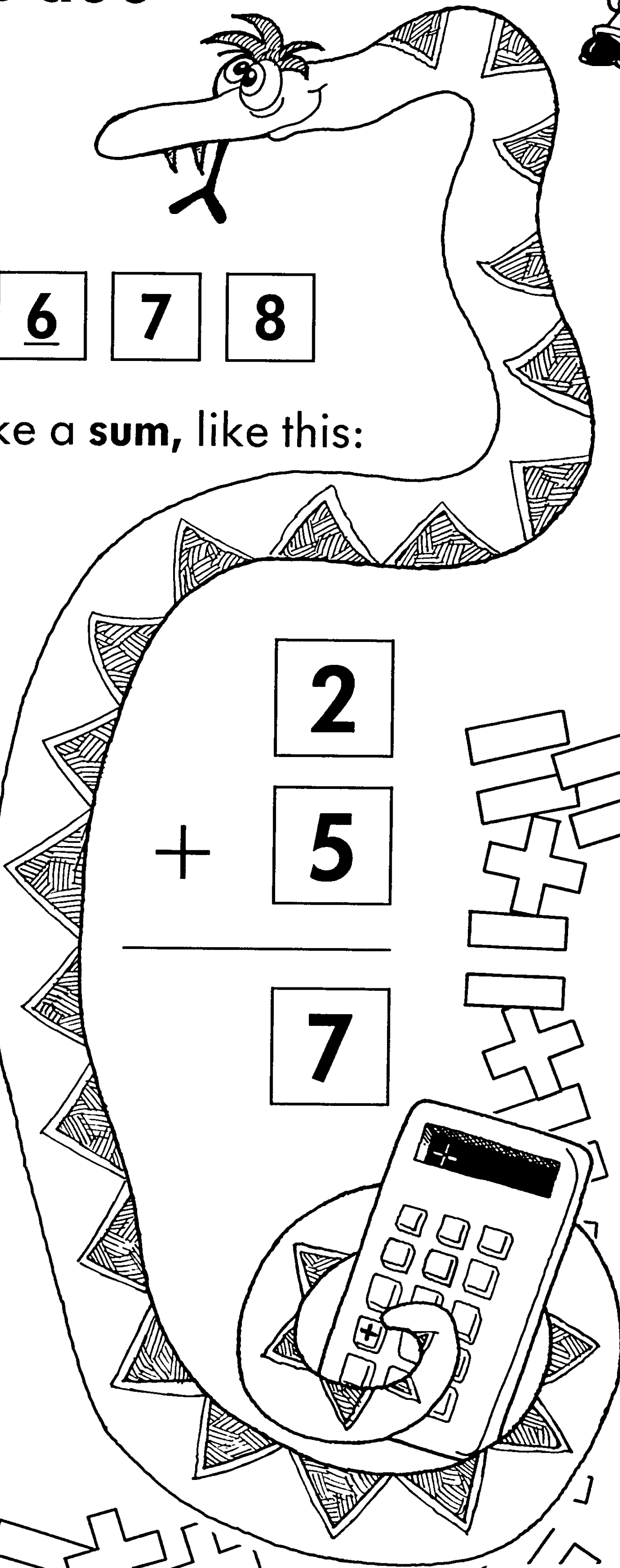
You will need

cards numbered 2 to 8

2	3	4	5	6	7	8

Use three cards to make a **sum,** like this:

$$\begin{array}{r} \square \\ + \square \\ \hline \square \end{array}$$



Find some more **sums**.

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Domino dots

ADDITION

Finding different pairs of dominoes with a fixed overall dot total.

Apparatus

A full set of 28 dominoes is required. These can be made from card if necessary.
Domino paper (special paper 3).

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- Addition facts.
- Addition patterns.
- Recording results.

On domino paper. The dominoes can be recorded using either dots or numbers.

To make a total of 7 dots, the pairs must be 0 and 7 dots, 1 and 6 dots, 2 and 5 dots, or 3 and 4 dots.

0 and 7	[|]	[|]	[|]
	[1|6]	[2|5]	[3|4]

1 and 6	[|1]	[|1]	[|1]	[|1]
	[|6]	[1|5]	[2|4]	[3|3]

2 and 5	[|2]	[|2]	[|2]	[1|1]	[1|1]	[1|1]
	[|5]	[1|4]	[2|3]	[|5]	[1|4]	[2|3]

3 and 4	[|3]	[|3]	[|3]	[1|2]	[1|2]	[1|2]
	[|4]	[1|3]	[2|2]	[|4]	[1|3]	[2|2]

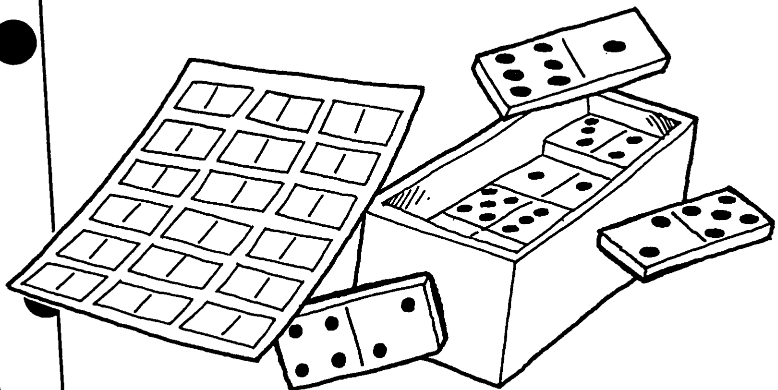
QUESTIONS

- ? How many pairs have a 'double'?
- ? Which pairs have a dot difference of 3?
- ? What dot differences can you find if the dot total is 7?
- ? What is the highest/lowest possible dot total?

EXTENSIONS

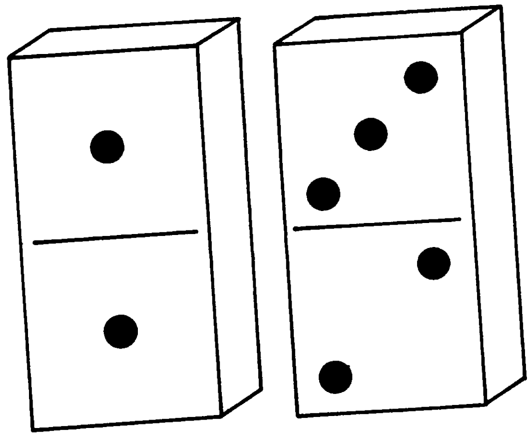
- Try other dot totals.
- Try dot differences.
- Try removing the blanks.

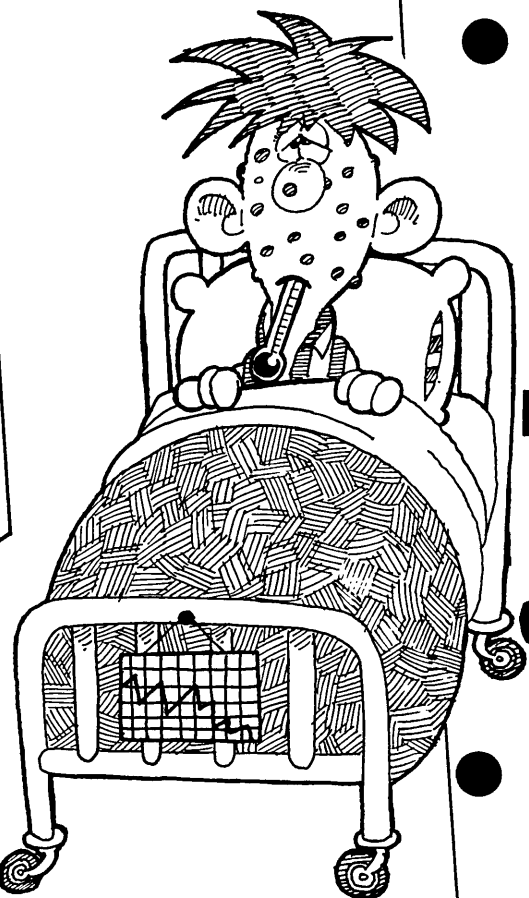
Domino dots



You will need

dominoes
domino paper





Total dots 7

This **pair of dominoes** has **7 dots** altogether.

Find some more **pairs of dominoes** with **7 dots** altogether.

31 Matching

ADDITION

Partitioning of a number in all possible ways.

Apparatus

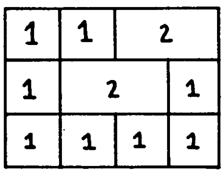
Use number rods to make the different matches.
Squared paper.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

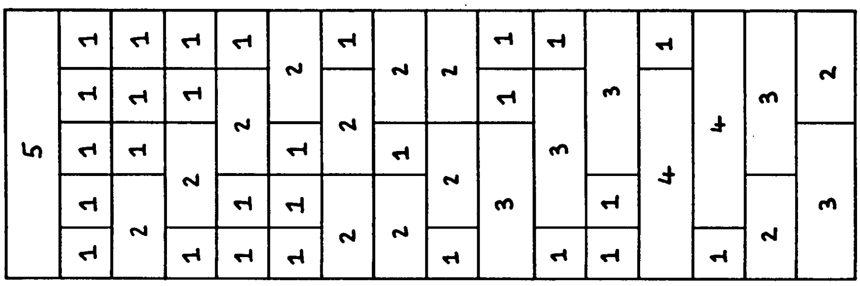
- Addition of two-digit numbers.
- Number patterns.
- Recording results.

On squared paper. The rods can be outlined and then coloured.

The other 4-rod matches are:



There are 15 different ways to match the 5-rod.



Note that the total number of matches for a 4-rod is 8, and the total number of matches for a 5-rod is 16.

QUESTIONS

- How many different ways are there of matching the 4-rod?
- Which matches are the same, but back to front?
- How many matches do not use a 1-rod?

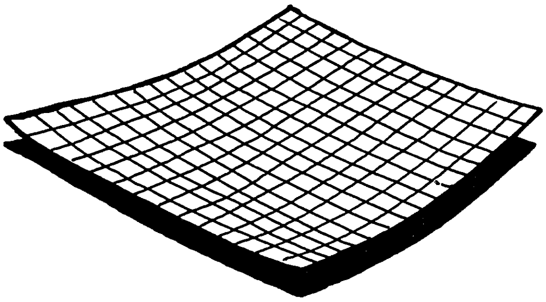
EXTENSIONS

- Try matching other number rods.
- Try matching the longer rods without using 1-rods.

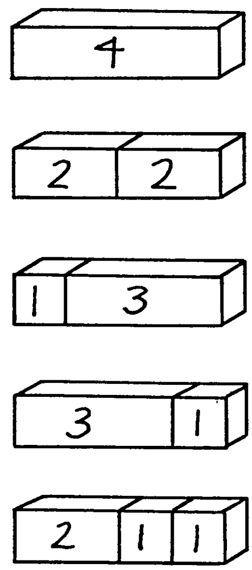
Matching

You will need

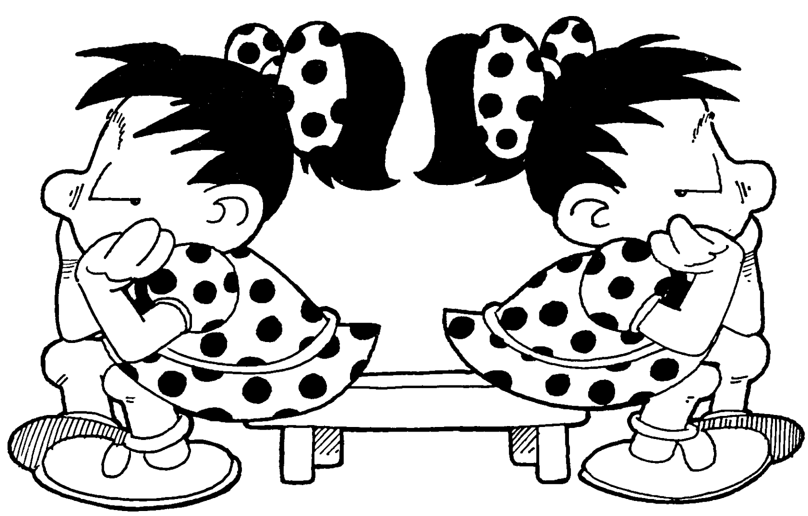
one **4-rod**
other **number rods**
squared paper



Match it with other rods.



Find some more ways of **matching**.
Try matching a **5-rod**.



starting INVESTIGATIONS SPECTRUM MATHS

32 Caterpillars

SHAPE PATTERNS

Making different shapes by joining four regular hexagons edge to edge.

Apparatus

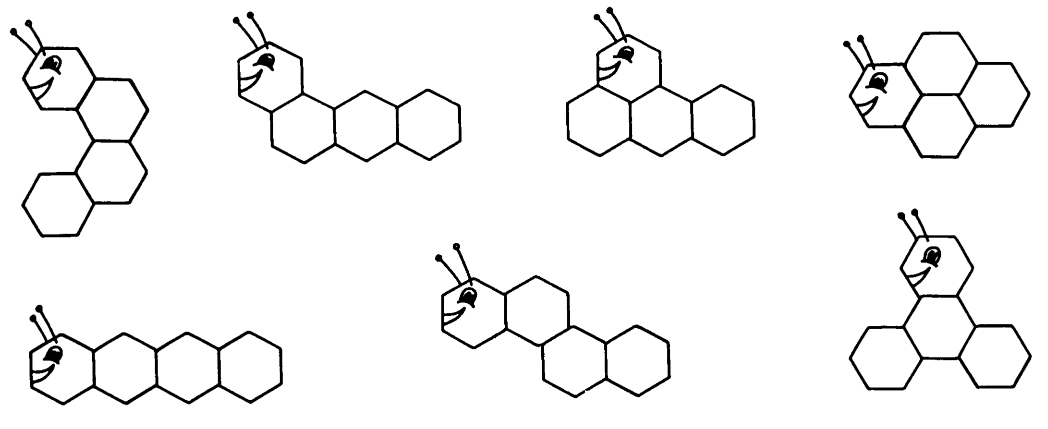
Use regular hexagons cut from card, or hexagonal Logiblocs.
Hexagon paper (special paper 2).

LEVEL	UA	N	SSM	HD
1	●			
2	●	●	●	
3	●			
4				
5				
6				

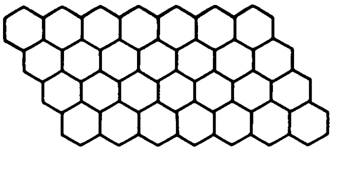
- Patterns using 2D shapes.
- Rotations and reflections.
- Recording results.

On hexagon paper. The caterpillars can be coloured differently.

There are seven different possible caterpillars:



Pupils can cut out the shapes. The seven different caterpillars can be joined to make shapes, e.g.:

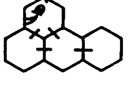


QUESTIONS

? Which caterpillar takes up most/least room on the paper?

? How many joins are there in each caterpillar?

e.g. 4 joins

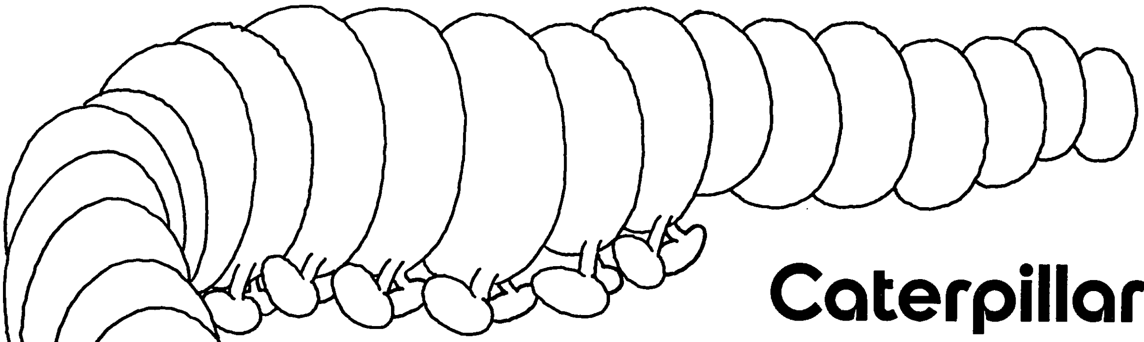


EXTENSIONS

→ Try with 3 or 5 hexagons.

→ Try joining shapes other than hexagons, e.g. squares, triangles.

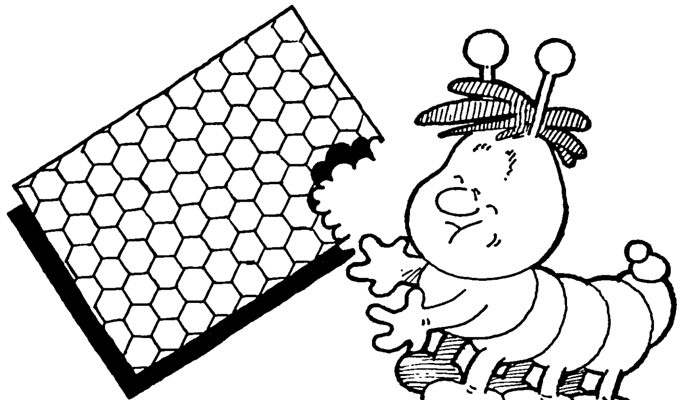
→ Try making different arrangements of hexagons of two separate colours in the same caterpillar shape.



Caterpillars

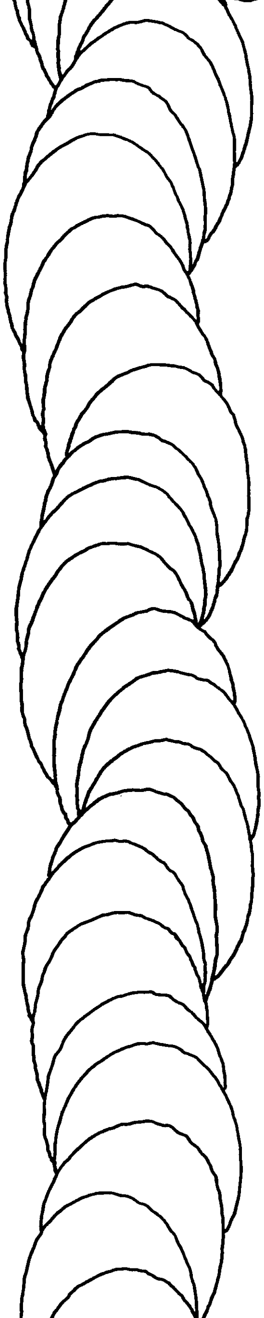
You will need

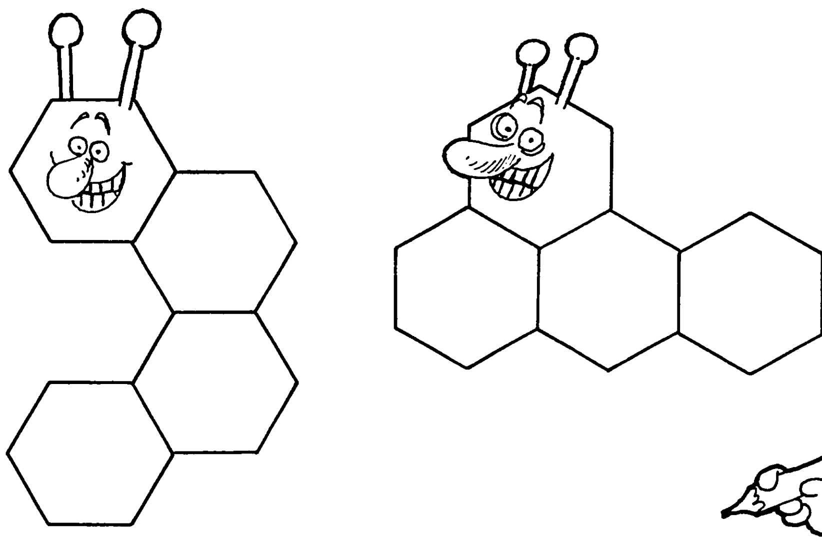
4 hexagons
hexagon paper



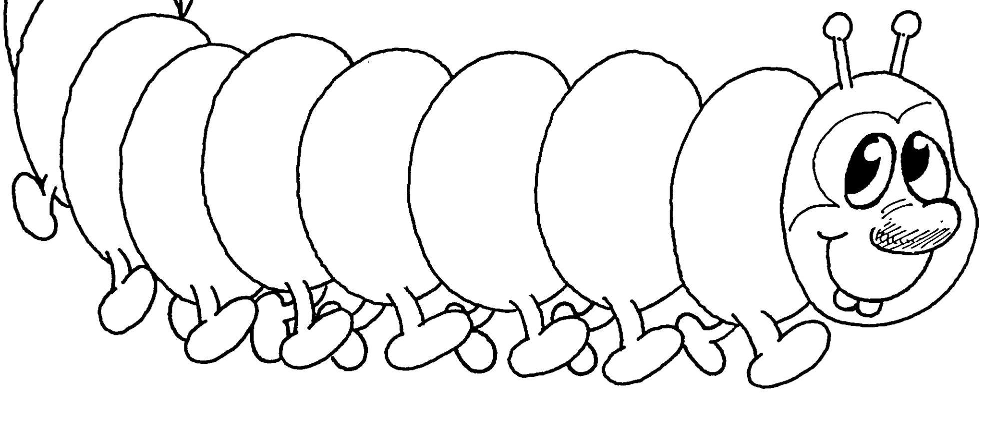
Join them to make **caterpillars**.

Here are two caterpillars:





Join the hexagons to make some more **caterpillars**.



starting
INVESTIGATIONS
SPECTRUM MATHS

33 Finding six

ADDITION

Different ways of expressing a given number in terms of whole numbers and addition signs.

Apparatus

Provide number rods if necessary.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- Addition facts.
- Addition patterns.

On paper or in a book. $6 = 1 + 1 + 4$.

Valuable discussion can arise from considering:

$6 = 1 + 2 + 3$ and $6 = 3 + 2 + 1$

Are they the same?

To work systematically start by adding two numbers, then three numbers, then four, and so on. Number rods can be used to illustrate each sum.

$6 = 1 + 5$ $6 = 2 + 4$ $6 = 3 + 3$	$6 = 1 + 1 + 1 + 3$ $6 = 1 + 1 + 2 + 2$
$6 = 1 + 1 + 4$ $6 = 1 + 2 + 3$ $6 = 2 + 2 + 2$	$6 = 1 + 1 + 1 + 1 + 2$ $6 = 1 + 1 + 1 + 1 + 1 + 1$

QUESTIONS

? What is the greatest number of signs that can be used?

? How many different ways use three signs?

? How many different ways can a 4 be used?

EXTENSIONS

→ Try for different target numbers.

→ Try allowing each number (apart from the target number) to be used up to three times, but not more.

→ Try introducing subtraction signs.

Finding six

Use whole numbers: 1, 2, 3, and so on,

and addition signs: +, +, + . . .

Here are some ways of writing **5**.

$5 = 1 + 4$

$5 = 2 + 3$

$5 = 1 + 1 + 1 + 1 + 1$

$5 = 1 + 2 + 1 + 1$

$5 = 1 + 2 + 2$

Find ways of writing **6**.

starting INVESTIGATIONS SPECTRUM MATHS

34 Towers

COLOUR PATTERNS

Using cubes of two separate colours to build different towers each 3 cubes high.

Apparatus

Use interlocking cubes such as Multilink.
Squared paper.

LEVEL	UA	N	SSM	HD
1	●		●	
2	●	●	●	
3	●			
4				
5				
6				

- Building 3D shapes.
- Patterns based on cubes.
- Recording results.

On squared paper. Draw strips and colour them.

There are eight different possible towers.

R	R	R	W
R	R	W	R
R	W	R	R

R	W	W	W
W	R	W	W
W	W	R	W

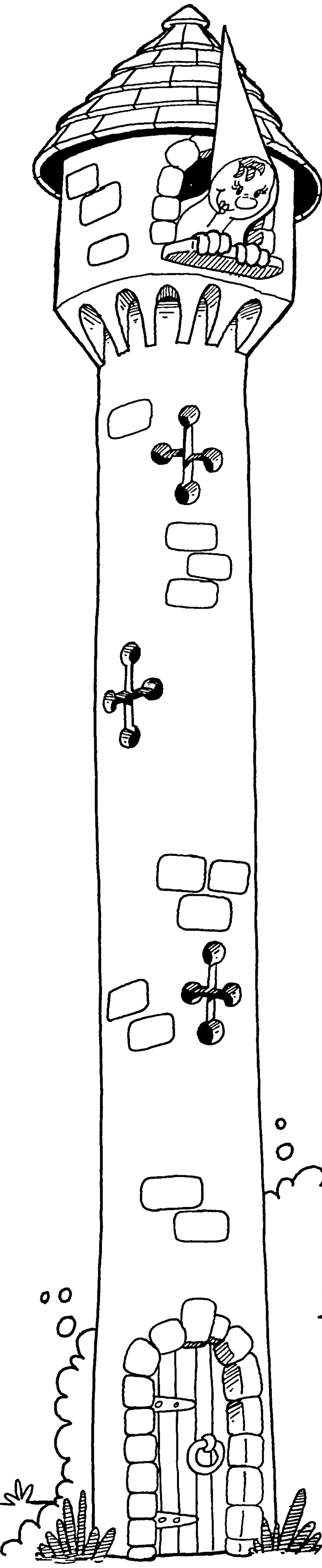
QUESTIONS

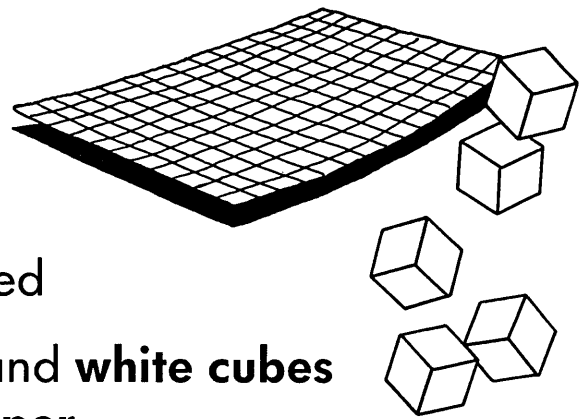
- How many different towers can you make?
- How many different towers have 1 red and 2 white cubes?
- Find two towers that would be identical if one was turned upside down.

EXTENSIONS

- Try towers of height 4 cubes.
- Try using cubes of three separate colours.

Towers



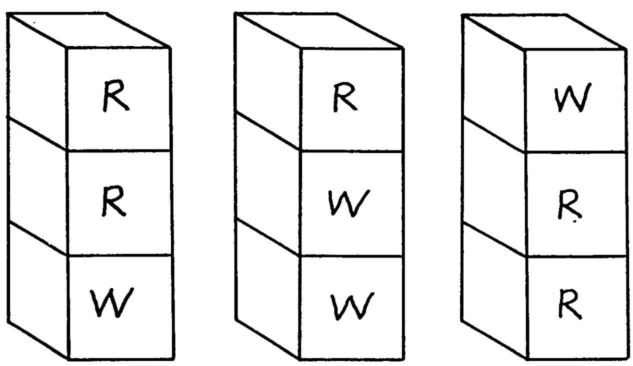


You will need

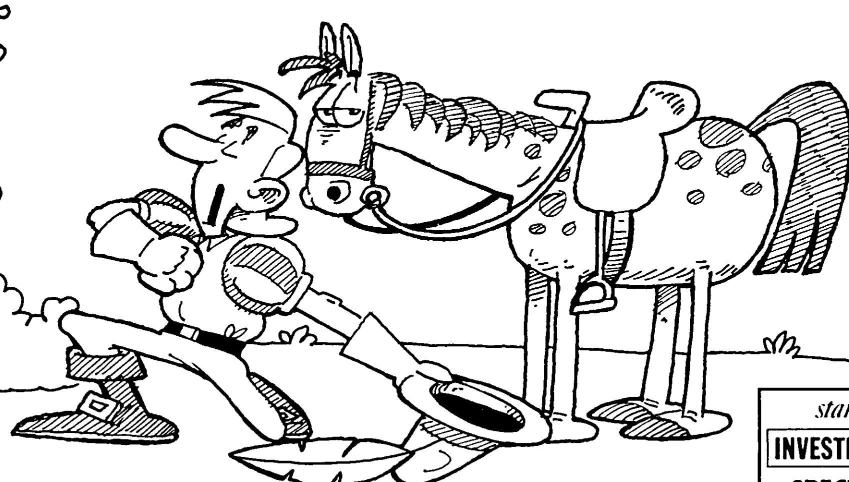
red cubes and **white cubes**
squared paper

Build different **towers** with **3 cubes** each.

Here are some:



Find some more **towers** with **3 cubes.**



starting
INVESTIGATIONS
SPECTRUM MATHS

35 In the window

ADDITION

Addition bonds for numbers up to 18. Sums of pairs of adjacent numbers on a 4 by 4 square.

Apparatus

Use the window paper (special paper 6). If possible, stick the outline windows onto thin card.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●	●		
4				
5				
6				

- ● Addition facts up to 10.
- ● Addition facts up to 20.
- ● Recording results.

On paper or in a book. $5 + 9 = 14$

Encourage a systematic approach. For example, consider horizontal pairs first and start at the top left.

There are 12 different horizontal pairs:

Pair	Total	Pair	Total	Pair	Total
5 9	14	9 2	11	2 6	8
1 5	6	5 4	9	4 1	5
8 2	10	2 0	2	0 4	4
6 3	9	3 7	10	7 3	10

There are 12 different vertical pairs:

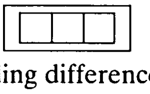
Pair	Total	Pair	Total	Pair	Total	Pair	Total
5 1	6	9 5	14	2 4	6	6 1	7
1 8	9	5 2	7	4 0	4	1 4	5
8 6	14	2 3	5	0 7	7	4 3	7

The following totals are possible: 2, 4, 5, 6, 7, 8, 9, 10, 11, 14.

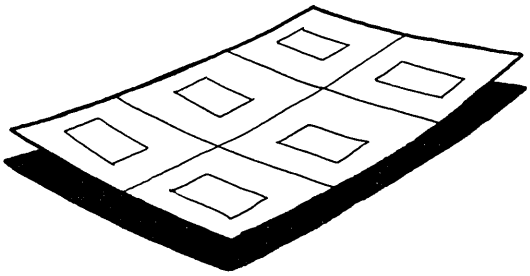
QUESTIONS

- Is it possible to find a total of 14?
- What totals are not possible?
- How many different windows give a total of 9?

EXTENSIONS

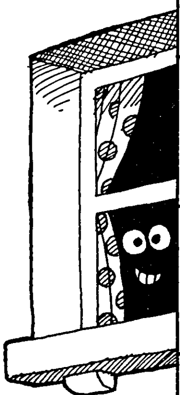
- Try with different arrangement of numbers.
- Try with a different shaped window, e.g. 
- Try finding differences instead of sums.

In the window

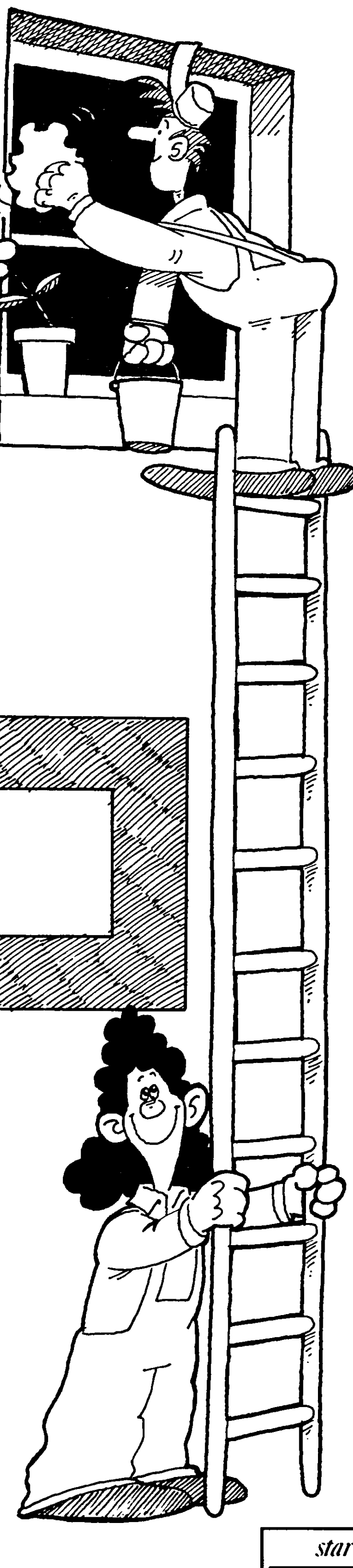


You will need

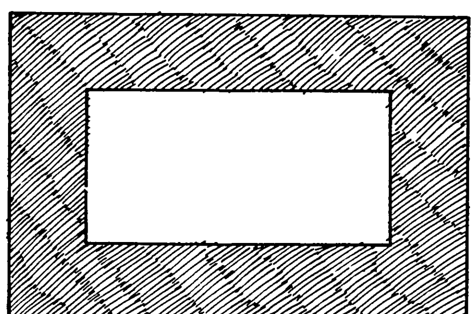
window paper



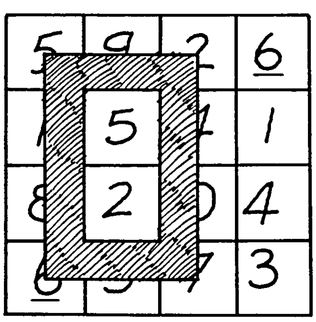
5	9	2	6
1	5	4	1
8	2	0	4
6	3	7	3



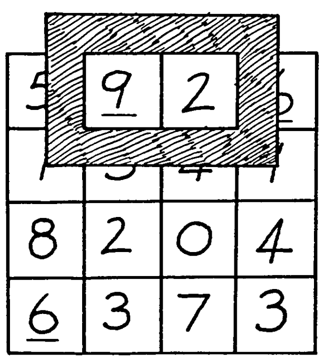
Cut a hole in card
to make a **window**
large enough to show
two squares only.



You can find **totals** like this:



Total 7



Total 11

Find some more **totals**.

36 Lines

NUMBER PATTERNS

Different arrangements of three number rods in a line.
Conservation of number. Conservation of length.

Apparatus

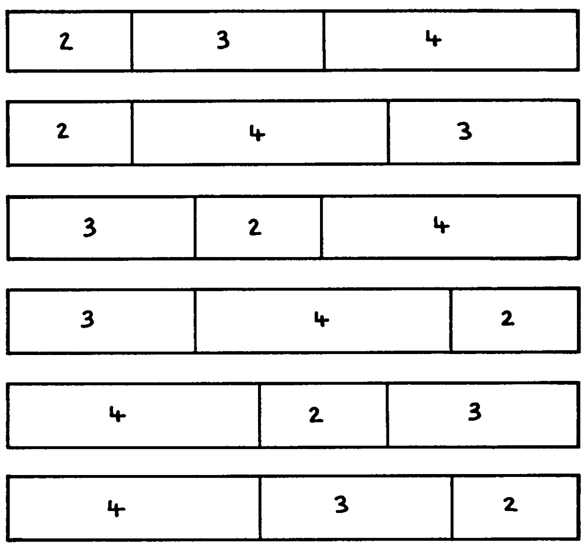
Use 2-rods, 3-rods and 4-rods from either Cuisenaire or Colour Factor.
Squared paper.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●		●	
4				
5				
6				

● Addition facts.
● Number patterns.
● Sorting shapes in different ways.
● Recording results.

On squared paper. The rods can be outlined and then coloured.

There are six different ways of making a line:



Note that 4, 3, 2 and 2, 3, 4 are defined as different ways. The arrangements can be coloured according to the colours of the rods.

QUESTIONS

? What is the total of the three rods?

? How many different lines have the 2-rod in the middle?

? How many have the 3-rod in the middle?

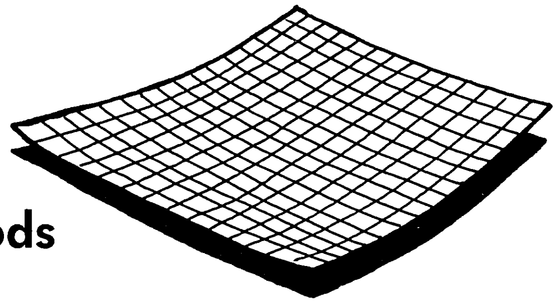
EXTENSIONS

→ Try with three different number rods, e.g. 3, 5 and 8.

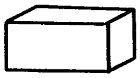
→ Try with four rods.

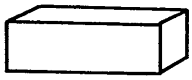
Lines

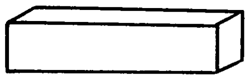
You will need



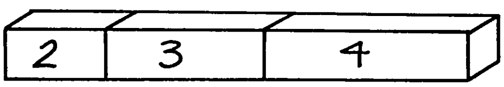
squared paper
these **number rods**

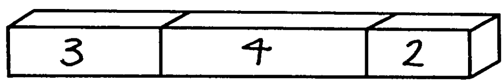
 a **2-rod**

 a **3-rod**

 a **4-rod**

Make a **line** in different ways.





Find other ways of making a line.

One, two

SHAPE PATTERNS

Making different shapes using 1-rods and 2-rods.

Apparatus

Use rods from Cuisenaire or Colour Factor.
Alternatively make a unit square and two 2 x 1 rectangles from card.
Squared paper.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●		●	
4				
5				
6				

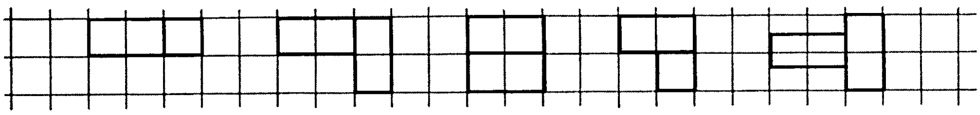
- Arranging shapes in different ways.
- Recording results.

On squared paper. The shapes can be drawn on squared paper and then coloured. For example, the 2-rods can be in one colour and the 1-rod in another.

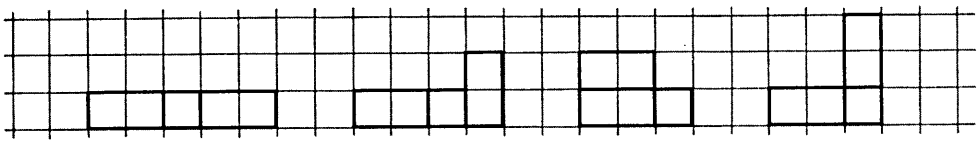
These notes assume that the shapes are flat (two dimensional).

A systematic approach is to first consider shapes made from just 2 rods.

These include:

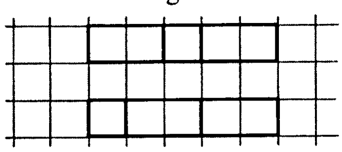


Then, taking each shape in turn, the third rod can be fitted in every possible position:

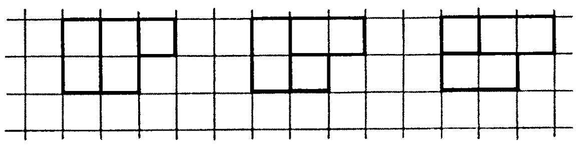


and so on.

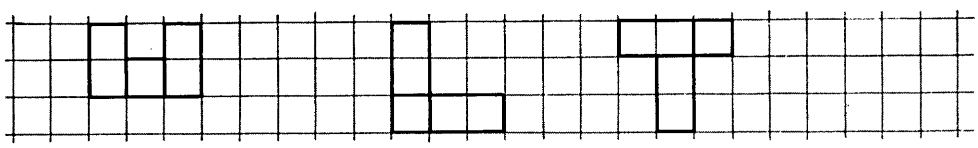
Also, there are different arrangements which make the same shape, for example:



and



Some arrangements make letters of the alphabet.

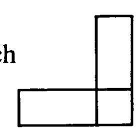


QUESTIONS

- Which shape takes most/least room on the paper?
- How many different L shapes can you make?
- Can you make other letters of the alphabet, e.g. Z, W, V, H?

EXTENSIONS

- Try finding different ways of making the same shape.
- Try not letting the 2-rods touch each other, e.g.:

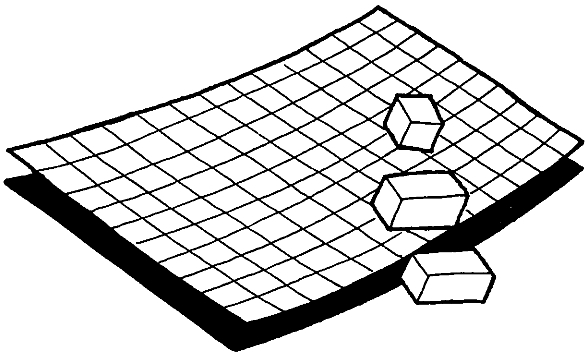


- Try with two 1-rods and two 2-rods.
- Try with one 1-rod, one 2-rod and one 3-rod.

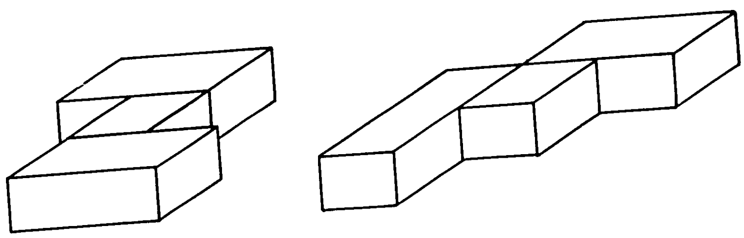
One, two

You will need

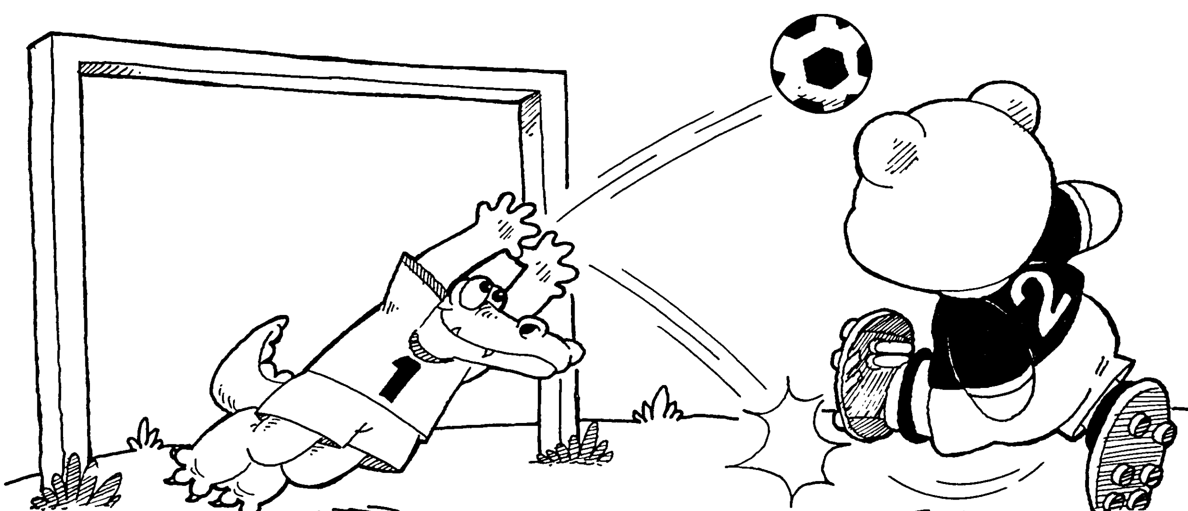
one **1-rod**
and two **2-rods**
squared paper



Join them to make **shapes**.



Find some more **shapes** like this.



38 Can you pay

MONEY

ADDITION

Investigating how many different sums of money can be made when selecting from 4 coins – one each of 1p, 2p, 5p, and 10p coins.

Apparatus

Use 1p, 2p, 5p and 10p coins.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- Addition of money.
- Patterns in addition.
- Recording results.

On paper or in a book. Draw round coins, use sticky circles, or use rubber stamps. Alternatively, write 13p = 10p + 2p + 1p.

The different amounts possible are:

1p	1p	11p	10p, 1p
2p	2p	12p	10p, 2p
3p	1p, 2p	13p	10p, 2p, 1p
4p	Not possible	14p	Not possible
5p	5p	15p	10p, 5p
6p	5p, 1p	16p	10p, 5p, 1p
7p	5p, 2p	17p	10p, 5p, 2p
8p	5p, 2p, 1p	18p	10p, 5p, 2p, 1p
9p	Not possible		
10p	10p		

QUESTIONS

- What amounts cannot be paid?
- How many different amounts can be paid?
- What is the greatest amount possible?
- What amounts can be paid with just two coins?

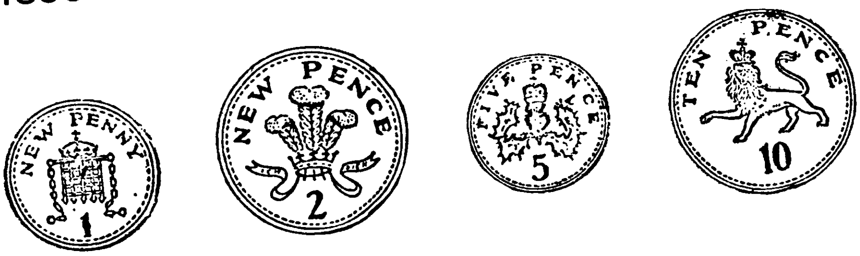
EXTENSIONS

- Try with a different set of coins, e.g. 5p, 10p, 20p, 50p.
- Try with just three coins.
- Try with two coins the same, e.g. 5p, 5p, 2p, 1p.

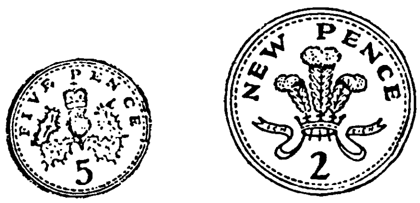
Can you pay

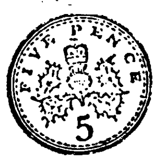
You will need

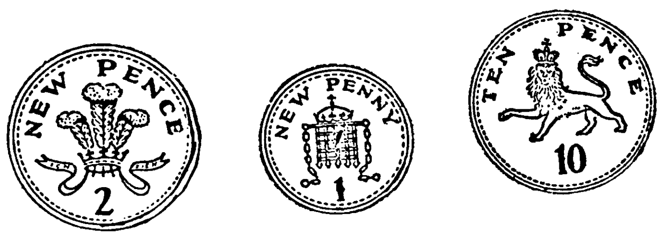
these **four coins** only



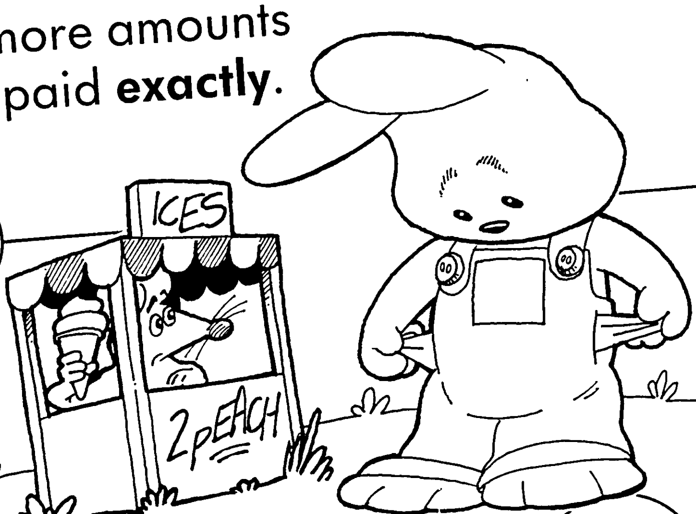
What amounts can be paid **exactly**.

7p with 

5p with 

13p with 

Find some more amounts
that can be paid **exactly**.



starting
INVESTIGATIONS
SPECTRUM
MATHS

39 Spot the difference

SUBTRACTION

Investigation of all the possible differences between the numbers of dots on the faces of two thrown dice.

Apparatus

Use standard 1 to 6 dice.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- Subtraction facts.
- Subtraction patterns.
- Recording results.

On paper or in a book: Difference 3   6−3   5−2   4−1

On squared paper: Difference 3   6 3   5 2   4 1

or 

There are four different ways of obtaining a difference of 2:

Differences 0 to 5 are possible, in the following ways:

0	6 6	5 5	4 4	3 3	2 2	1 1
1	6 5	5 4	4 3	3 2	2 1	
2	6 4	5 3	4 2	3 1		
3	6 3	5 2	4 1			
4	6 2	5 1				
5	6 1					

QUESTIONS

- Which differences are possible?
- How many throws give a difference of 0?
- What differences are possible if the total dots on the two faces of the thrown dice is 8?

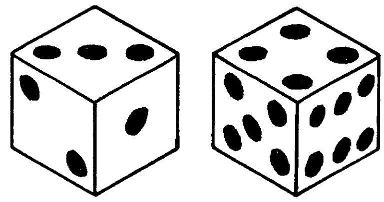
EXTENSIONS

- Try with different dice, e.g. 3, 4, 5, 6, 7, 8 written on the faces of blank cubes.
- Try finding dot totals instead of differences.

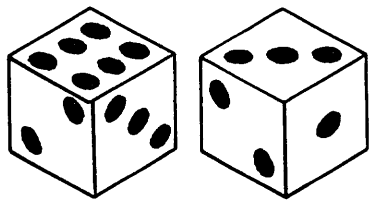
Spot the difference

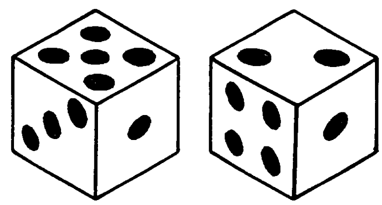
You will need

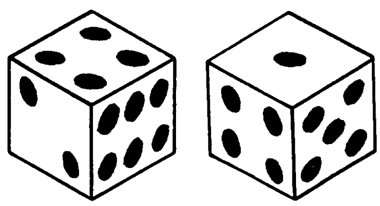
2 dice



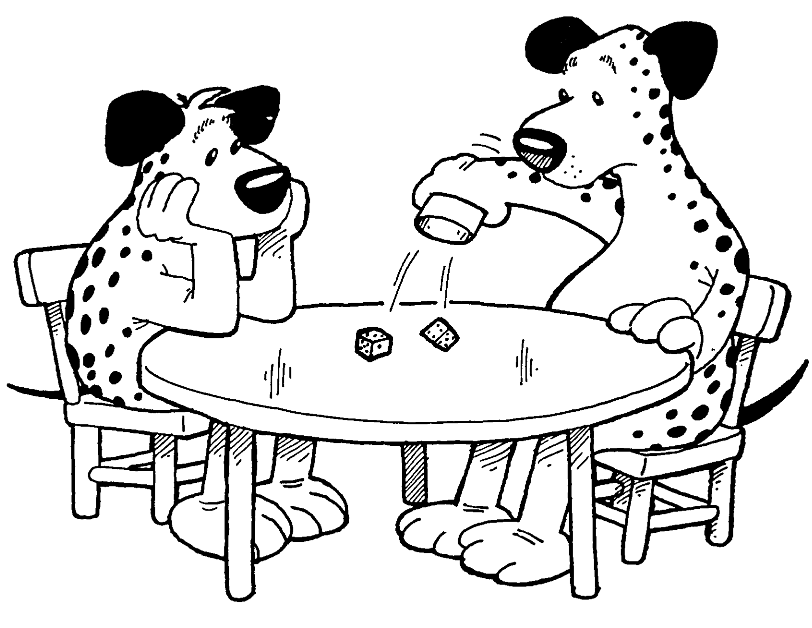
These throws have a **difference of 3**.







Find throws with a **difference of 2**.



40 Twos

PLACE VALUE

Selection of two digits from four to make different two-digit numbers.

Apparatus

Use cards numbered 1 to 9.

LEVEL	UA	N	SSM	HD
1	●			
2	●	●		
3	●			
4				
5				
6				

- Ordering numbers.
- Place value.
- Recording results.

On paper or in a book.

12
14
.
.
.

Twelve different two-digit numbers are possible:

$\boxed{1}\,\boxed{2}$

$\boxed{1}\,\boxed{4}$

$\boxed{1}\,\boxed{5}$

$\boxed{2}\,\boxed{1}$

$\boxed{2}\,\boxed{4}$

$\boxed{2}\,\boxed{5}$

$\boxed{4}\,\boxed{1}$

$\boxed{4}\,\boxed{2}$

$\boxed{4}\,\boxed{5}$

$\boxed{5}\,\boxed{1}$

$\boxed{5}\,\boxed{2}$

$\boxed{5}\,\boxed{4}$

These can be ordered:

12, 14, 15, 21, 24, 25, 41, 42, 45, 51, 52, 54.

QUESTIONS

- How many different two-digit numbers are possible?
- What is the smallest/largest possible number?
- How many numbers have a $\boxed{4}$ in the tens place?
- How many have a $\boxed{5}$ in the units place?
- Which number is nearest to 30?, to 40?, . . .

EXTENSIONS

- Try with a different set of four cards.
- Try with only three cards.
- Try choosing from five cards.
- Try with two cards the same.

Twos

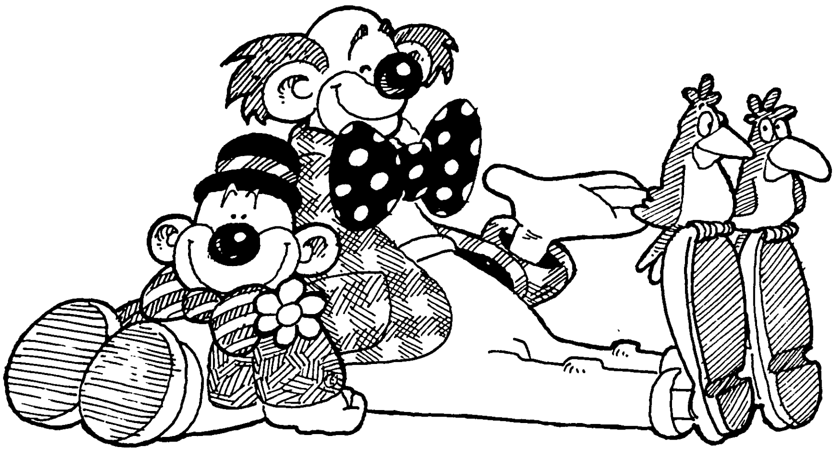
You will need these cards

2	4	1	5

Make **two-digit numbers.**

2	4
4	1

Find some more **two-digit numbers.**



Special paper 1

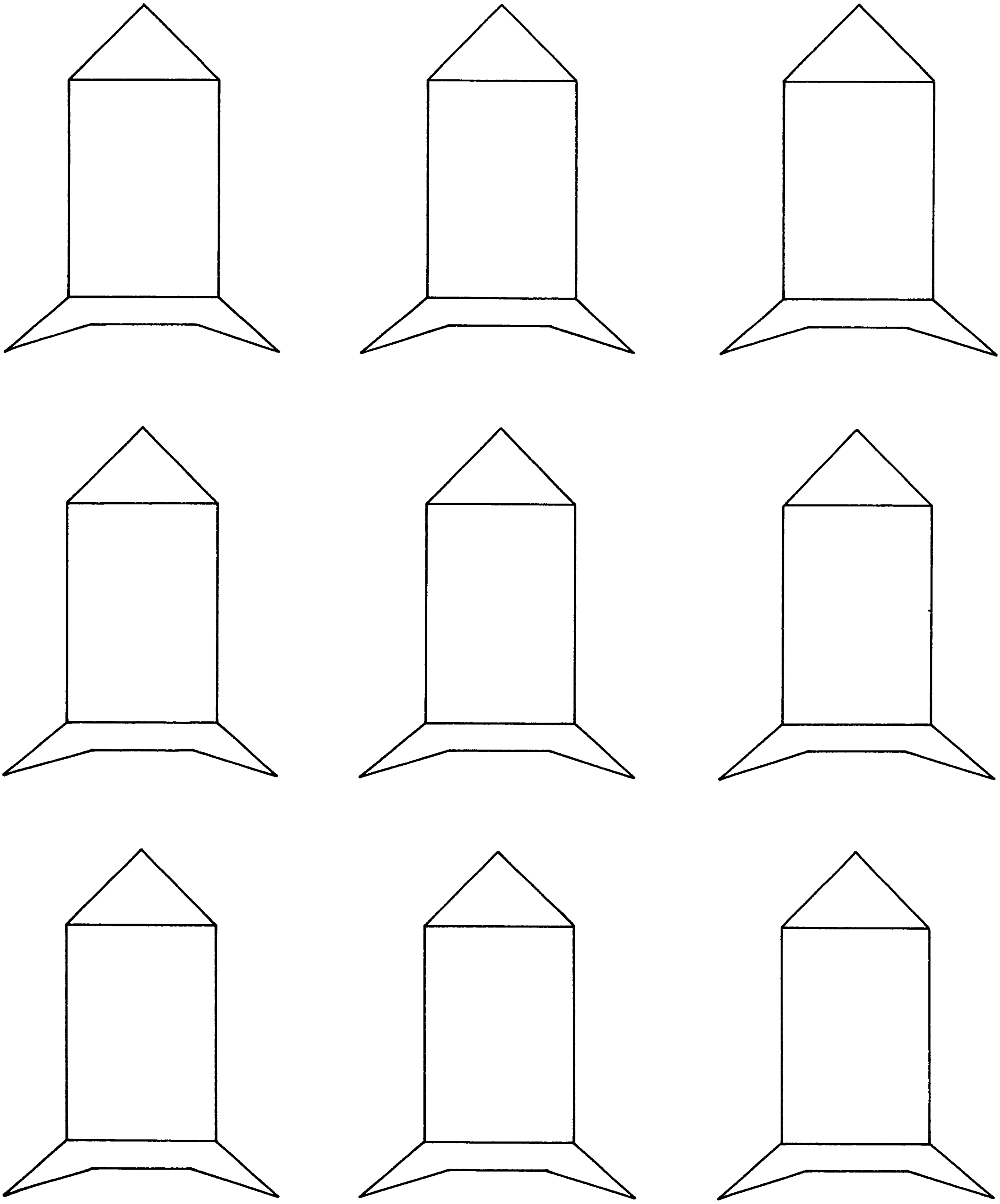
Special paper 2

 (see page 2)

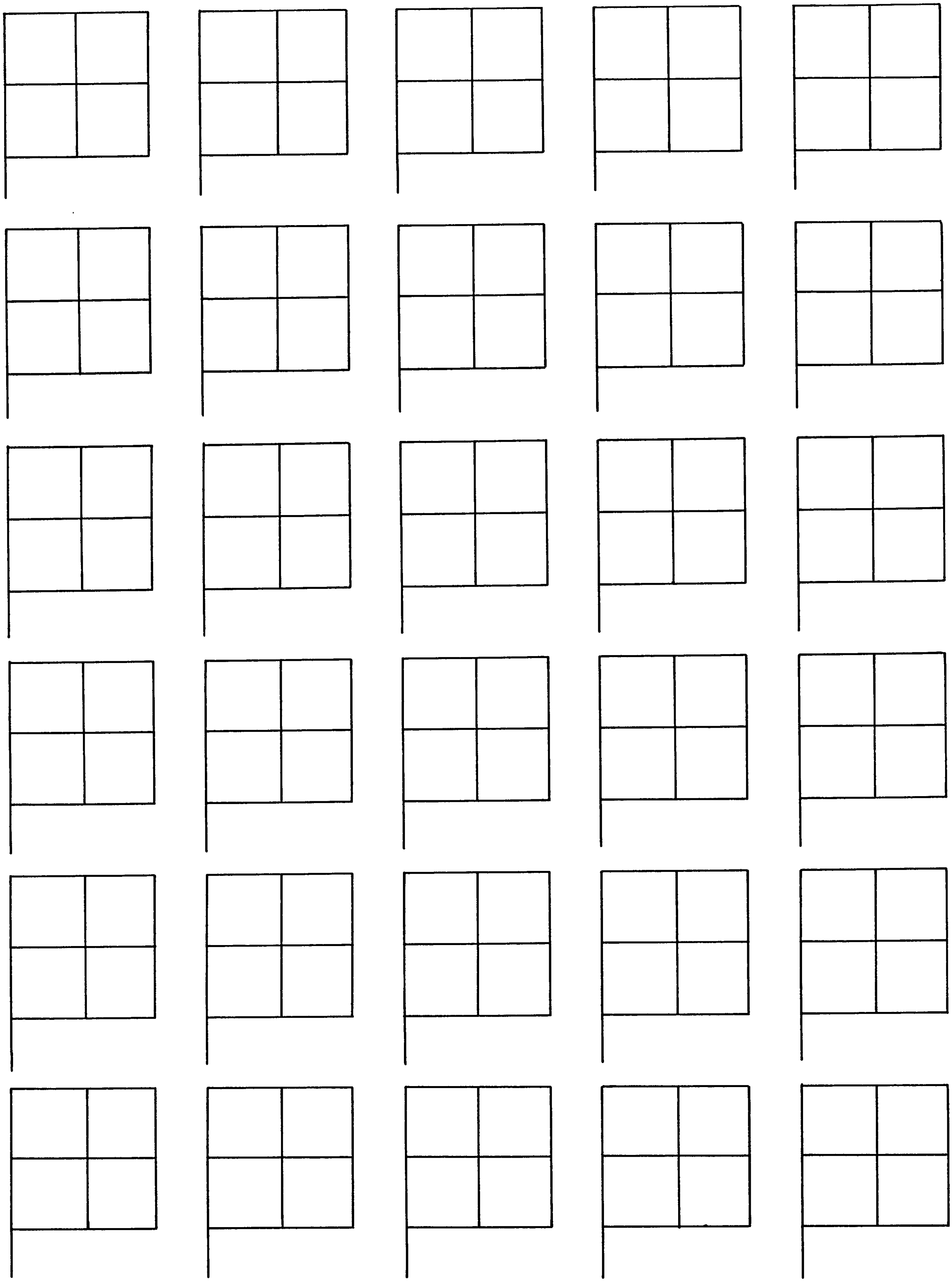
Special paper 3

 (see page 2)

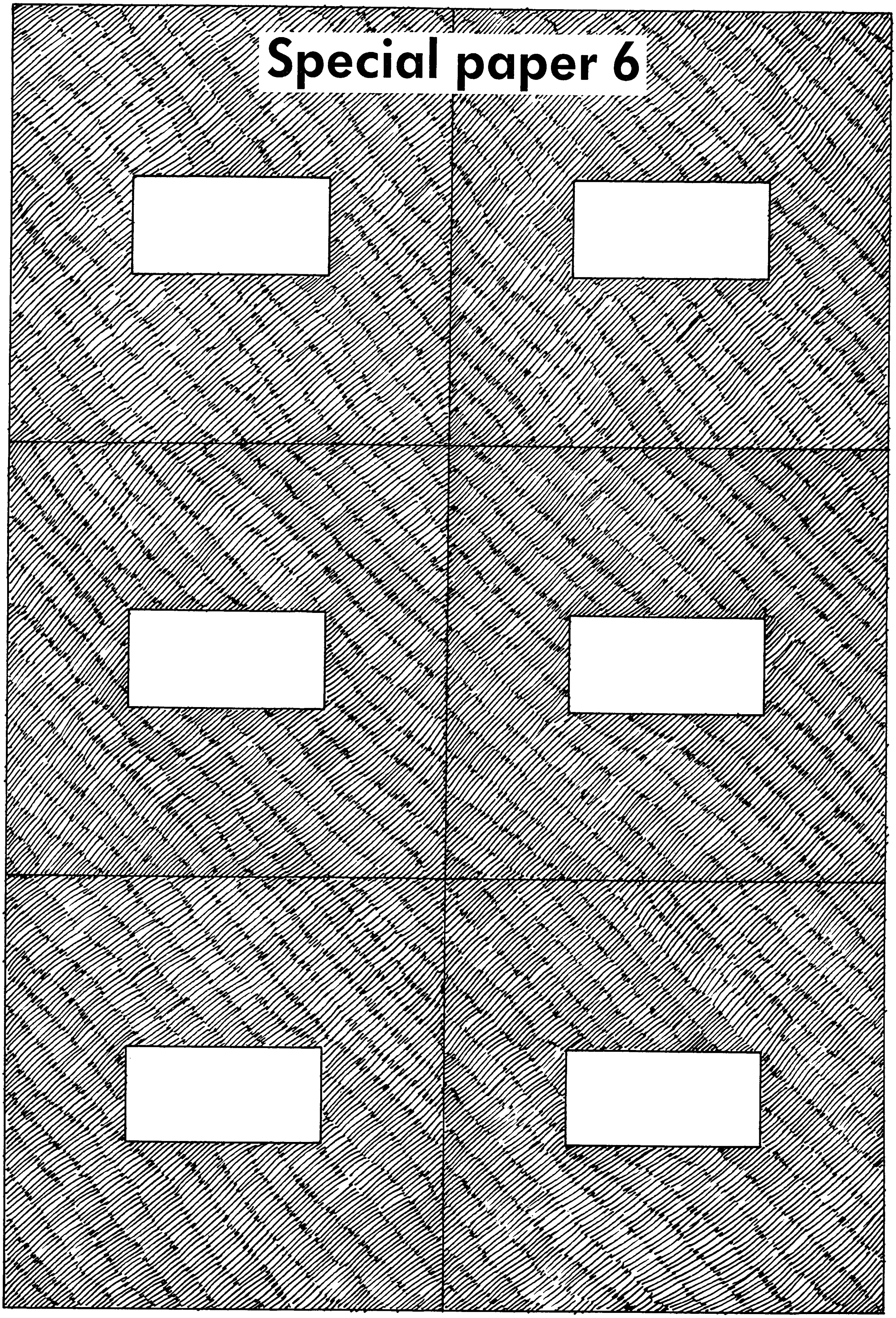
Special paper 4



Special paper 5



Special paper 6



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