Chapter 3

Scale and proportion

Giant earthworm

Human
Size and scale of numbers

Students sometimes struggle to understand the scale of numbers we use in science. As well as the technical process of managing very large and very small numbers there is also the challenge of understanding scale and developing a sense of relative size. The following activities are designed to develop students’ understanding of this concept.

ACTIVITY: Powers of ten
Think about how, in the course of a science lesson, we range from talking about things at human scale to microscopic, to atomic and to astronomical. Students need to develop a similar sense of scale.

• A few years ago a video was produced to address this very issue, called ‘Powers of Ten’. It starts off with a view of people having a picnic on the shores of a lake, and zooms out and then in. It presents a neat view of how scale varies over the range of scales we use in science.

• As a precursor, students can be asked to rank order, in terms of relative size, some of the objects - see the additional document: 3_objects_from_Powers_of_Ten.docx that will be seen in the video. It is useful to do this before students see the video and then let them correct it whilst (or after) watching it. A poster showing different objects from the sequence is also available to support this discussion.

• Draw out from discussion any points they were unaware of or surprised by.

Make a note of any points that arise that may have implications for teaching subsequent topics and share these with colleagues.

ACTIVITY: Sense of scale wall display
This activity is designed to develop a sense of scale in students as they proceed through studies in science. It is taken from the kind of display that is not uncommon in history classrooms, with a timeline running along one (or more) walls.

The idea is to set up a sequence of points along the wall at regular intervals with lengths (shown as numbers) going up in powers of ten, i.e. 1m, 10m, 100m, 1km, and down as well, i.e. 100mm, 10mm, 1mm, etc. This is extended up to astronomical distances and down to atomic ones. These are then supplemented with examples of things with that length as topics are considered.

For example, when working on a topic such as Cosmology, measures such as diameter of Earth’s orbit, diameter of the Solar System, diameter of the Milky Way can be referred to, as
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well as ones such as wavelength of red light. Students can be challenged to research examples of objects to include on the display; they can estimate and then find out where things would fit on the scale.

The aim is to develop students’ sense of scale by repeated reference to visual cues.

The LSIS engineering resource ‘Estimating length using standard form’ provides a set of cards which students can sort into the correct order to increase their understanding of the sense of scale. These could also form items on the wall display.

ACTIVITY: Prefixes
The aim is to gradually develop an understanding of standard prefixes.

- Make the point that the same prefix means the same thing when applied to a different unit, so one set of prefixes is all that is needed.
- Also explain that prefixes increase by a factor of a thousand (or \(10^3\)) each time. The prefixes poster - see the additional document: 3_prefixes_poster.docx may help to support this process.

You may decide that expressions in relation to 1m (e.g. 1mm = \(10^{-3}\)m, 1µm = \(10^{-6}\)m, 1nm = \(10^{-9}\)m) should be on permanent display or that they should be added when appropriate. On the one hand there is merit in familiarising students with powers; however some students find them off-putting and ‘developing a sense of scale’ is not synonymous with ‘learning prefixes’.

ACTIVITY: Standard form
This develops from the previous point and supports students to express large and small numbers in standard form.

- Remind students, if necessary, of the meaning of indices and make sure they understand the significance of numbers such as \(10^2\) and \(10^6\).
- Now explain that their challenge is to turn each of the following numbers into something called standard form, in which it has to be two numbers, multiplied by each other.
- The first one has to be between 1 and 10 and the second one 10 to a certain power.

Demonstrate a couple and then ask them to work in small teams to try these:

90, 28, 14, 14.5, 14.25, 750, 745, 6, 6.2

- Take feedback and ask students, as appropriate, to explain their reasoning.
- If appropriate then progress to multiplication and division.

Reinforce points about standard form with wall displays and occasional activities.

The ‘Indices and Standard Form’ unit from CMP is designed to support the development of these skills.

See the additional document: 3_scale_poster.docx
Strategies to develop skills of estimation

Estimation is a key skill. It is a quick and easy way of coming up with a ‘ball park figure’ which may be sufficient to make a decision. For example a quick estimate of total appliance current ratings may show that the maximum load for a circuit will be significantly exceeded. Students also need a quick way of checking that what their calculator says is a plausible answer to the calculation they entered.

The following activities are designed to develop the dialogue with students with respect to the skills of numeracy, allow them to make connections with other areas of the curriculum and inform your planning and delivery.

ACTIVITY: Making the case for estimation

Ask a group of students to suggest whether there is merit in being able to estimate the answer to a calculation.

Ask whether they’ve ever made a mistake through a calculated answer being out by a factor of ten and suggest that doing a quick estimate is a useful check.

Now ask for suggestions for useful strategies. If necessary provide some examples of calculations for which an estimated answer would be useful. Examples might include:

- 194 x 314
- 200 x 3000
- 626 + 594 + 608 + 612 + 589
- 348 + 49 + 355 + 47 + 341 + 53
- 9% of 538
- 1.4 x 500

Be aware that students may have been taught and mastered different ways to carry out multiplication and division, all of which are correct – here we are trying to get them to put into words the strategies they use to estimate the answer.

Scientists often find it useful to have an idea of the scale of something, often making use of rough and ready measurements. The next activity may also be helpful in securing the idea of estimation.

ACTIVITY: Rounding off and counting zeros

Explain to students that one of the most common strategies is rounding off, so 34 becomes 30 and 35 becomes 40. The purpose is to get to numbers that are reasonably close to the original but much easier to work with.

For example, 22 x 38 becomes 20 x 40, which is 800.
Points to watch:
• There’s no point in rounding off 3647 to 3650, as it’s not much easier. It’s 4,000, or maybe 3,600 (especially if the other number is really easy, such as 10)
• Watch the first number – it’s the most significant
• Provide students with some examples – or, even better, ask them to produce examples. This will reveal more of their understanding.

Explain about ‘counting zeros’. For example, 700 x 5000 becomes 35, followed by five zeros.

Points to watch:
• Sometimes multiplication of the significant numbers produces further zeros. For example, 800 x 5000 becomes 40 followed by five zeros, i.e. 4,000,000 (with six zeros).

For example, estimate and then calculate the weight of a mass of 3200 kg if \( g = 9.81 \text{ m/s}^2 \)

The ‘Rounding and Estimating’ resource from MEP includes further examples and activities around rounding and estimating.

ACTIVITY: Similar numbers and grouping
Sometimes there may be a similarity between numbers. For example, 686 + 702 + 714 + 696 + 712 + 688 can be estimated as 700 x 6. Similarly 252 + 496 + 510 + 255 + 491 can be approached as (3 x 250) + (3 x 500).

What is important here though is trying to move students from a procedural approach (“We do this type of sum like this”) to a problem solving approach (“Have a look at this and see if you can suggest a way of approaching it”). This not only encourages students to be more creative but also to articulate why they are doing what they are doing.

Students may well learn more from each other’s explanations; the skill of the teacher is to facilitate the discussions.

For example, estimate and then calculate the total external surface area of a building with the following areas open to the air: 32 + 18 + 32 + 18 + 42 + 42

ACTIVITY: Decimals, percentages and fractions
1.6 x 32 can be seen as close to ‘one and a half’ times 32, i.e. 32 + 16.
Similarly 0.9 x 44 is close to 1 x 44.
If students are happier working in percentages or decimals they can easily switch, so 10% is 0.1 (and also 1/10).
For example, if a light bulb is 19% efficient and is supplied with 2400J of energy, estimate and then calculate the useful output and the wasted output.

Resources which may help develop this area include:
• The MEP resource ‘Arithmetic: Addition and Subtraction of Decimals’
• The Mathcentre’s ‘Numeracy Refresher’ publication provides more examples of calculations using estimation
Proportion and ratio are important ideas in science, yet some students find them tricky. They may understand very well that one thing is greater than another but not so well that it is the factor by which it is greater that is important. Some students will use the term ‘directly proportional’ to mean simply that one factor increases if another one does.

ACTIVITY: Ratio – the Cornish Cooler
Introduce this recipe for a (non-alcoholic) cocktail:
4 parts cherries
6 parts aloe vera juice
4 parts cranberry juice
2 parts lime juice
1 part honey

• Ask why the recipe uses parts rather than specific volumes and draw out that people might want to make different quantities.
• Explain that whether you’re making one glass or a large jug that the ratio of the different ingredients is what is important.
• Ask students to identify proportions of one ingredient to another, such as “the volume of cherries is twice the volume of lime juice.”
• Explain that ratio is expressed as two numbers, e.g. cherries to lime juice is 2:1.
• Add that the ratio is in the smallest pair of numbers that represents that relationship, so aloe vera juice to cranberry juice is 3:2 rather than 6:4.

Example: if laboratory chemicals, such as sulphuric acid of a particular concentration, are being made up, the ratio of the chemicals needs to be right. A certain concentration of acid always needs the same proportion of water.

The ‘Ratio Makes Sense’ materials offer further examples of the use of this concept.

ACTIVITY: The X factor
Explain that ratio doesn’t tell us how much bigger one thing is than another, but how many times bigger. This multiplication factor is critical. If we were investigating, for example, the hypothesis that ‘taller babies grow into taller adults’ it would be useful to know the ratio of the baby’s height to the adult height. If the research indicated that all people were, generally 3.6 times taller at 18 years than they were at 7 days, that would be a useful finding.
Understanding proportion and ratio

A useful starting point with this is to say ‘if I double this factor, will that one double as well?’ This is important when it comes to experimental design and selecting the size of variables.

Strategies:
- Test ideas out in practical contexts, e.g.
  - Will doubling the load on the spring double the extension?
  - Will doubling the voltage across a resistor double the current flowing through it?
  - Will doubling the force acting on a mass double its acceleration?
- Thinking ideas through by reasoning and then proving by calculation, e.g.
  - If I double the distance I travel at a certain speed double the time for the journey?
  - If I double the force acting on an area will it double the pressure?

Example: Hooke’s Law. If it is possible to multiply each load by the same factor and arrive at the extension, then the relationship is directly proportional.

The ‘Ratio Makes Sense’ materials offer further examples of the use of this concept.

**ACTIVITY: Ratio of area and volume**

Some ratios are relatively easy to identify, such as the relationship between the height of a seedling and the height of a full grown plant. However sometimes students need to explore ideas such as the ratio of areas or volumes.

- Ask students to gather, or present them with, leaves from the same species of plant but of different sizes.
- Ask them to select a pair, with similar shapes and different sizes and estimate how many times greater the surface area of the larger one is than the smaller one.
- Then ask them to check their estimation. A good way is to use squared paper, draw round the leaf and count the number of squares covered or more than half covered. Students may well be surprised that a leaf doesn’t even have to be half as wide again and half as long again to have twice the surface area.

Example: This process could be used when exploring the factors affecting the effectiveness of plants in gathering light for photosynthesis.

There is a similar argument relating to volume.

Ask students to make two cubes of plasticine, one of side 1cm and the other of side 2cm. They should now work out the ratio of surface area to volume. Again if they say beforehand what they think the comparison will be, the answer will be more significant.

Example: limiting factors on cooling and size of animals.

The ‘Boxes’ resource includes a number of practical activities around surface area and volume which you may wish to use to secure students’ understanding.
Understanding proportion and ratio

**ACTIVITY: Proportion and the big picture**

Students often work with sets of numbers related to the scale of the classroom (for the good reason that we give them data generated from their own activities). However this may mean that they don't gain experience of working with large sets of numbers.

- Ask students to imagine that they were trying to identify popularity of birth months.
- Ask them to indicate which month of the year they were born in and display this as a bar chart.
- Ask students to identify which were the more popular months and which the less popular. It is likely that some months are underrepresented and others overrepresented.
- Now ask them if this could be used to draw conclusions about the population overall. Draw out that it would have limitations, principally due to the sample size.
- Ask what would happen if data from the whole school population was used and draw out that it would be more representative. The point may be raised that using a school population is flawed as it only includes people of a certain age range. Acknowledge this as a fair point and say that if such surveys are being done professionally they have to be balanced against a range of factors.
- Explain that sampling is used widely; it is unusual for any large scale research to count everyone eligible.

Example: understanding how sampling data can be used to identify trends across a whole population, such as the proportion of people of a certain age who suffer from heart condition.

There are more activities using ratios and proportions in the Mathcentre’s *Numeracy Refresher* publication.
Proportion, inverse proportion and graphs

One way of testing for proportionality is, of course, to look at a graph of the data. Some factors in science, however, are inversely proportional and students also need to understand these, including ways of interpreting this data by graphing. The following activities explore key features of this process.

**ACTIVITY: Proportion and graphs**

A good way of identifying whether two factors are directly proportional is by drawing a graph. Students may think that simply having a positive gradient suggests such a relationship: it should be made clear that it needs to be a straight line of positive gradient going through the origin (refer to other sections in this publication on lines of best fit and the use of the origin).

*Hooke’s Law* provides a good example of this. The graph of load against spring length does not go through the origin but that of load against extension does (as long as the elastic limit is not exceeded). Both are straight lines with positive gradients but only the latter goes through the origin.

The SEP/Gatsby resources on [Interpreting Graphs](#) can be used to support the development of the skill of interpreting graphs further.

**ACTIVITY: Inverse proportion**

Students will also meet situations in which variables are inversely proportional.

- Ask students to imagine that they were travelling to a place which was, say 10km away. If there is a convenient location then use this but try to make it one in which the distance is an easy number to manipulate.
- Ask if you were to cycle there at 10km/hour how long the journey would take. Display the figures for speed and time.
- Now ask students to repeat the process by imagining that they are travelling there on a scooter at 20km/hour, car at 40 km/hour, motorbike at 80km/hour, etc. Draw out how the figures are changing.

Responses might be:

- As one increases the other decreases
- As one doubles, the other halves
- Multiplying one by the other always give the same answer

Explain that such patterns indicate inverse proportionality.
‘The Greenest Route’ resource provides a number of activities that involve students using a range of data, converting units and using inverse proportion.

You might also like to consider the example of \( V = IR \). Students can explore how using a fixed voltage supply to make current flow through various different resistors results in a different sized current flow.

**ACTIVITY: Inverse proportion and graphs**

If students have carried out an experiment into variables that are inversely proportional they can be asked to look at the data and suggest what the graph might look like. Constructing such a graph is a good exercise in drawing a line of best fit that is a curve and be the basis for useful discussions about whether sufficient data points have been gathered and whether any look to be anomalous. However as the result is a curve with a negative gradient it doesn’t prove inverse proportionality.

Example: \( pV = constant \). See the sections on interpreting graphs to support students in making sense of the graph produced: they should be relating this to particle behaviour. The Practical Physics website provides an explanation of the relationship \( pV = constant \).

**ACTIVITY: Reciprocals**

Students may not have met this concept previously, in which case ask them to evaluate \( 1/x \) if \( 1 \leq x \leq 10 \) and look at the pattern in the numbers.

Then ask them to find the reciprocals of one of the sets of data from an experiment in which the variables are inversely proportional.

For example, with comparing pressure and volume of a fixed mass of gas, find values of \( 1/p \).

Finally ask them to graph the reciprocal of one variable against the other variable (e.g. \( 1/p \) against \( V \)). The result should be a straight line with a positive gradient, going through the origin.

Part two of the SMILE resource on graphs includes resources to support the drawing of reciprocal graphs.

The BBC GCSE Bitesize also provides maths resources which students may have met previously.
What are your priorities or next steps in exploring the development of the students’ use of scale and proportion in your department?

After reflecting on the activities you have explored in this chapter, and the errors that your students commonly make, you might consider:

- Trying out various strategies in different contexts, perhaps with different groups of students
- Providing feedback to the rest of the science team, sharing your experiences and discussing consistent approaches to aspects of numeracy that your students find difficult
- Engaging in a dialogue with colleagues in the maths team to consider common approaches to this aspect of numeracy, when relevant aspects are taught in mathematics lessons and how this matches the mathematical requirements of the science GCSE and GCE courses.

Chapter 4 of this iBook explores some possible next steps in more detail and encourages you to create an action plan to secure further development.