

SMILE WORKCARDS

Similarity and Enlargement

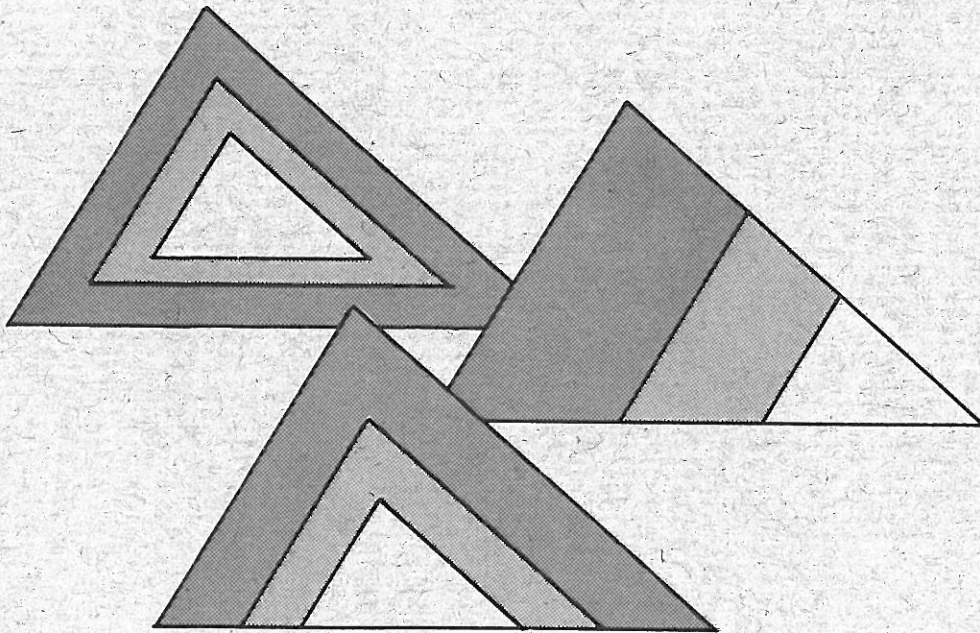
Contents

	Title	Card Number
1	Sorting Triangles	1905
2	Domino	46
3	Double-Up	1388
4	Shapes That Can Grow w/s	1759
5	Scale Factor	838
6	Four Pentominoes	1928
7	Areas of Similar Shapes	1559
8	Similar Triangles	2027
9	Lengths of Similar Objects	1259
10	Nine Pentominoes	1929
11	Similarity Problems	1560
12	Negative Scale Factor	845
13	Similar Solids	1261
14	Matrices and Area	1922

Smile 1905

Sorting Triangles

Sort the triangles in this pack into sets of similar triangles.

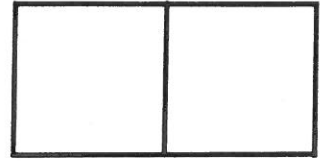


What is special about similar triangles?

You will need 2cm squared paper.

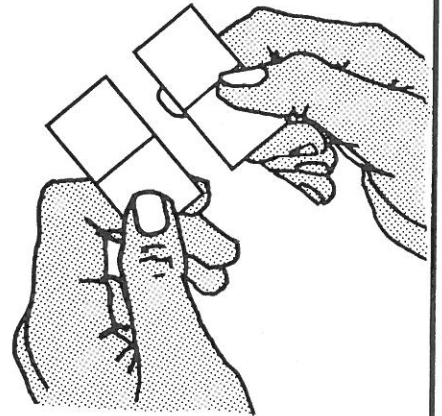
DOMINO

A domino is the shape made by joining 2 squares, edge to edge.

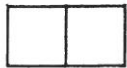


A domino is a special sort of rectangle. Make some dominoes.

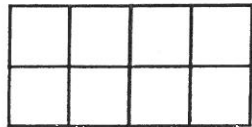
- 1) Fit 2 dominoes together to make a 2×2 square.
Draw this square in your book.
- 2) Fit dominoes together to make some larger squares.
Which squares can you make? *Draw them.*
Which squares can't you make? *Why not?*
- 3) Dominoes are always twice as long as they are wide.
*Things which are the same shape but a different size are called **similar**.*



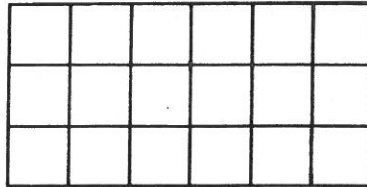
1×2



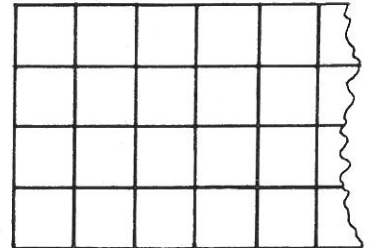
2×4



3×6



$4 \times$ ■



(a) How long is this domino? →

(b) If a domino is 10cm wide, how long is it?

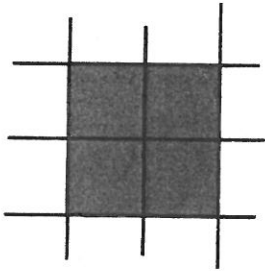
- 4) How many small dominoes do you need to make the similar larger ones, which are drawn above?
Draw sketches to show your answers.

How many small dominoes would you need to make a similar 5×10 domino? A similar 6×12 domino?

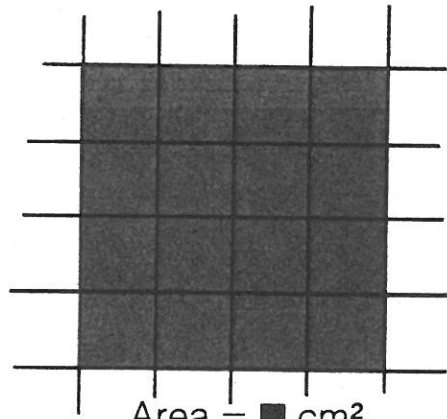
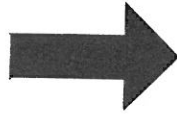
Double Up

Double the lengths of the sides of a square you get a bigger square!

1. Copy these and write down the areas



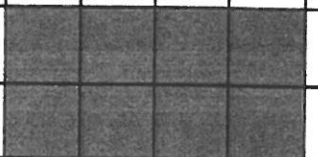
Area = 4 cm²



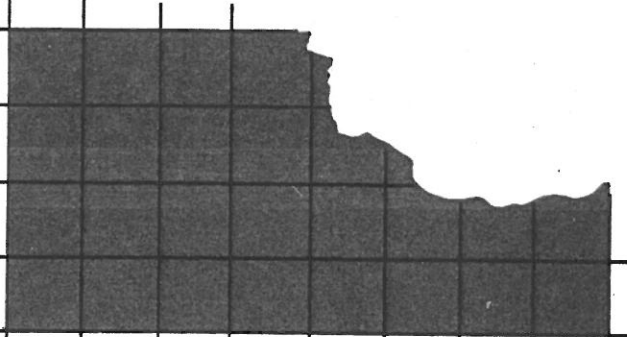
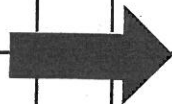
Area = ■ cm²

Copy these shapes. Next to each one draw the shape with sides twice as long.

2.



Area = ■ cm²

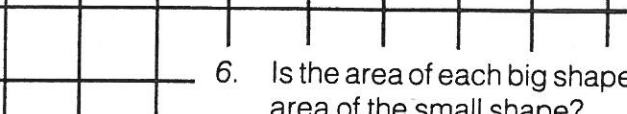


Area = ■ cm²

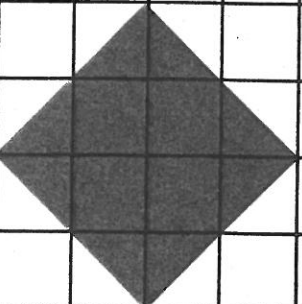
3.



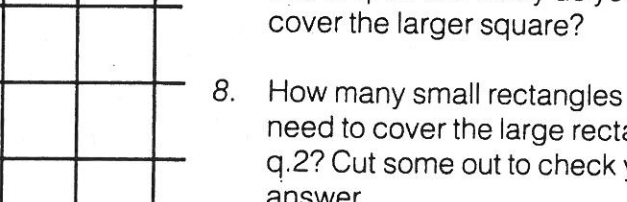
Area = ■ cm²



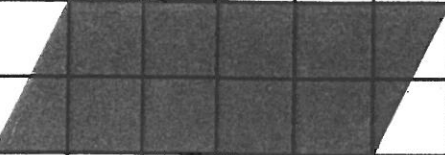
4.



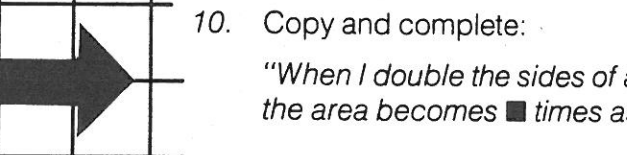
Area = ■ cm²



5.



Area = ■ cm²



6. Is the area of each big shape double the area of the small shape?

7. Cut out some squares like the smaller one in q.1. How many do you need to cover the larger square?

8. How many small rectangles do you need to cover the large rectangle in q.2? Cut some out to check your answer. Repeat for the shapes in q.3, 4 and 5.

9. Try doubling the lengths of some shapes of your own.

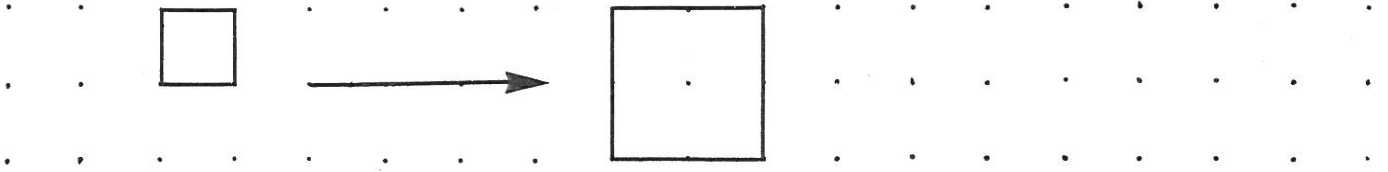
10. Copy and complete:

"When I double the sides of a shape, the area becomes ■ times as big."

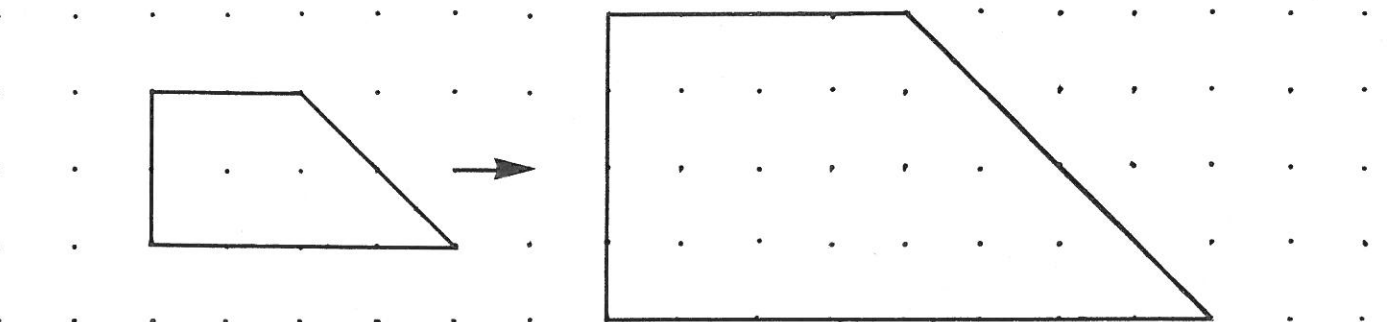
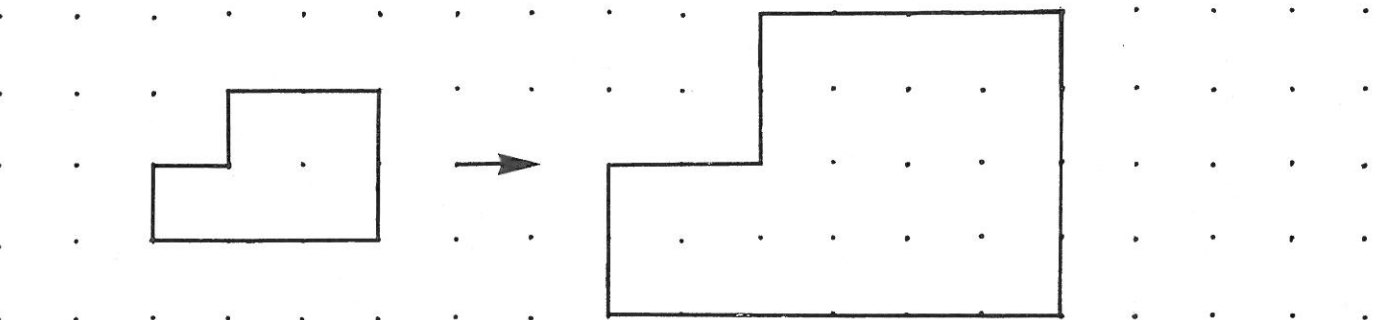
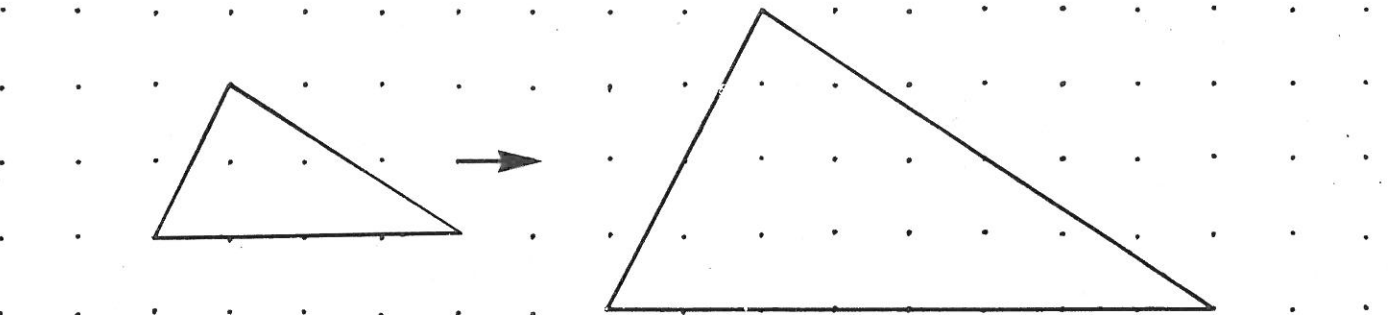
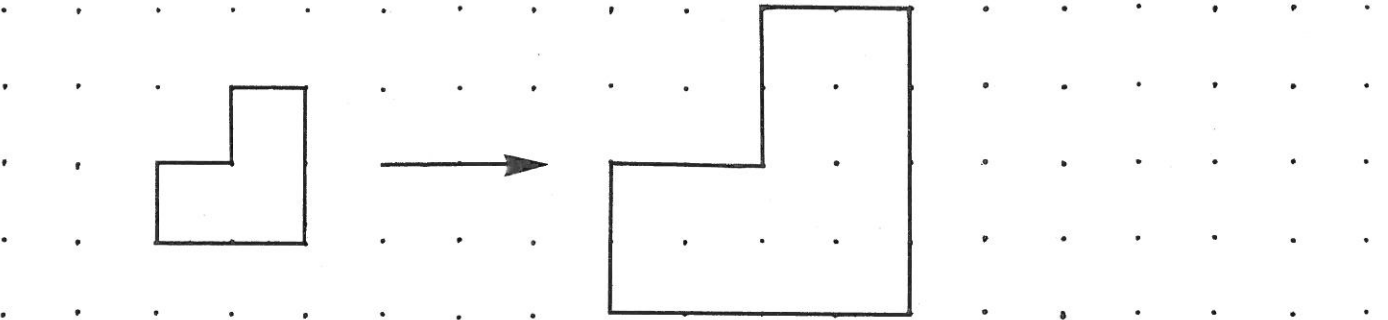
What happens to the area if you **treble** the sides of these shapes?

Shapes that can grow

Putting four small squares together to make a large square is easy.

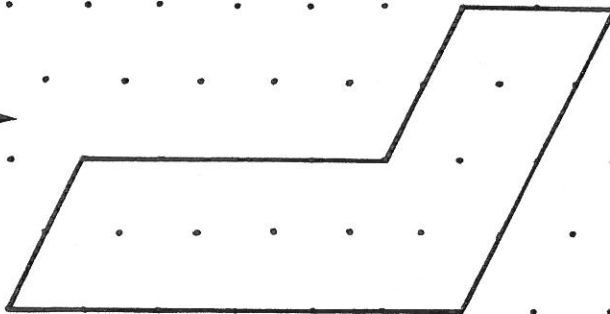
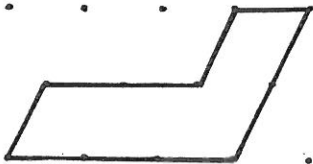
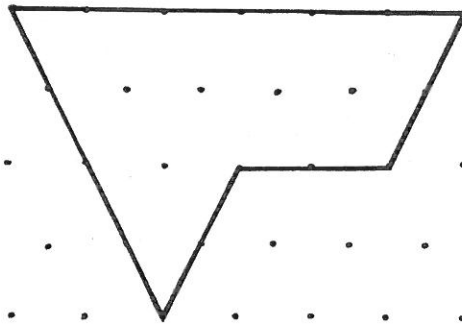
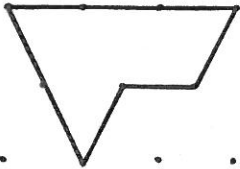
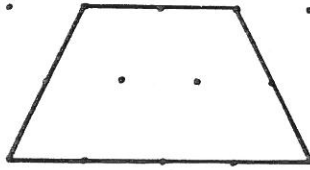


Try these:

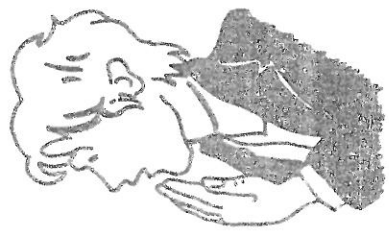


Turn over

Now try these:



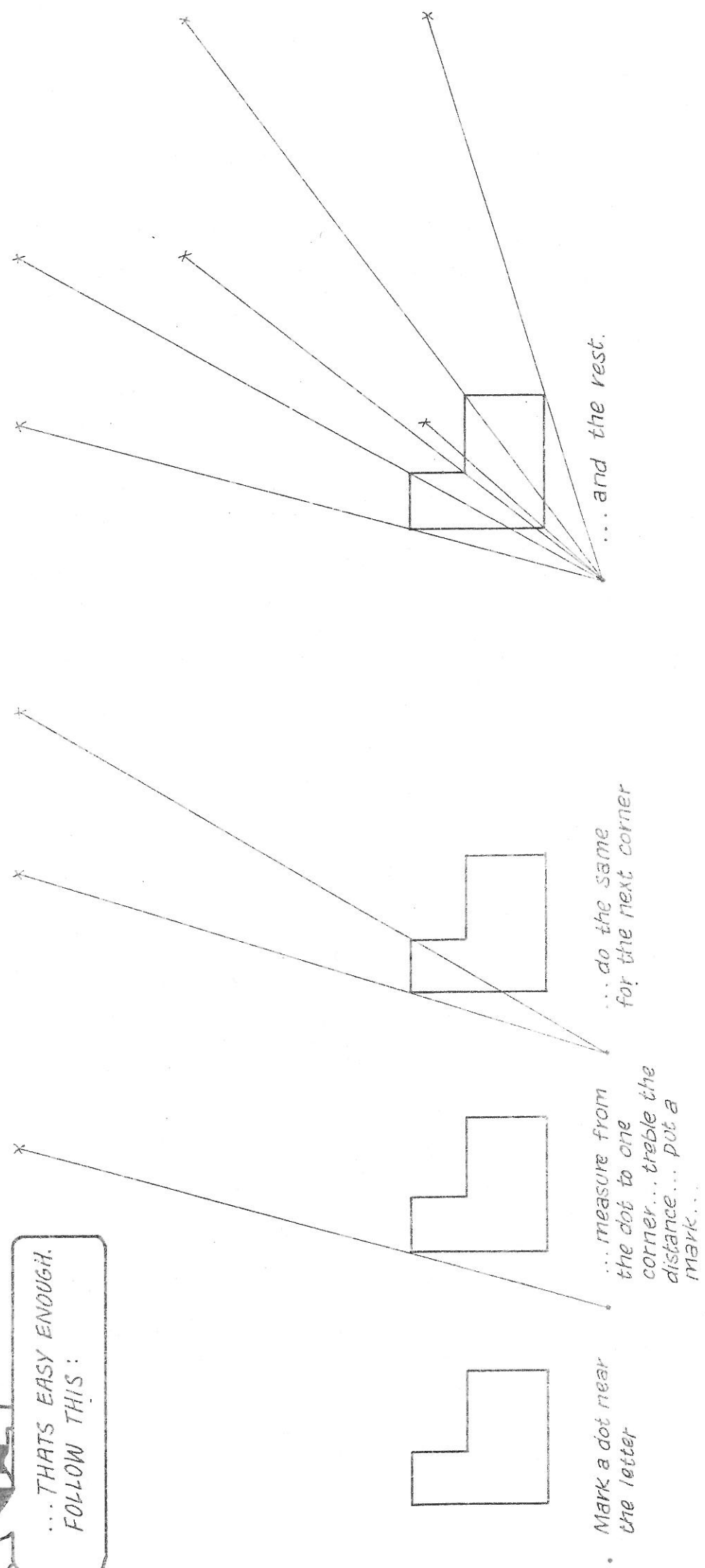
SCALED FACTOR



WE'VE GOT A NEW DESIGN FOR THE CLUB

IT'S TOO SMALL! WE WANT THAT DESIGN THREE TIMES AS BIG

...THATS EASY ENOUGH. FOLLOW THIS:



When you've done each corner, join them up. Measure the sides of each letter. What do you notice?



EACH SIDE IS TREBLED!
THAT'S A GOOD METHOD.
DOES IT MATTER WHERE
YOU PUT THE DOT?

A 1) USE THIS METHOD TO MAKE A LETTER **T** THREE TIMES AS BIG. 2) IS EVERY SIDE OF THE **T** TREBLED?

B THERE ARE IN FACT, SEVERAL QUESTIONS TO ANSWER ABOUT THIS METHOD:

- 1) DOES IT MATTER WHERE YOU PUT THE DOT? TRY SEVERAL POSITIONS AROUND A SIMPLE SHAPE.
. . . . TRY INSIDE THE SHAPE.
. . . . TRY ON THE EDGE OF THE SHAPE.
. . . . TRY ON THE CORNER OF THE SHAPE.

2) DOES IT WORK FOR ANY SHAPE?

- 3) DOES IT WORK FOR DOUBLING (2x) AS WELL AS TREBLING (3x)? TRY 2x
. . . . TRY 4x
. . . . TRY 10x

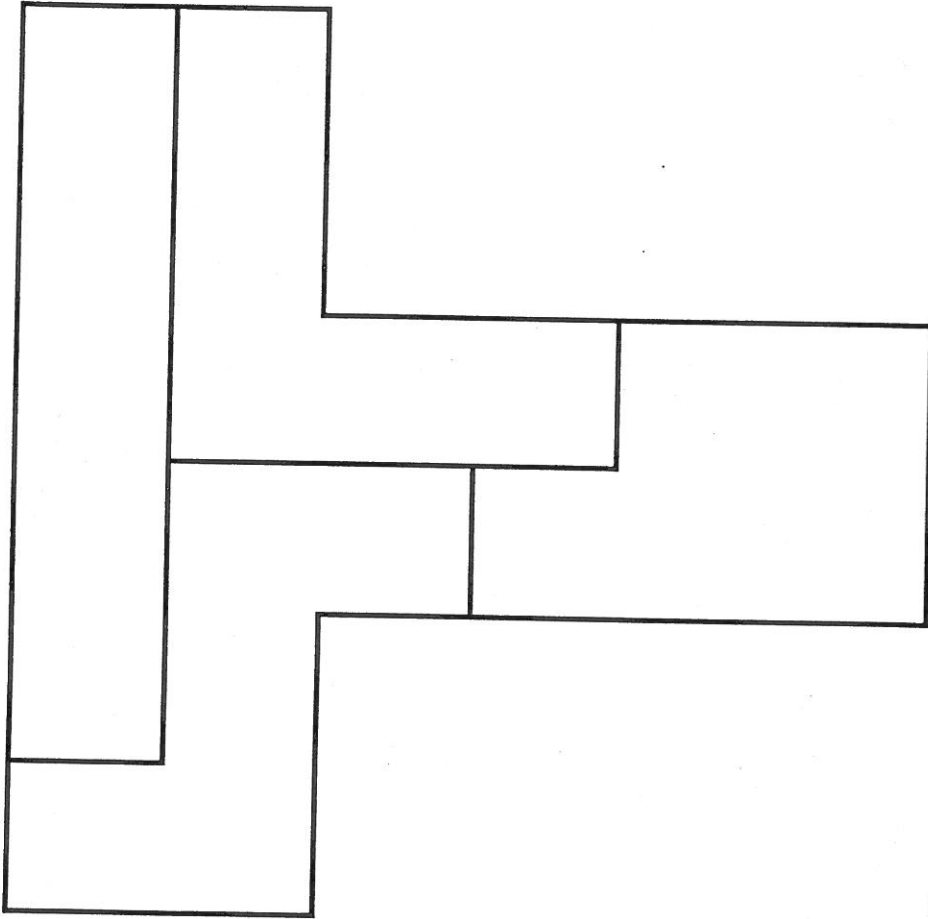
4) DOES IT WORK FOR $1\frac{1}{2}x$?

5) THE LAST QUESTION IS USUALLY THE MOST DIFFICULT! DOES THE SCALE FACTOR METHOD WORK FOR REDUCING SIZE? TRY $\frac{1}{2}x$
. . . . TRY $\frac{1}{4}x$

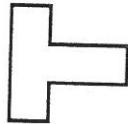
C HOW DOES THE AREA CHANGE EACH TIME?

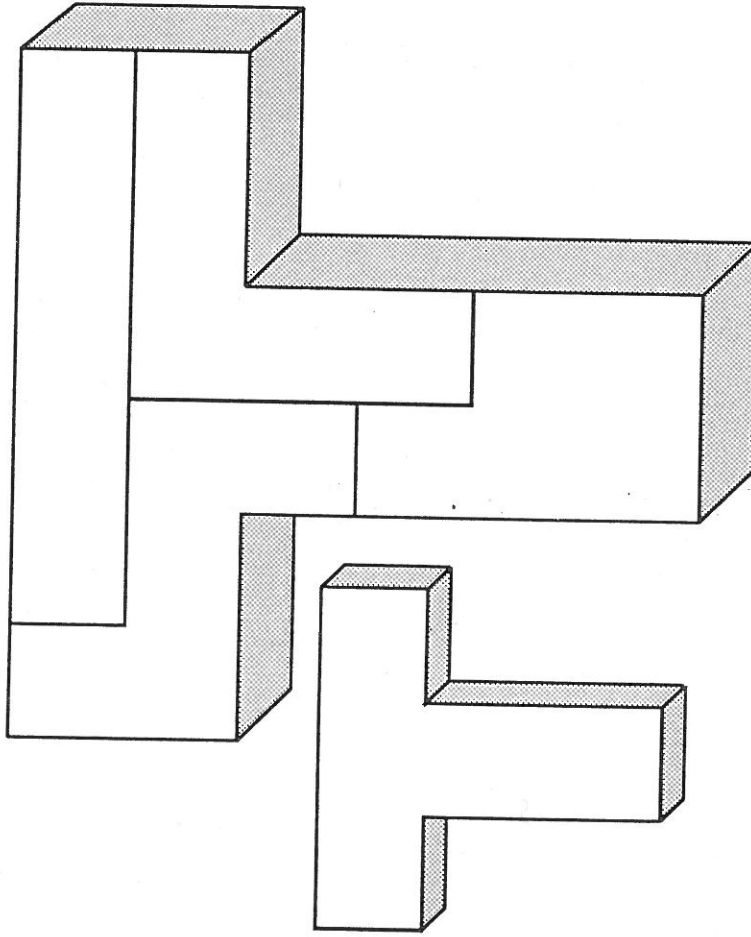
You will need a set of 12 wooden pentominoes.


Smile 1928


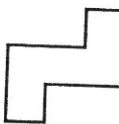


four pentominoes

An enlarged copy of the  pentomino can be made using four other pentominoes.



1. Make an enlarged copy of the  pentomino using four other pentominoes.

2. Do the same for  and 

Draw your results on 2cm squared paper.

3. a) How do the sides of the enlargement compare with the sides of the single pentomino?
- b) How does the area of the enlargement compare with the area of the single pentomino?

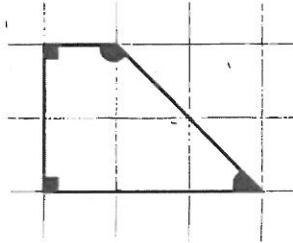
Turn over

One other pentomino can be enlarged in this way.

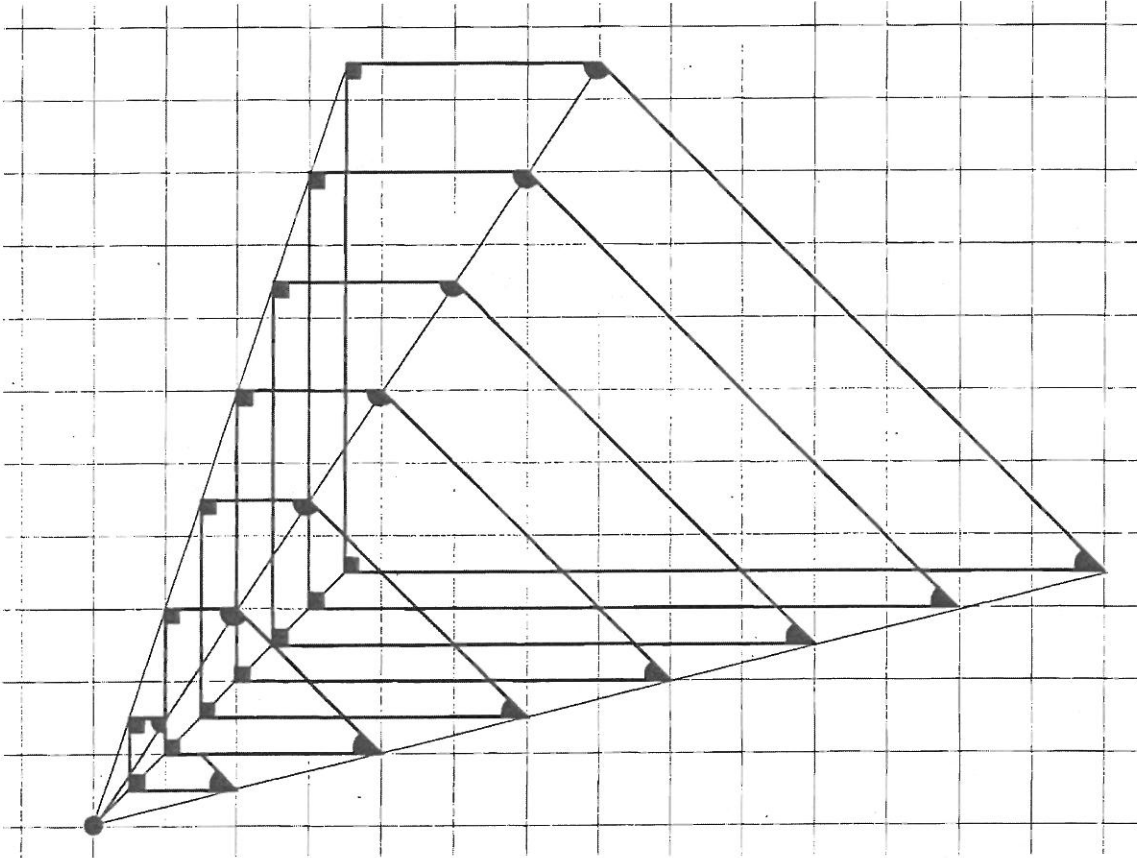
Can you find it?

Areas of Similar Shapes

The trapezium



has been enlarged by scale factors,
 $\frac{1}{2}$, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$
 to give a set of similar trapezia.

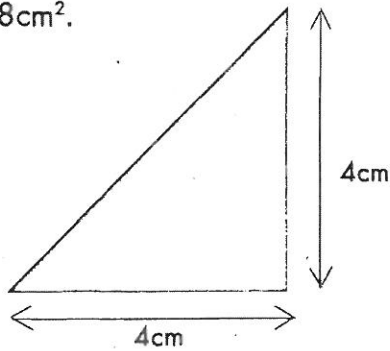


- Use a rotogram to check the corresponding angles of each trapezium are equal.
- Copy and complete this table.

a)	b)	c)	d)	e)
Scale factor	original length : corresponding new length	original area (cm ²)	new area (cm ²)	original area : new area
$\frac{1}{2}$	$1 : \frac{1}{2} = 2 : 1$	4	1	4 : 1
$1\frac{1}{2}$	$1 : 1\frac{1}{2} = 2 : 3$	4	9	4 : 9
2	1 : 2	4	16	4 : 16 = 1 : 4
$2\frac{1}{2}$	$1 : 2\frac{1}{2} = 2 : 5$	4	■	■ : ■
3	1 : 3	4	■	■ : ■ = ■ : ■
$3\frac{1}{2}$	$1 : 3\frac{1}{2} = \text{■} : \text{■}$	4	■	■ : ■

- What do you notice about the ratios in columns b) and e)?

This triangle has an area of 8cm^2 .



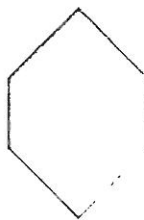
- Enlarge it by scale factors $\frac{1}{2}$, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$.
You may wish to use *MicroSMILE TRANSFORM*.
- Complete a table of results.

a)	b)	c)	d)	e)
Scale factor	original length : corresponding new length	original area (cm^2)	new area (cm^2)	original : new area
$\frac{1}{2}$	4:2 = 2:1	8	2	8:2 = 4:1

What do you notice about the ratios in column b) and e)?

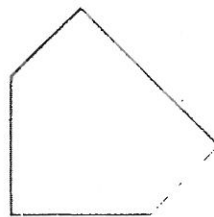
This shape has an area of 4cm^2 .

- What would its area be if it was enlarged by scale factor
 - a) 2
 - b) $\frac{1}{2}$
 - c) $3\frac{1}{2}$?



This shape has an area of 6cm^2 .

- What would its area be if it was enlarged by scale factor
 - a) 2
 - b) $\frac{1}{2}$
 - c) $3\frac{1}{2}$?



- Copy and complete this summary:

When a shape is enlarged by scale factor n

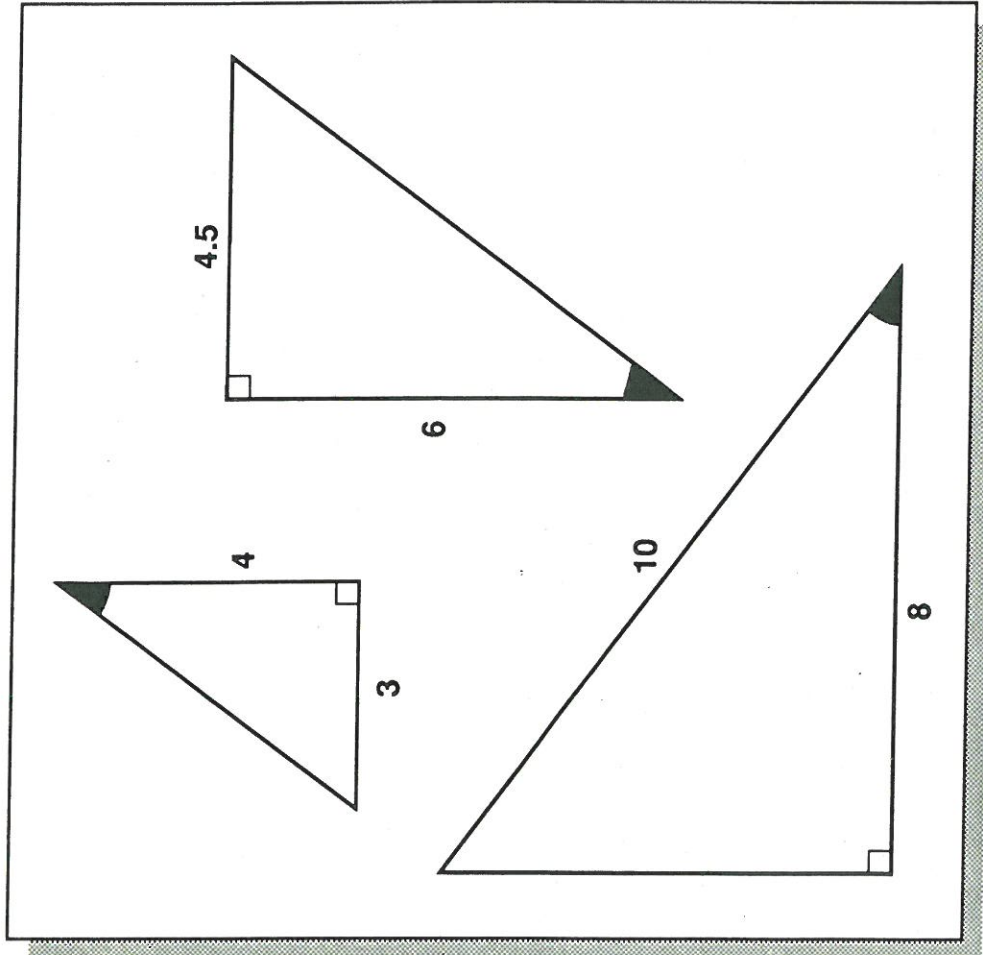
- the corresponding angles are
- the ratio of the sides is 1:
- the ratio of the areas is 1:

SIMILAR TRIANGLES

Smile 2027

These are similar triangles.

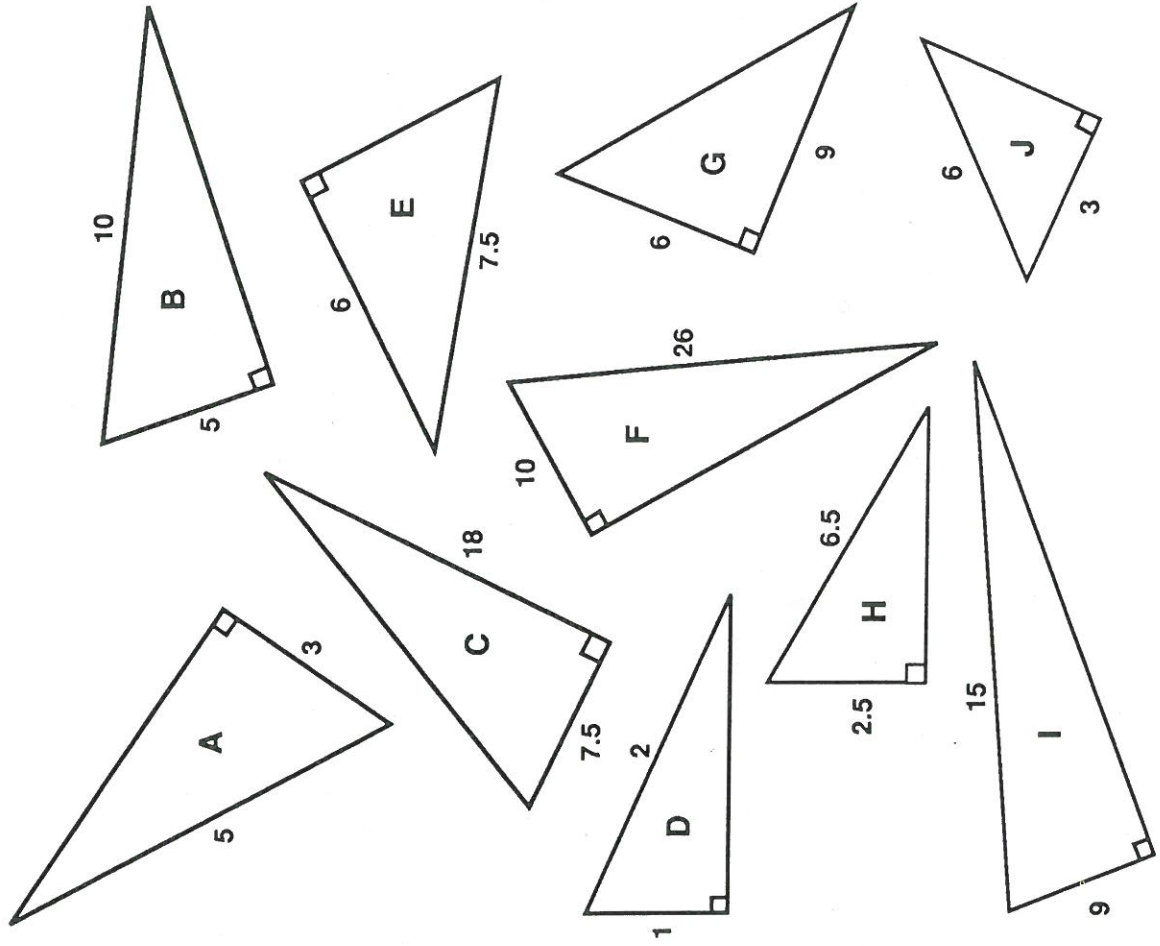
Explain why.



Turn over

Find three groups of similar triangles.
Which triangle is the odd one out?

(The triangles are not drawn to scale.)



Lengths of *Similar* Objects

A scale model or scale drawing is *similar* to the original object.

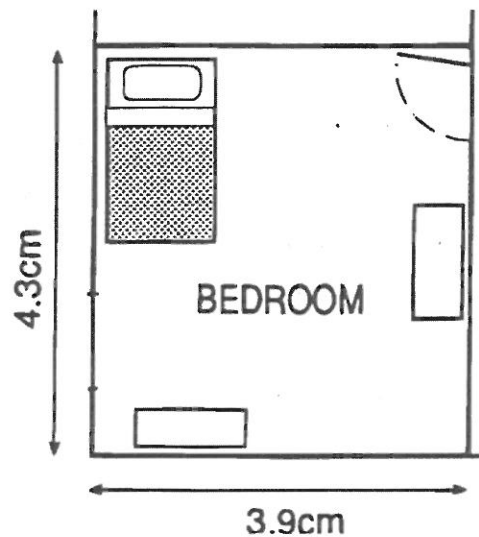
- Two objects are *similar* when:
- all the lengths of one object are a fixed number of times the corresponding lengths of the other object.
 - corresponding angles are equal.

1. A scale drawing of a bedroom uses a scale of 1 to 100 (1:100).

The length of the bedroom on the drawing is 4.3cm.
 The length of the real bedroom is $100 \times 4.3\text{cm} = 430\text{cm}$
 $= 4.3\text{m}$

The width of the bedroom on the drawing is 3.9cm.

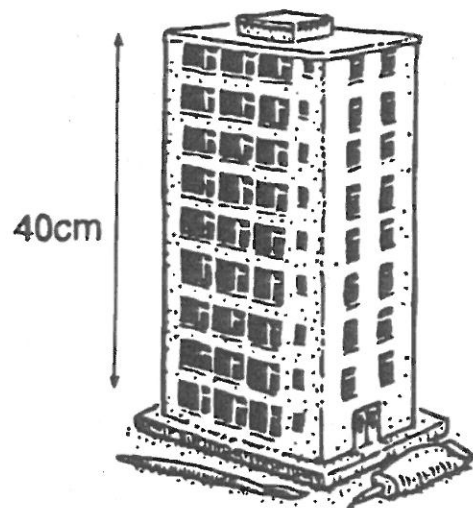
What is the width of the real bedroom?



2. An architect's model of a block of flats is 40cm tall.

It was made using a scale of 1 to 50 (1: 50).

How tall is the real block of flats?



The drawings are not to scale.

3. Copy and complete this table of corresponding lengths of a real object and a scale model.

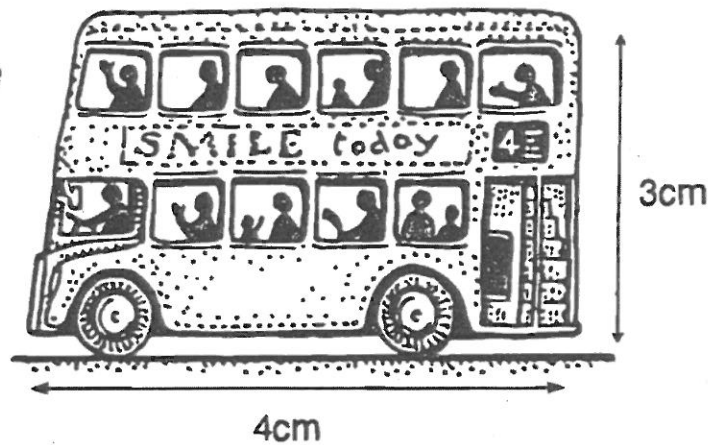
The ratio of the real object : scale model is 200:1

real object	scale model
320m	1.6m
35m	
	5cm

4. A bus is 4.8m long.

This scale model is 4cm long. The corresponding lengths are 4cm to 4.8m

The ratio of the model length : bus length is $4:480 = 1:120$



The bus is 3.6m high. The model is 3cm high.

Is the model bus similar to the real bus?

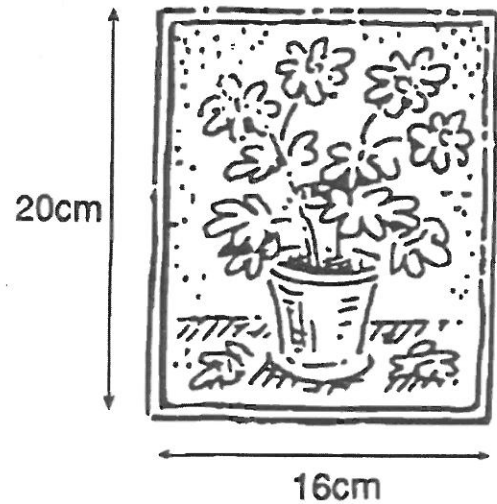
5. Another model of the same bus is made on a scale of 1 to 80 (1:80).

What are the dimensions of this model if it is similar to the real bus?

6. Calculate the dimensions of the following models:

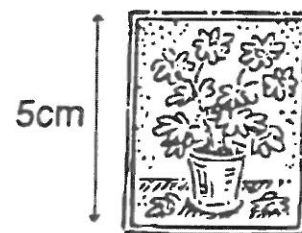
	Description	Real size	Scale of model
a	cuboid	60cm by 50cm by 35cm	1 to 10
b	rectangle	1.8m by 0.8m	1 to 10
c	cuboid	2.4m by 2.5m by 1m	1 to 20

7. A painting is 20cm long and 16cm wide.



- a) A similar postcard of the painting is 5cm long.

What is its width?

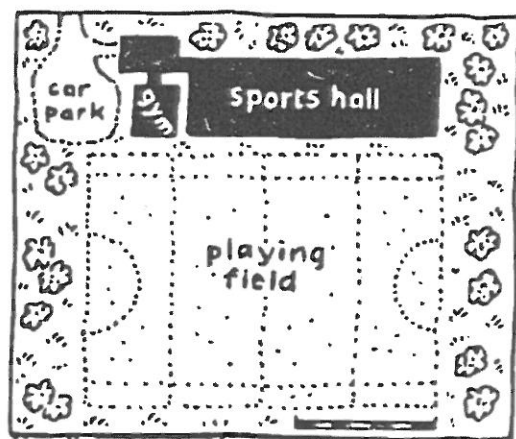


- b) A similar stamp features the painting. It is 2cm wide.

What is the length of the stamp?



8. A map of a recreation centre has a scale of 1 to 1000.



What is the size on the map of:

- the gym 10m by 12m?
- the sports hall 66m by 20m?
- the playing field 120m by 100m?



9. a) Some ordnance survey maps use a scale of 1 to 50 000.

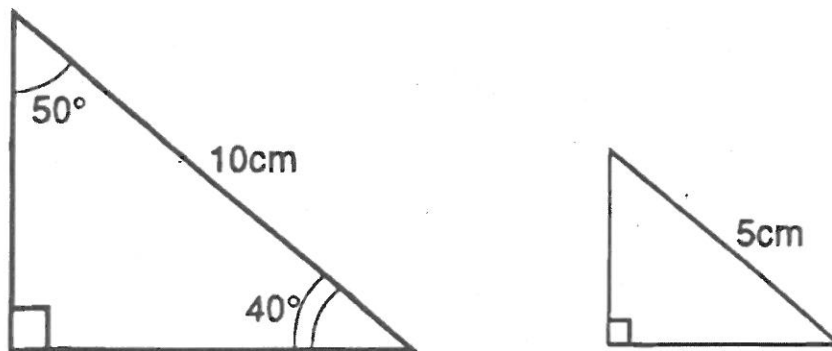
What distance in kilometres is represented on these maps by i) 1cm?

ii) 5cm?

b) Other maps use a scale of 1 to 20 000.

What distance in kilometres is represented by 8cm on these maps?

10. These two triangles are similar.



What are size of the angles of the smaller triangle?

Smile 1929

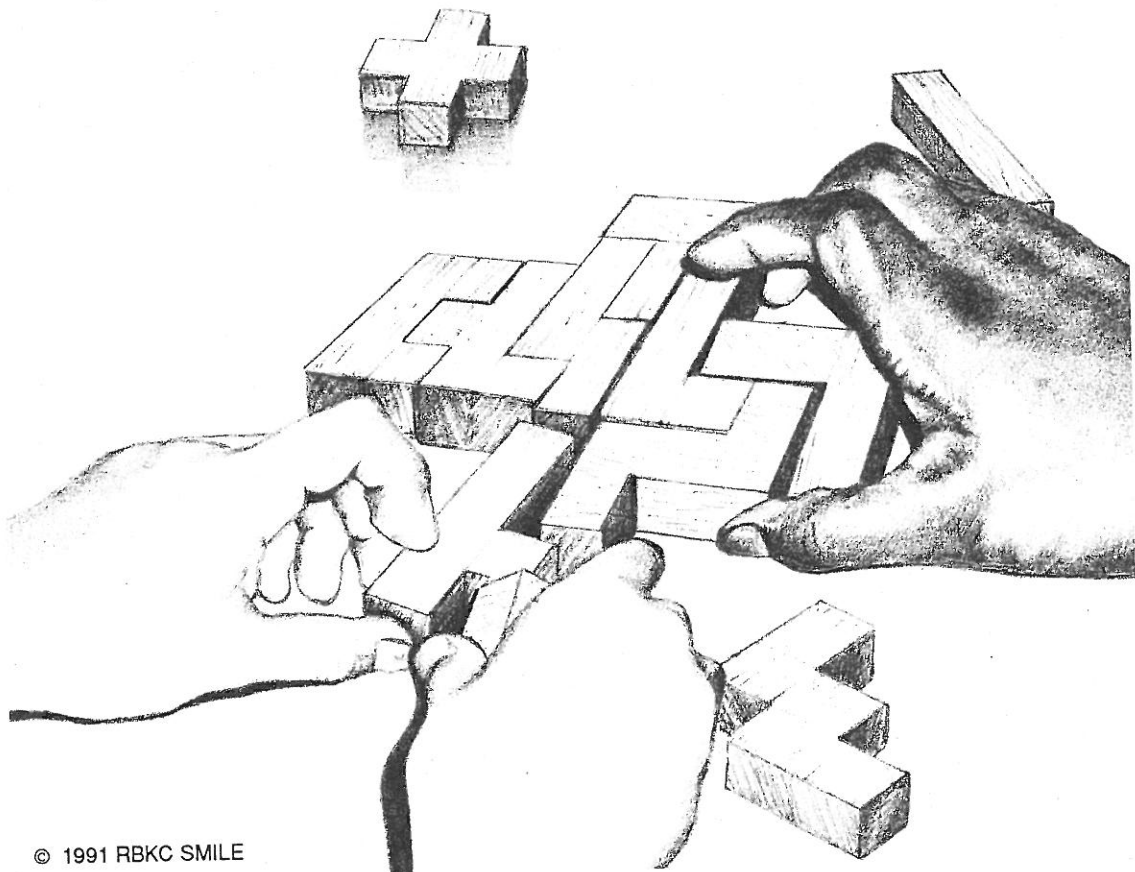
Nine pentominoes

You will need a set of 12 wooden pentominoes and 2cm squared paper.

Select **one** pentomino.

Use **nine** of the other pentominoes to make an enlargement of it.
Draw your result.

Try some more.

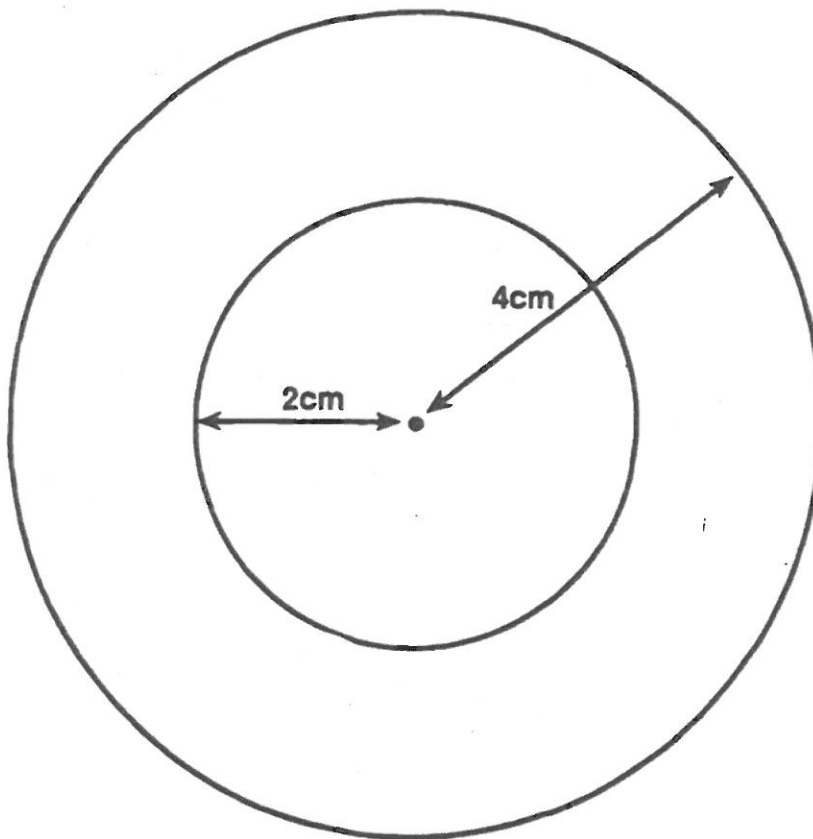


Similarity Problems

When a shape is enlarged by a scale factor n , the original shape and the new shape are **similar**.

- The ratio of each original side to the corresponding new side is $1:n$.
- The ratio of the original area to the new area is $1:n^2$.

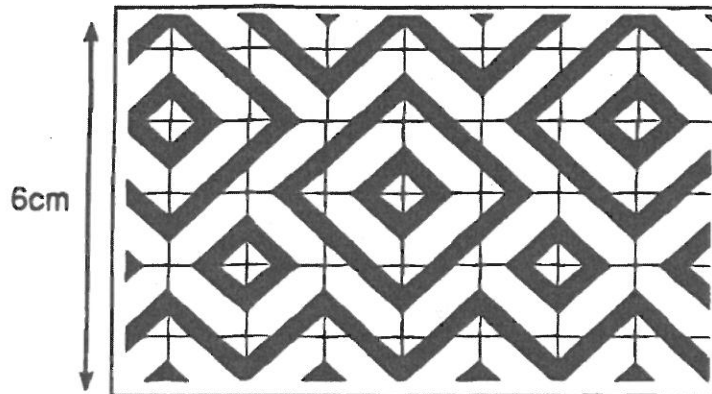
1. A circle with radius 2cm is enlarged by scale factor 2.



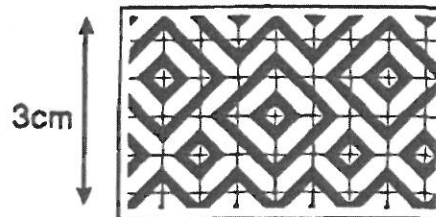
- Calculate the diameter, circumference and area of each circle.
- Do your results agree with the summary in the box?

Drawings are not to scale.

2. The area of this postcard is 60cm^2 .

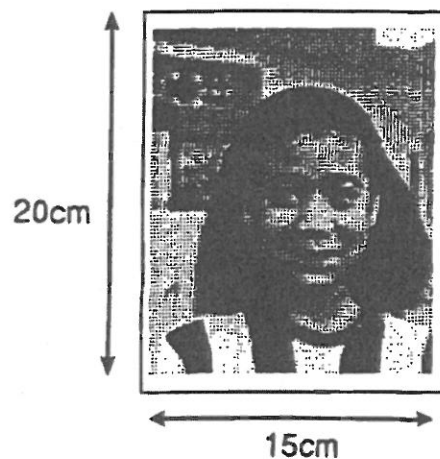


A similar version of the same postcard has a height of 3cm.



- What is the area of the small version?

3. A photograph is 20cm long and 15cm wide.

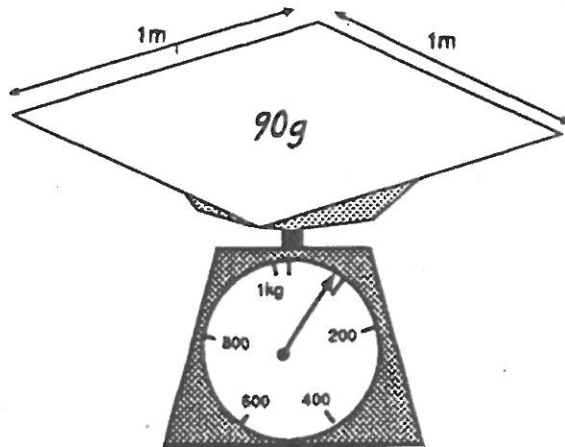


The length of a similar, smaller photograph is 5cm.

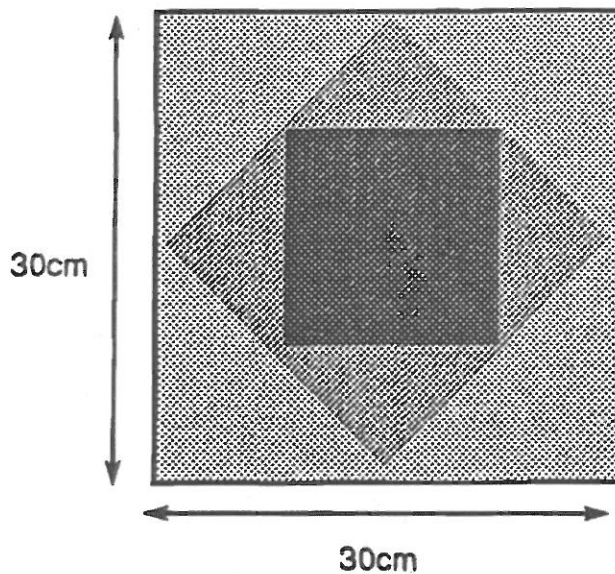


- What is the width of the smaller photograph?
- What is the area of the smaller photograph?

4. One square metre of paper weighs 90g.



- How many kilograms would a 10 metre by 10 metre square of paper weigh?
5. 18 square floor tiles each 30cm long are needed to cover a floor.



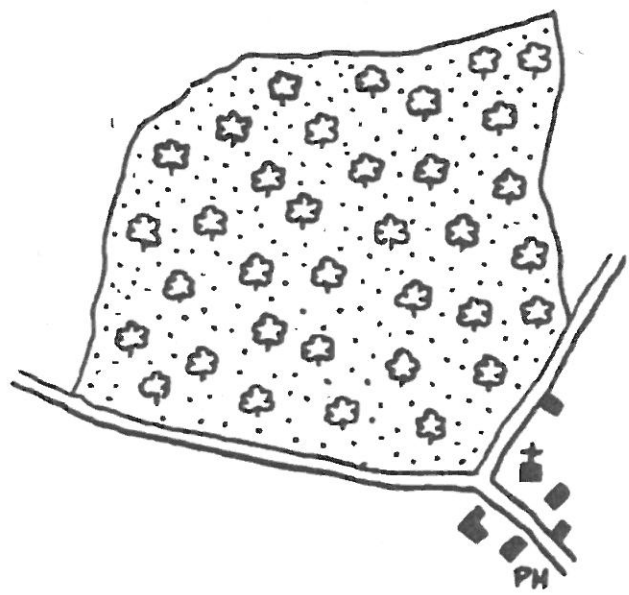
- How many tiles would be needed if square tiles of 7.5cm long are used?



6. This is the map of a deciduous forest.

Scale

1:50 000



- Calculate the area of the forest in km^2 .

NEGATIVE SCALE FACTOR

1) Use the SCALE FACTOR method (see card 0838) to complete the enlargement on worksheet 0845A by a factor of 4x

2) On the same worksheet change the shape using scale factors :- 2x

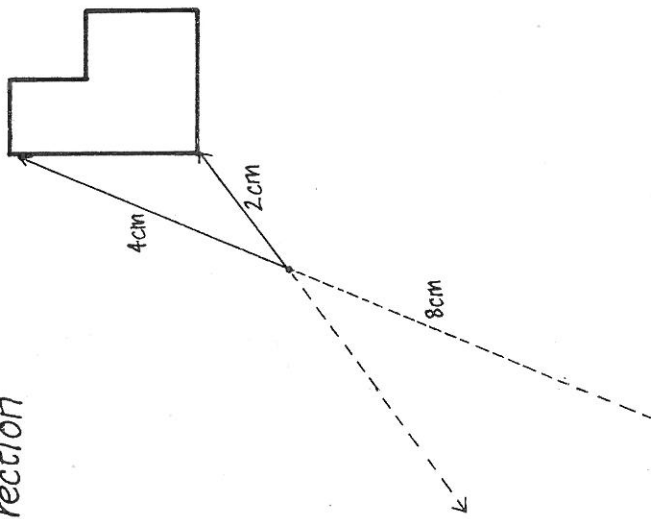
- 1½x
- ½x
- ⅛x
- 0x

3) What happens to the shape as the scale factor diminishes ?

4) Think about the number line as it passes below zero: where do you think the shape on the worksheet will be if we change it by a factor of -2x ?

Scale Factor -2

A negative scale factor like -2 instructs you to double the distance from the dot, but in the opposite direction



5) Draw the shape on the worksheet, -2x
Does your answer to question (4) agree?

6) What effect does the scale factor -1 have?

7) Does the position of the dot affect the negative scale factor?

SCALE FACTORS & ARROWS



8) Draw an arrow +2cm long.



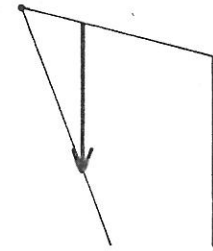
Change the arrow +2 by a scale factor +3



Measure the new arrow. Record your result

$$+3 \times +2 = \blacksquare$$

9) Now to change an arrow -2 by a scale factor +3:

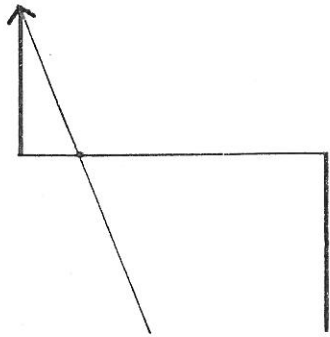


Can you predict the answer?

$$+3 \times -2 = \blacksquare$$

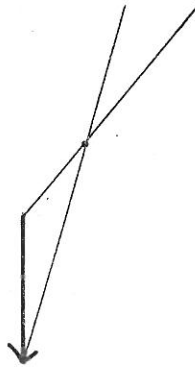
Draw it; check your result.

- 10) Change the arrow +2 by the negative scale factor -5



Record your result $-5 \times +2 =$

- 11) Now to change the arrow -2 by the negative scale factor -5



Can you predict the result?

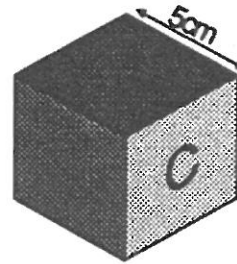
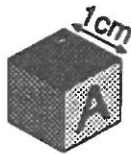
$$-5 \times -2 =$$

Draw it: check your answer.

- 12) Try several positive and negative scale factors: what affect do they have on (a) positive arrows? (b) negative arrows?

Volumes of Similar Objects

All cubes are similar.



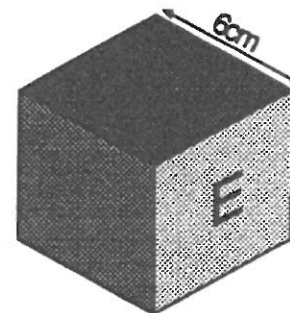
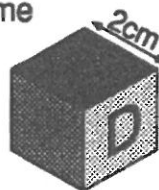
1. Copy and complete the table for cubes A, B and C.

	Side Length	Surface Area	Volume
A	1cm	■	■
B	3cm	■	■
C	5cm	■	■

2. Copy and complete the table of ratios of measurements.

	Side Length	Surface Area	Volume
A to B	1:3	■	■
A to C	■	1:25	■
C to B	■	■	125:27

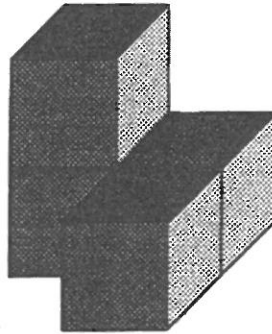
3. Write down the ratios of side length, surface area and volume for cubes D and E.



Calculate the surface areas and volumes to check whether you were correct.



Make a solid with four cubes.



Make a **similar** solid which is an enlargement of scale factor 2.

- How many cubes did you use?

The ratio of the corresponding side lengths is 1: 2.

- What is the ratio of the corresponding surface areas?
- What is the ratio of the corresponding volumes?

If you made a **similar** solid which is an enlargement scale factor 3.

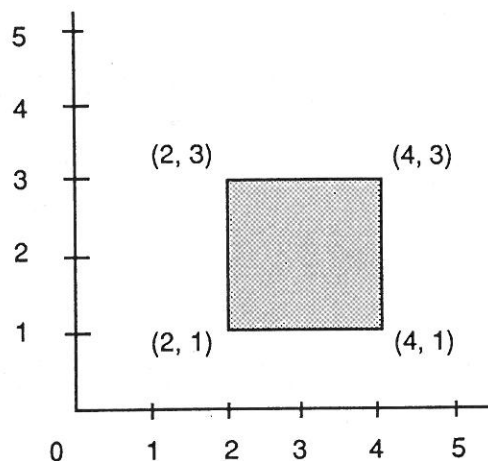
- What would be the ratio of the corresponding side lengths?
- What would be the ratio of the corresponding surface areas?
- What would be the ratio of the corresponding volumes?

Make solids with different numbers of cubes.

- Enlarge them by different scale factors to give similar solids.
- For each enlargement record the ratio of corresponding
 - side lengths.
 - surface areas.
 - volumes.

Generalise your results for **similar** solids.

Matrices and Area



1. Transform this square using the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

$$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 2 & 4 & 4 & 2 \\ 1 & 1 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 8 & \blacksquare & \blacksquare \\ 3 & \blacksquare & \blacksquare & \blacksquare \end{pmatrix}$$

What is the area of the **original** square?

What is the area of the **new** shape?

What is the ratio **New Area : Original Area**?

Turn over

2. Transform the same square using the following matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

3. **Can you find a connection between a matrix and the effect it has on the area of a shape?**

It may help to tabulate your results.

4. Try some more examples to test your hypothesis.

Will it work for other shapes ?