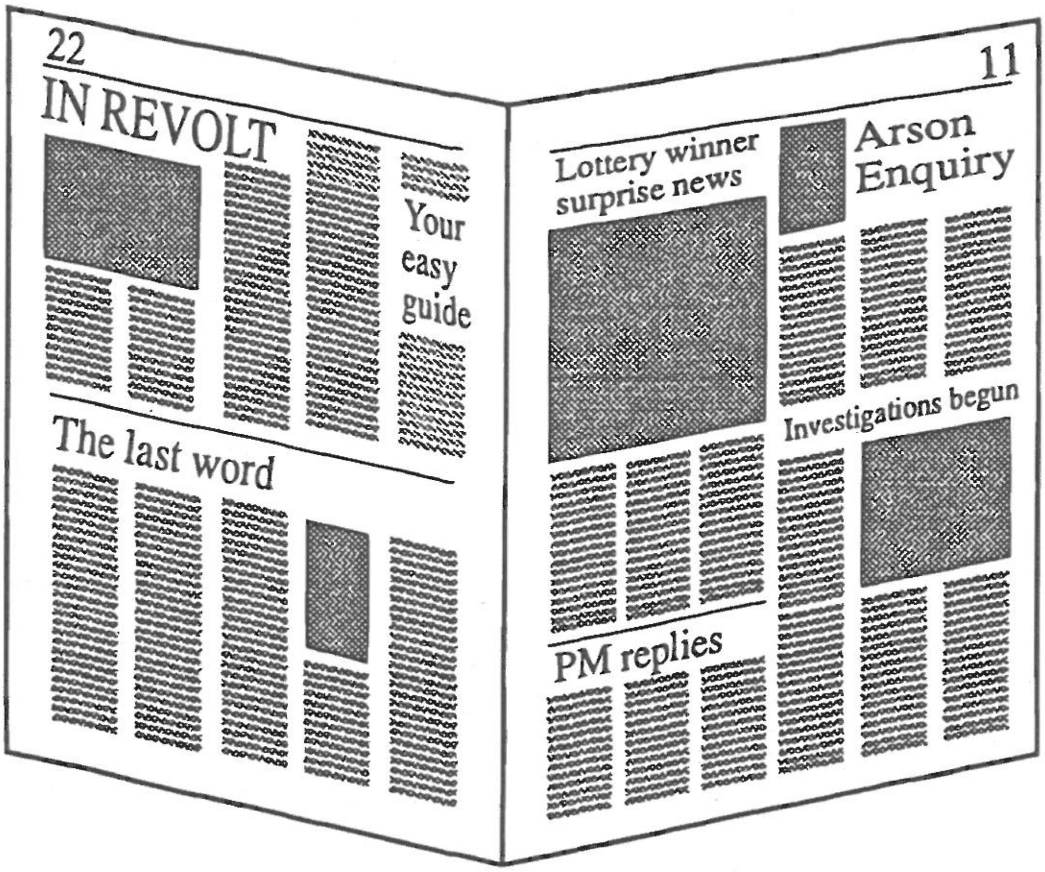
SMILE WORKCARDS

Patterns and Generalisations Pack Three

Contents

Numbering the Pages

This shows one sheet from a newspaper.



- How many pages were there in the complete paper?

- Make a small newspaper with 6 sheets of scrap A4 paper.
  Number the pages.
  What patterns do you notice in the page numbers?

- *Investigate page number patterns for other newspapers.*

142 857 times table

(1) Use your calculator to complete the first seven rows of this table.

x	142 857
1	
2	
3	
4	
5	
6	
7	
.	
.	
.	
.	
.	
.	
.	

(2) What do you notice about your answers so far?

(3) Use your calculator to extend your table for the next two rows.

(4) Investigate your table for number patterns and fill in the next five rows of your table without using a calculator.

(5) Check your results if you wish.

(6) Now use your calculator to work out

$\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, ---------

Any comments on your results?

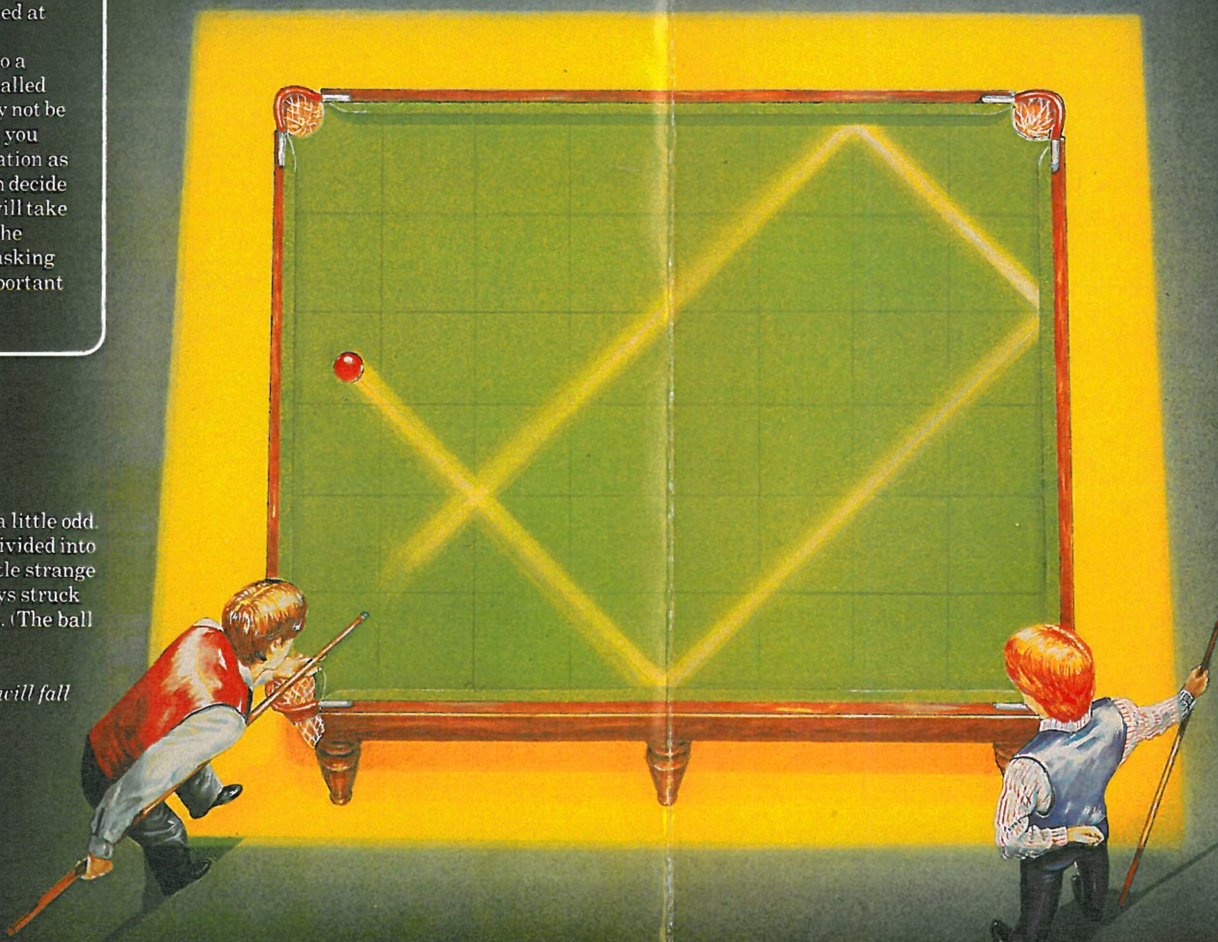
BILLIARDS

LET'S INVESTIGATE

A chance for you to explore mathematics

What is an investigation?

Investigations are the source of all mathematical discovery. When Isaac Newton observed the falling apple his imagination was fired to find out why. The result was the discovery of the laws of gravity. When Leonhard Euler looked at the problem of the seven bridges of Königsborg, his investigations led to a whole new branch of mathematics called 'topology'. Your results will probably not be so dramatic but with investigations you have the opportunity to use imagination as well as mathematical skills. You can decide how far and in which direction you will take a problem. You will also be asking the questions. You might find out that asking the right questions can be just as important as finding the right answers.



The Strange Billiard Table

The billiard table in the illustration is a little odd. It only has four pockets and the base is divided into squares. The rules of the game are a little strange too. Only one ball is used and it is always struck from the same corner at 45° to the sides. (The ball always rebounds at 45° to the sides.)

Can you work out which pocket the ball will fall into?

The Billiard Table Investigation

Below are a few questions to start you off, but remember that an important part of an investigation is asking your own questions.

What would happen if the billiard table was of a different size, say 2 × 6.5 × 10.4 × 8 etc?

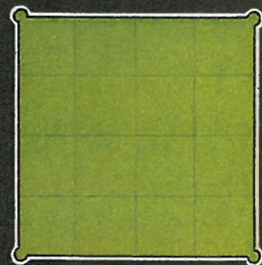
On some tables the ball will travel over every square. Which tables?

How many times does the ball hit the side of the table?

Which pocket does the ball fall into?

Use the drawings below or squared paper to investigate billiard tables. What answers do you think you will get ..? *What questions will you ask?*

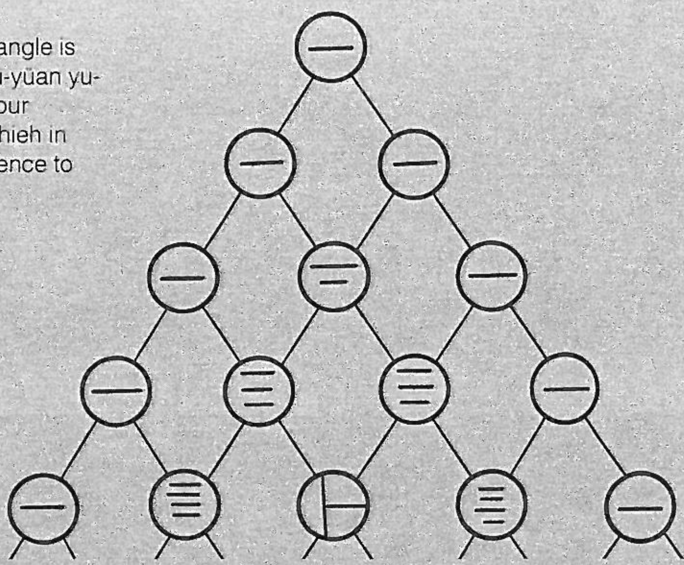
1x8



4x4

3x7

This illustration of the Chinese Triangle is based upon a diagram from "Ssu-yüan yu-chien" ("Precious Mirror of the Four Elements") written by Chu Shih-chieh in 1303. This is the first written reference to the triangle that has been found.



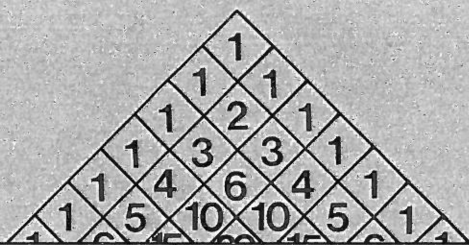
The Chinese Triangle

The number arrangement above is one of the most famous in mathematical history. It was known in China by 1100 AD and is mentioned by Omar Khayyam, the Arabian poet and mathematician, who lived from 1050 to 1123.

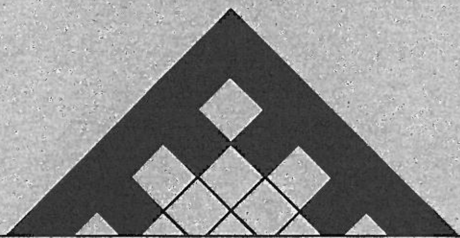
It is not known whether it was discovered in the two distant countries around the same time or whether information about it passed along the 'silk route' which connected China and Arabia.

In the early 15th century the French mathematician Pascal wrote a treatise on it and since then it has often been called Pascal's Triangle in European books.

The triangle is formed by successive additions: a number in any position is found by adding the two numbers directly above it. So the first five rows look like this:

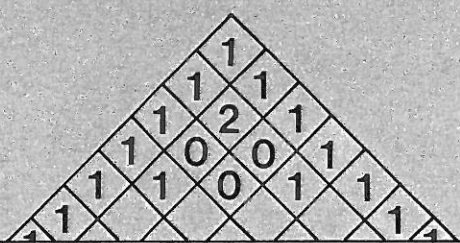


If you choose two colours, one for the odd numbers and a different one for the even numbers, the triangle looks like this:



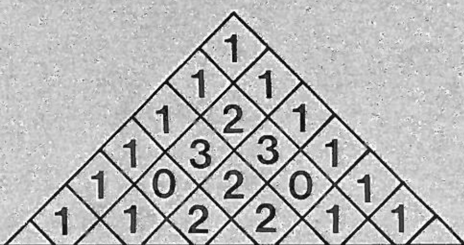
Continue the pattern: what do you notice?

If you write the triangle in mod 3, the first five rows look like this:



What happens if you continue?

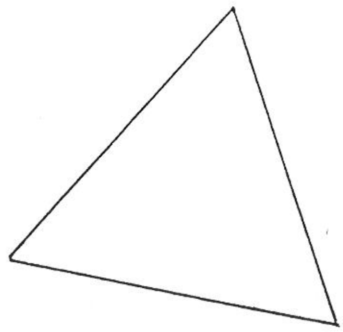
If the first six rows are written in mod 4, they look like this:



What happens if you continue?

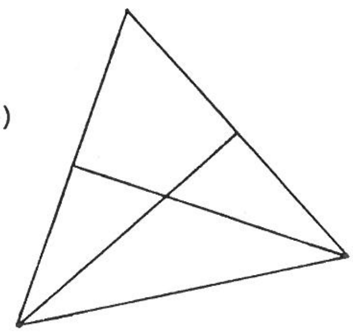
Can you find any other patterns in the Chinese Triangle?

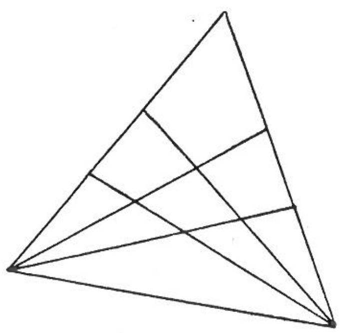
Cubes from triangles



There is only one triangle here.

(1) How many triangles here?
(there are many more than 3)

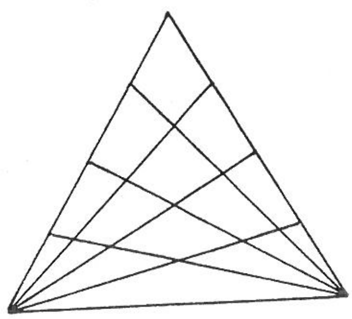




(2) How many in this one?

(3) Look at your answers so far. Do they help you to find the answer to this one?

(Hint: there is a clue in the title)



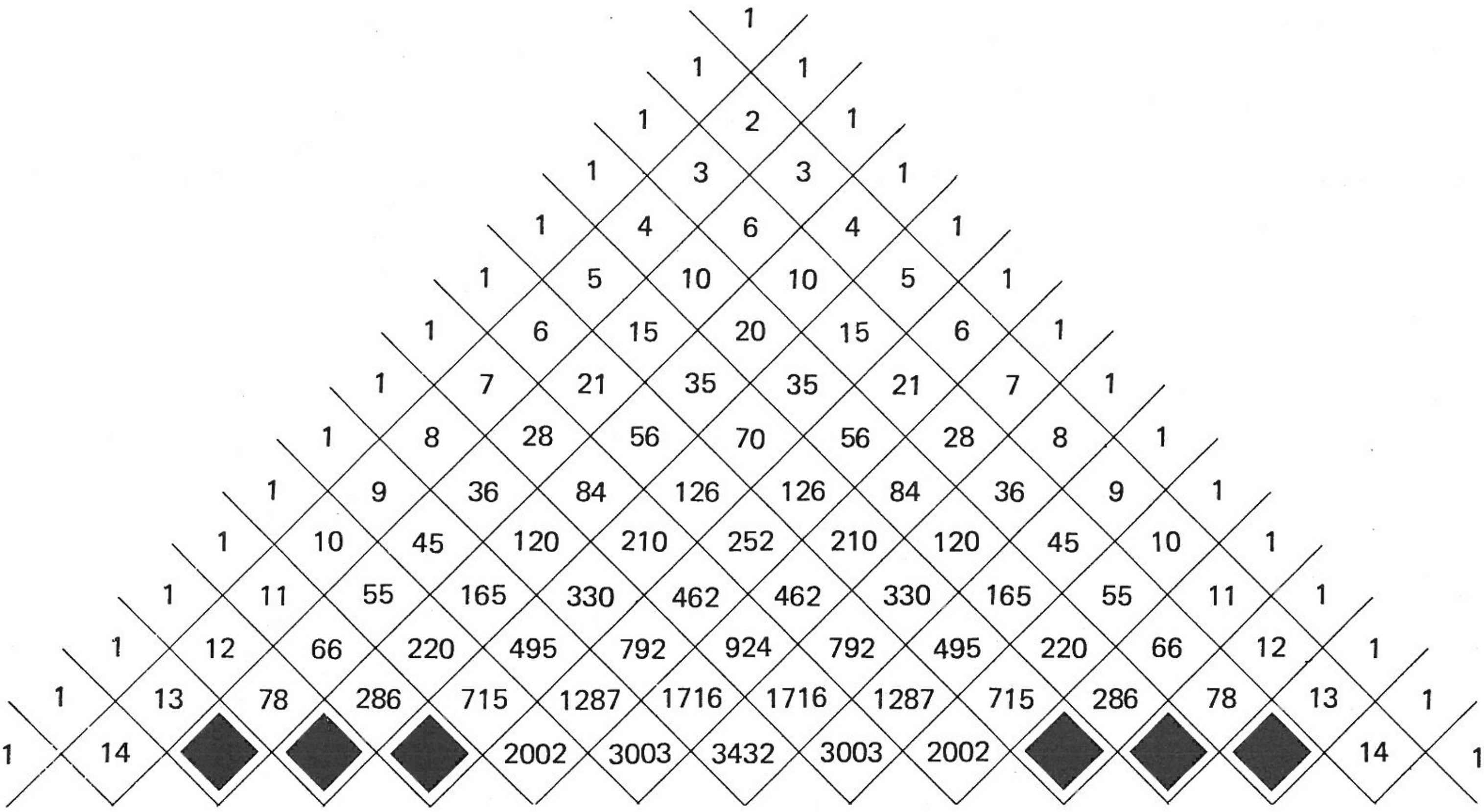
(4) Check your answer to question 3 by counting .

(5) Draw the next diagram and write down the number of triangles it contains.

Patterns in Pascal's Triangle

This is the beginning of Pascal's Triangle.

Within it there are many hidden sequences....



1. Write the missing numbers in your book.
2. Find the triangle numbers and describe where they appear.
3. Add the numbers in each row. Look at the totals. *Describe the sequence.*
4. Look at the row which begins 1,7,21..What do you notice?
   Which other rows have the same property?
5. Calculate $11^2, 11^3, 11^4$. Where can you find these numbers in the triangle? Calculate 11^5. Try to work how you could get 11^5 from the triangle. Use the triangle to obtain 11^6 and 11^7. *Check your method by calculation.*

Turn over

$$1$$

$$1 \qquad \frac{1}{1}$$

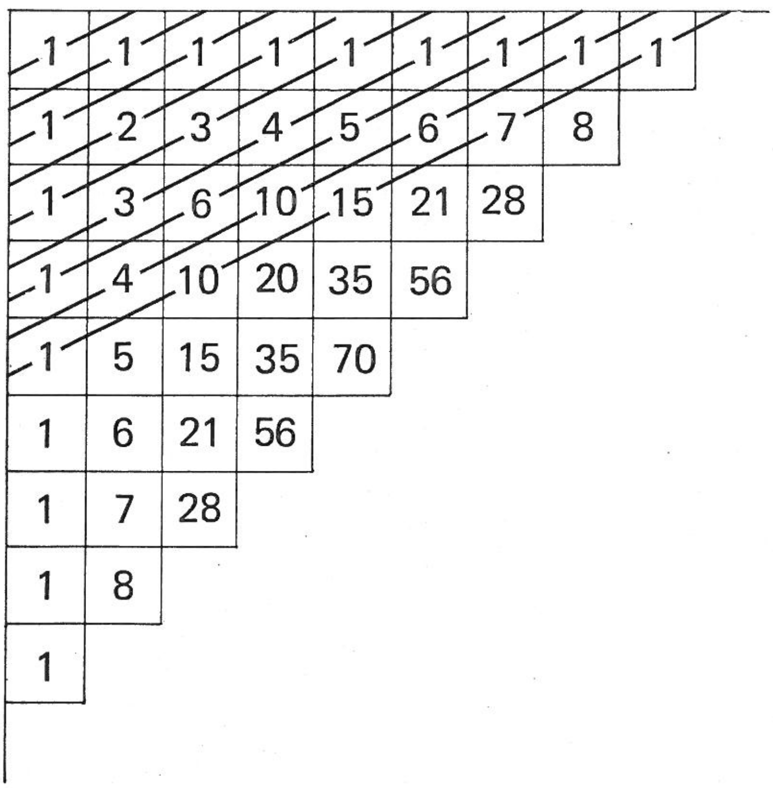
$$1 \qquad \frac{2}{1} \qquad \frac{2\times1}{1\times2}$$

$$1 \qquad \frac{3}{1} \qquad \frac{3\times2}{1\times2} \qquad \frac{3\times2\times1}{1\times2\times3}$$

$$1 \qquad \frac{4}{1} \qquad \frac{4\times3}{1\times2} \qquad \frac{4\times3\times2}{1\times2\times3} \qquad \frac{4\times3\times2\times1}{1\times2\times3\times4}$$

$$1 \qquad \frac{5}{1} \qquad \frac{5\times4}{1\times2} \qquad \frac{5\times4\times3}{1\times2\times3} \qquad \frac{5\times4\times3\times2}{1\times2\times3\times4} \qquad \frac{5\times4\times3\times2\times1}{1\times2\times3\times4\times5}$$

6. Find the first few terms of the row of Pascal's Triangle which begins 1,100,.....



7. Can you find the Fibonacci Sequence?

1,1,2,3,5,8,13,....

Check the next few terms.