SMILE WORKCARDS

Algebraic Structure Pack Three

Contents

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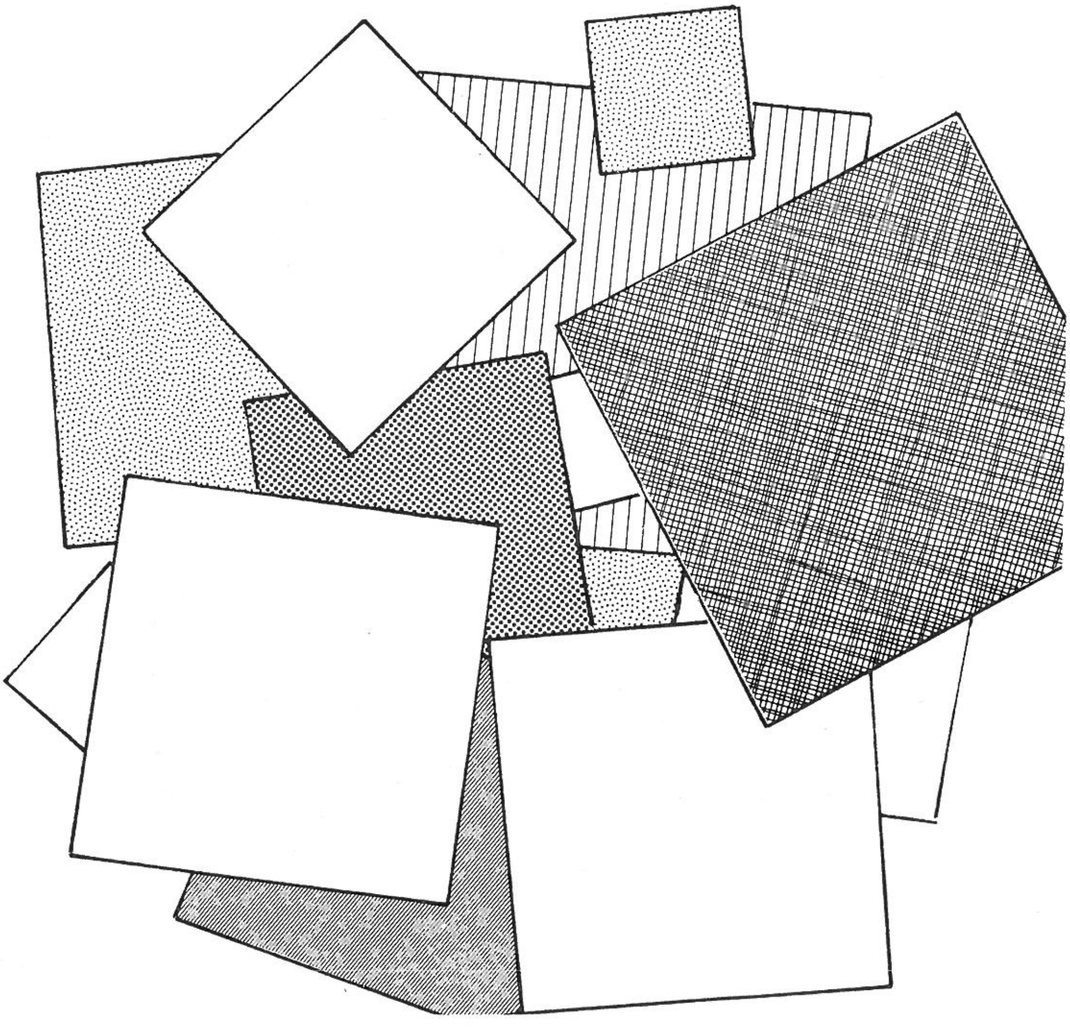
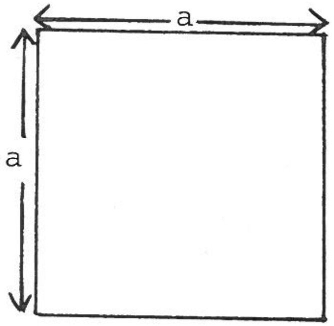
SMILE

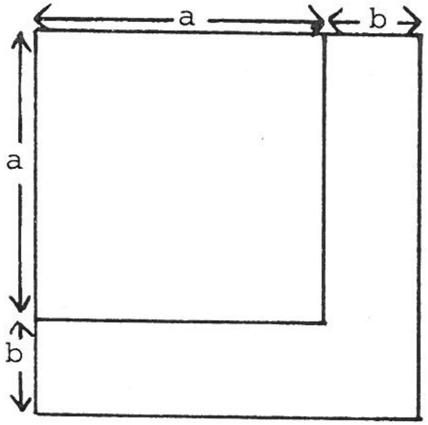
Start with a^2



Start with a^2

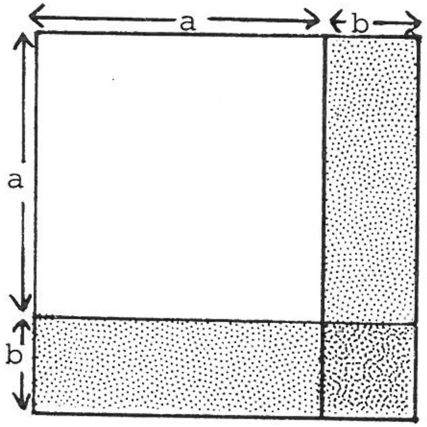


Expand this square to



(1) How much bigger is the (a+b) square than the (a) square?

This diagram will help:



Check that $ab = ba$

and you will find the increase in size is $(2ab+b^2)$

(2) Use your answer to copy and complete:

$$(a+b)^2 = ■ + ■ + ■$$

(3) If $a = 5$ and $b = 3$, check that your identity works.

(4) Use your identity to calculate $(102)^2$

(5) If $b = a$, verify that $(2a)^2 = 4a^2$

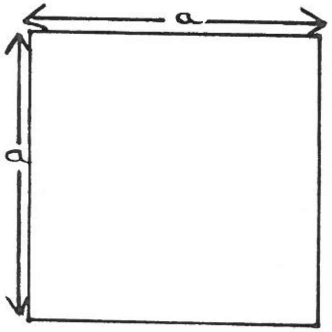
(6) What is the difference between $(a+b)^2$ and a^2+b^2?

(7) If $b = -4$, calculate $(96)^2$

(8) Put $b = -c$ and rewrite your identity

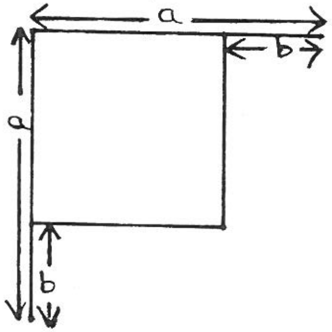
$$(a-c)^2 = ...$$

Start with a^2 again



Shrink this square

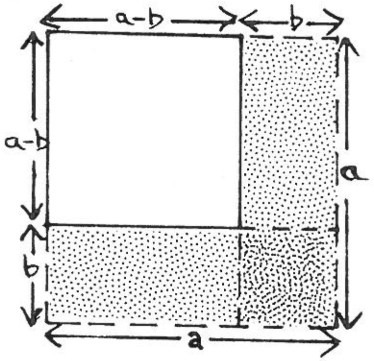
to



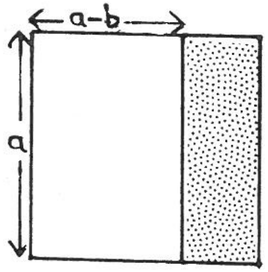
(9) What is the area of the (a-b) square?

This diagram will

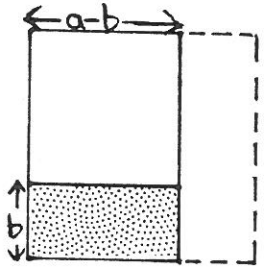
help:



Here are two ways of working out $(a-b)^2$:



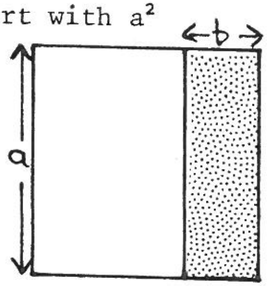
Remove right hand rectangle
Left hand rect. $= a.(a-b)$
$= a^2 - ab$



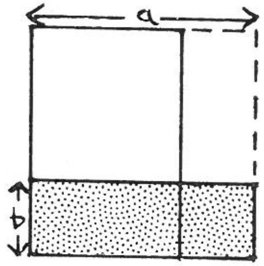
Remove bottom rectangle
Bottom rectangle $= b(a-b)$
$= ba - b^2$

∴ Unshaded square $= (a^2-ab)-(ba-b^2)$
$= a^2-ab-ba+b^2$
$= a^2-2ab+b^2$

Start with a^2



Remove ab from the right
Remainder $= a^2 - ab$



Remove ab from the bottom
$(a^2-ab)-ab$
$= a^2-2ab$

This has taken away the bottom right hand square twice, so add it back once:

$a^2-2ab+b^2$

Check this result with your answer to questions 8 and 9.

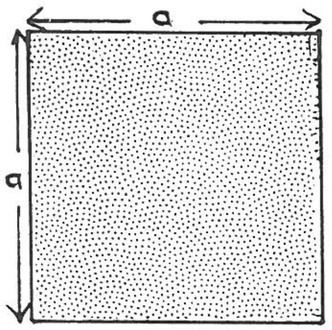
(10) If a=7 and b=4 check the identity
$(a-b)^2 = a^2 - 2ab + b^2$

(11) Use the identity to calculate 99^2

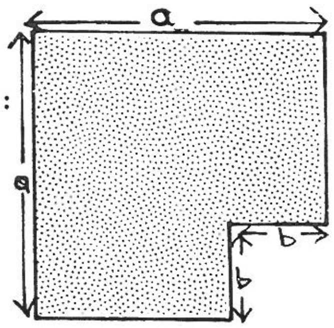
(12) What is the difference between (a^2-b^2) and $(a-b)^2$?

(13) What is the difference between $(a+b)^2$ and $(a-b)^2$?

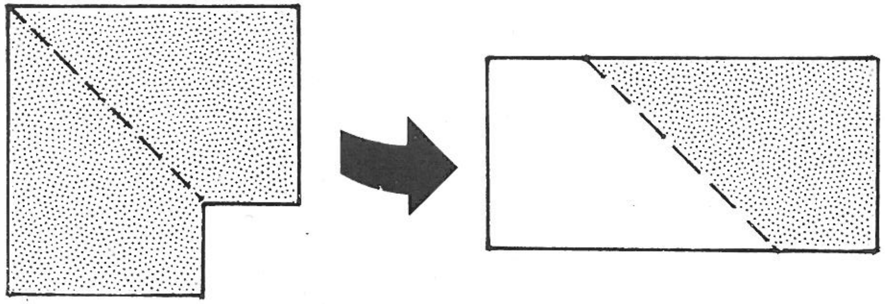
.... and start with a^2 again!



Remove b^2 from this:



Cut along the dotted line and refit:



(14) What are the length and width of the rectangle?

(15) Use the results of page 5 to write an identity for $(a+b) \cdot (a-b)$

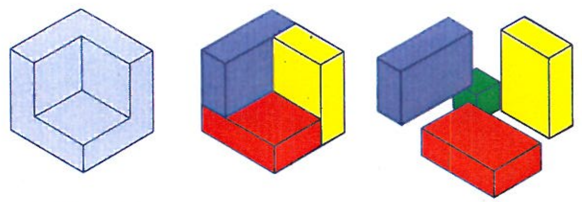
(16) If $a = 10$ and $b = 1$, check that your identity works.

(17) Use your identity to calculate 103 x 97.

(18) If $x = (a+b)$ and $y = (a-b)$ use the identities you have found to simplify
(a) $x^2 + 2xy + y^2$
(b) $(x + y)^2$

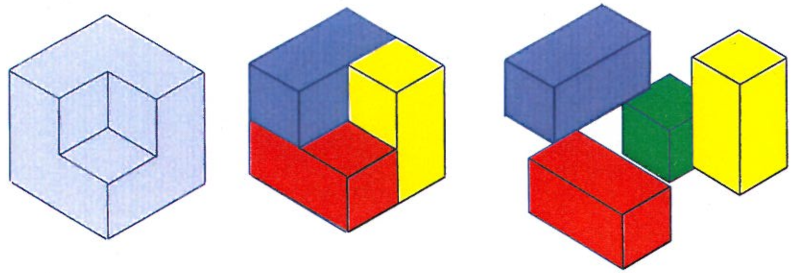
Identicubes

This diagram shows



$3^3 - 2^3 \quad = \quad 3(3 \times 2 \times 1) + 1^3$

Continue and generalise this sequence.



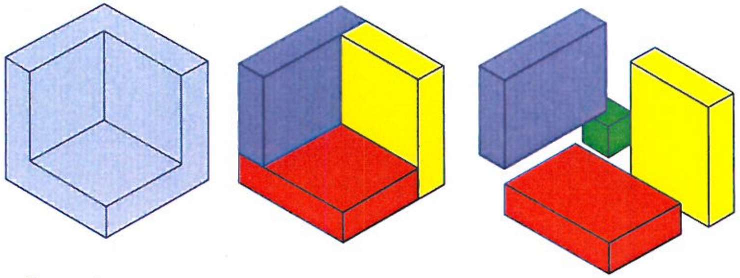
$4^3 - 2^3 \quad = \quad 3(\blacksquare \times \blacksquare \times \blacksquare) + \blacksquare^3$

5^3

$\vdots \qquad\qquad\qquad \vdots$

$x^3 - 2^3 \quad = \quad (\qquad\qquad) + \quad ^3$

Continue and generalise this sequence.



$4^3 - 3^3 \quad = \quad 3(\blacksquare \times \blacksquare \times \blacksquare) + \blacksquare^3$

5^3

$\vdots \qquad\qquad\qquad \vdots$

$x^3 - (x-1)^3 \quad = \quad (\qquad\qquad) + \quad ^3$

Use your two generalisations to find an identity for $x^3 - y^3$.

smile

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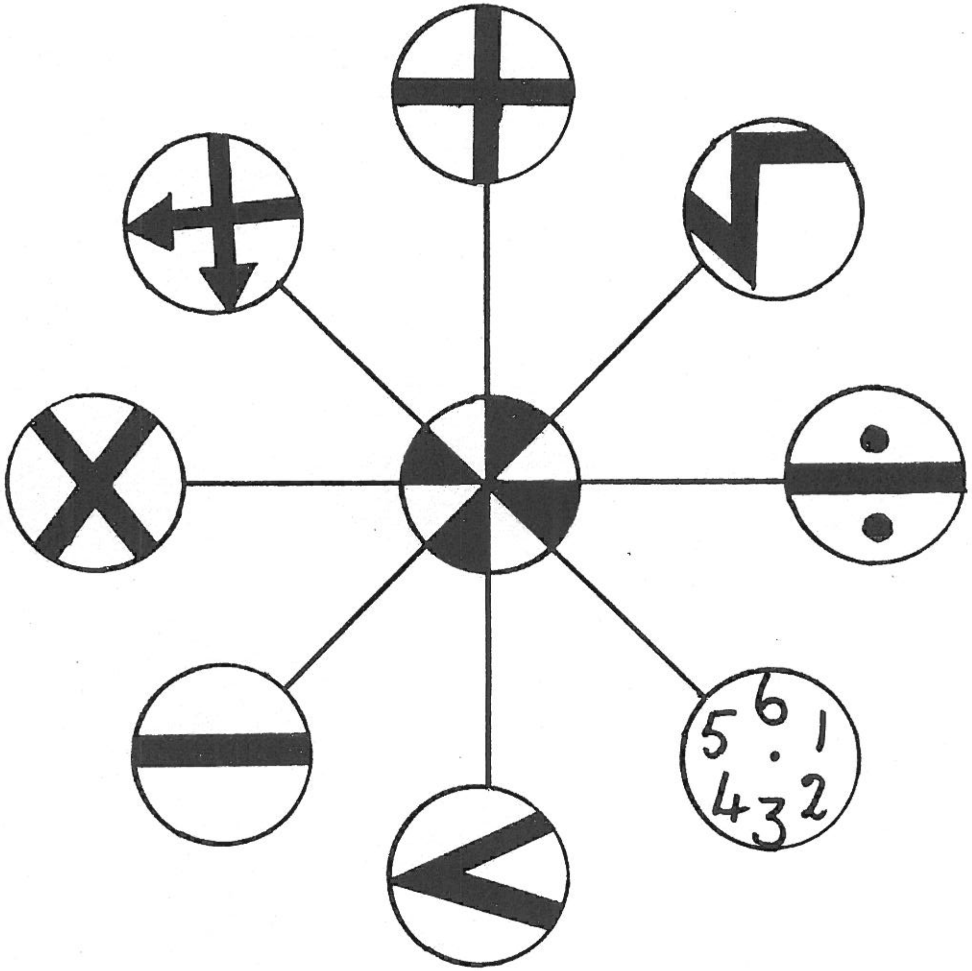
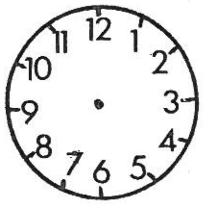
You will need:
cards A, B, C, D; card S; cards E, F, G, H, J, K

Operations



Work through this booklet in order.

Clock Arithmetic



(1) Draw a clock-face
If it is 10 o'clock now,
then 5 hours later it
will be 3 o'clock so.....

$$10 \oplus 5 = 3$$

(2) If it is 9 o'clock now, what time will
it be in 7 hours?
Write down 9 $\oplus$ 7 = 4

(3) If it is 4 o'clock now, what time will
it be in 10 hours?
Write down 4 $\oplus$ 10 = ■

(4) Work out:-

(a) 6 $\oplus$ 7 (c) 2 $\oplus$ 6 (e) 7 $\oplus$ 7
(b) 11 $\oplus$ 4 (d) 8 $\oplus$ 5 (f) 12 $\oplus$ 12

(5) Check your answers so far. They are all in the operation table on page 3.

(6) Copy and complete this table:-

Second Number

First number

$\oplus$	1	2	3	4	5	6	7	8	9	10	11	12
1												
2						8						
3												
4										2		
5												
6							1					
7							2					
8					1							
9							4					
10					3							
11				3								
12												12

(7) List the set of numbers which are <u>in</u> the table.

(8) Read this carefully and copy it:

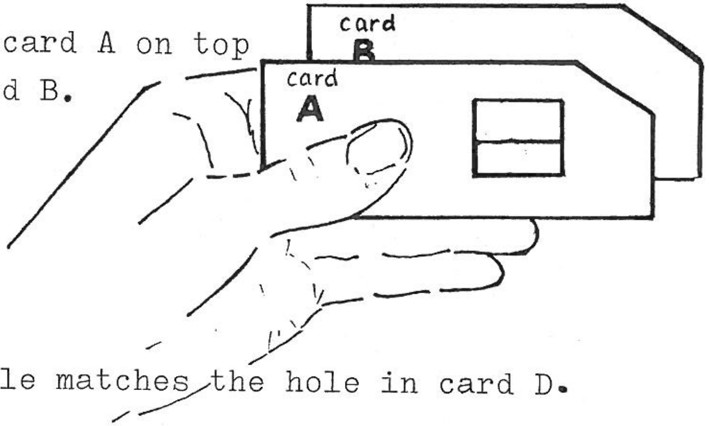
> All the numbers in the table come from the set of clock numbers, so the set is CLOSED under the operation $\oplus$

(9) Some people say that Mars has a 7 hour clock. Draw a Martian clock-face, make a table for 'addition' and say whether $\{1,2,3,4,5,6,7\}$ is closed under this operation or not.

Using Cards

Use cards A,B,C and D.

(1) Place card A on top of card B.



The hole matches the hole in card D.

(2) Place card B on top of card C.
Which card does the new hole match?

(3) Copy and complete this operation table.

Second Card

First card

On top of	A	B	C	D
A		D		
B			B	
C				
D				

(4) Is $\{A,B,C,D\}$ closed under the operation 'on top of' ?

(5) Copy and complete:-

C on top of A = ■ A on top of C = ■
C on top of B = ■ B on top of C = ■
C on top of C = ■ C on top of C = ■
C on top of D = ■ D on top of C = ■

(6) Which card combines with any of the cards without changing the hole?
This is called the identity.

(7) Copy and complete:-

■ is the identity of $\{A,B,C,D\}$ under the operation 'is on top of' because it causes no change.

(8) Look at the table.

What is special about the row and the column belonging to the identity?

(9) What is the identity for the table on page 3?

(10) What is the identity for Martian clock numbers under 'addition'?

Rotations

This section is about turning the square card S, clockwise about its centre.

(1) Copy and complete:

A	Rotate through 90°	
B	Rotate through 180°	
C	Rotate through 270°	
D	Rotate through 360°	

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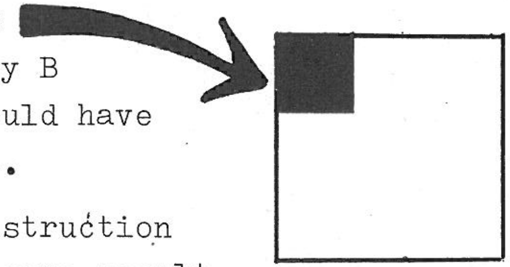
(2) Start like this



(3) Do A followed by B

(4) Check that C would have the same result.

(5) Which single instruction would have the same result as C folowed by B?

(6) Copy and complete this operation table.

(7) Is {A,B,C,D} closed under the operation followed By'? Give your reason.

Second Instruction

First Instruction	A	B	C	D
A		C		
B				
C		A		
D				

(8) Which is the identity for the set {A,B,C,D} ? Explain your answer.

Ordinary Addition

(1) Copy and complete these addition tables:-

+	1	3	5	7
1				
3		6		
5				12
7				

+	0	2	4	6
0				
2		4		
4				10
6				

(2) Is {1,3,5,7} closed under addition?

(3) Is {0,2,4,6} closed under addition?

(4) Is there an identity for {1,3,5,7} under addition? If so, what is it?

(5) Is there an identity for {0,2,4,6} under addition? If so, what is it?

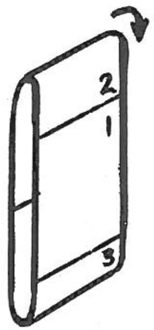
---

(6) Find out about {1,3,5,7} and {0,2,4,6} under MULTIPLICATION:

(a) Are the sets closed?

(b) Do they have identities?

Drawing operation tables will help you.



Rotating Blackboard

Operation A : move on 1 panel
Operation B : move on 2 panels
Operation C : move on 3 panels

For two operations combined the table is

Second Operation

First Operation

✳	A	B	C
A	B	C	A
B	C	A	B
C	A	B	C

(1) Copy the table.

(2) Is $\{A,B,C\}$ closed under the operation ✳ ?

(3) Explain why C is the identity.

---

(4) Shade in each square where the identity C appears.

(5) Copy and complete:-
A ✳ B = ■
B ✳ A = ■

> A and B combine in either order to give C, the identity.
> So A and B are the INVERSES of each other.

(6) Copy this and find the inverse of C.

(7) For the table on page 6, what is the inverse of **A**?

(8) For the table on page 3, what is the inverse of 7?

Remainders

Second Number

First Number

⊗	1	2	3	4	5	6
1						
2						
3					1	
4						
5						
6		5				

⊗ means multiply then divide by 7, and find the remainder.

6 x 2 = 12
12 ÷ 7 = 1, remainder 5

3 x 5 = 15
15 ÷ 7 = 2, remainder 1

(1) Copy and complete the table.

(2) Is $\{1,2,3,4,5,6\}$ closed under the operation ⊗ ?

(3) Which is the identity?

(4) Write out the 6 members of the set together with their inverses.

☒ means multiply, then divide by 4, and find the remainder.

(5) Make a table for $\{1,2,3\}$ under the operation ☒

(6) Write as much as you can about
(a) closure (b) the identity (c) inverses.

An Exercise

For each of the tables below,

(a). Is the set closed under the operation?

(b) Which member (if any) is the identity?

(c) Write the inverses of all the members (if they exist).

(1)

	X	Y	Z
X	Z	X	Y
Y	X	Y	Z
Z	Y	Z	X

(2)

	p	q	r
p	p	r	q
q	r	q	p
r	q	p	r

(3)

	5	6	7	8
5	7	8	5	4
6	4	7	6	5
7	5	6	7	8
8	8	4	8	7

(4)

	A	B	C	D
A	A	B	C	D
B	B	D	A	C
C	C	A	D	B
D	D	C	B	A

(5) Make an operation table for the set $\{K, L, M\}$ under the operation ⦶ if

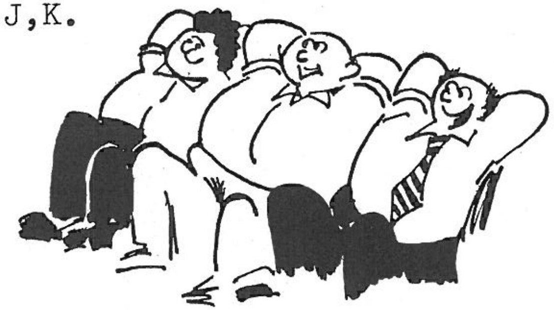
M is the identity

K and L are inverses of each other

K ⦶ K = L and L ⦶ L = K

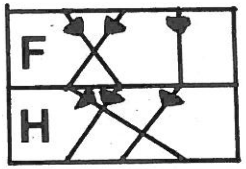
Permutations

Use cards E,F,G,H,J,K. They show how Ed, Fred and Ned, who sit in a row, can swap places.



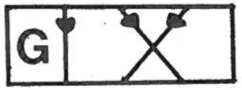
Use 3 counters (of different colours) to see how the cards work.

Ed, Ned and Fred start in the same places every day.



One day they used F followed by H.

The next day, they just used G.



CHECK THAT THE RESULT WAS THE SAME.

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(1) Draw a complete operations table for $\{E, F, G, H, J, K\}$ under 'followed by'.

(2) Write as much as you can about:-

(a) Whether the set is closed or not,

(b) an identity (if it exists),

(c) inverses and self inverses.

smile

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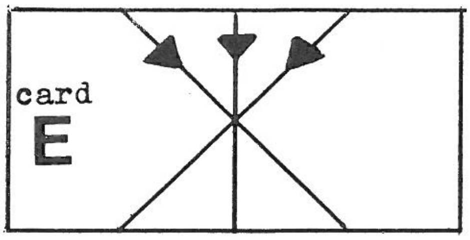
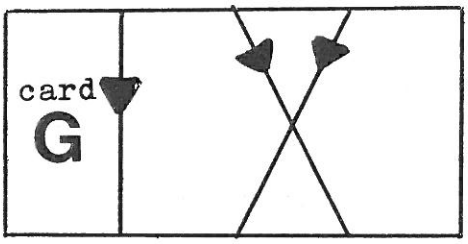
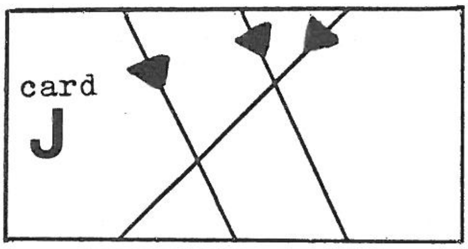
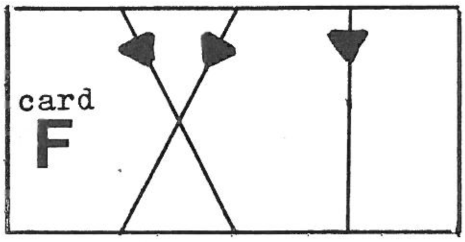
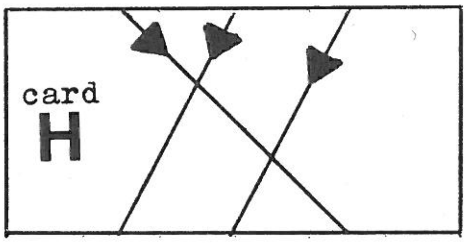
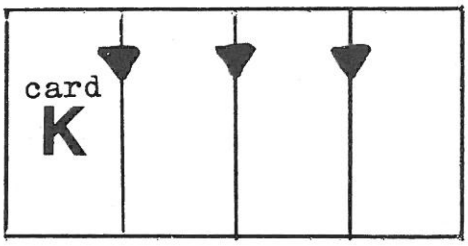

card
E

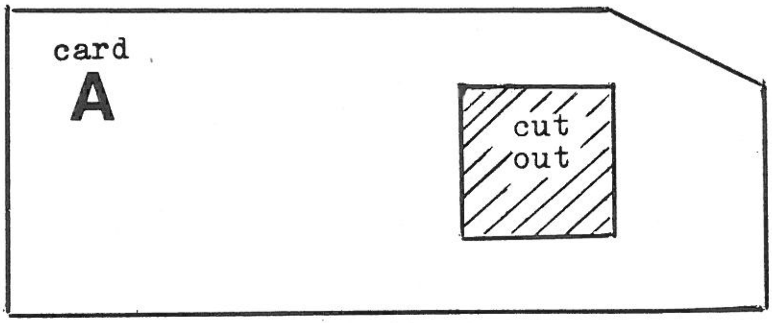

card
G

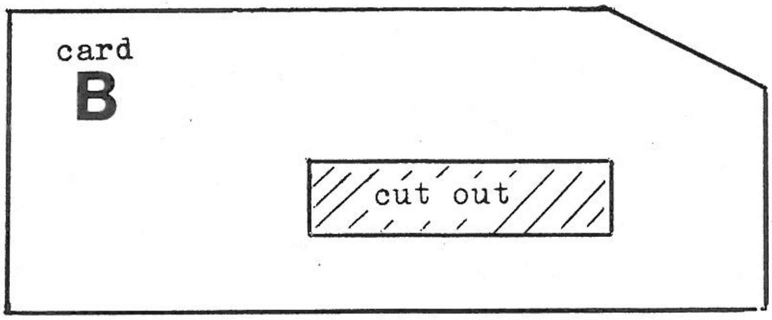

card
J

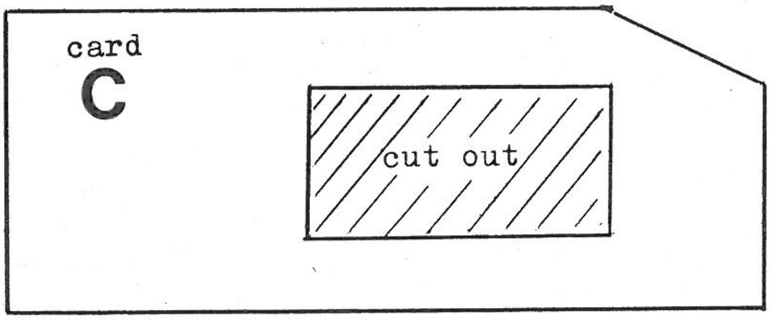

card
F

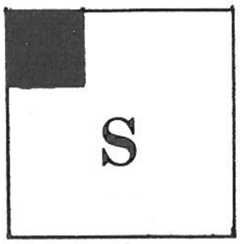

card
H

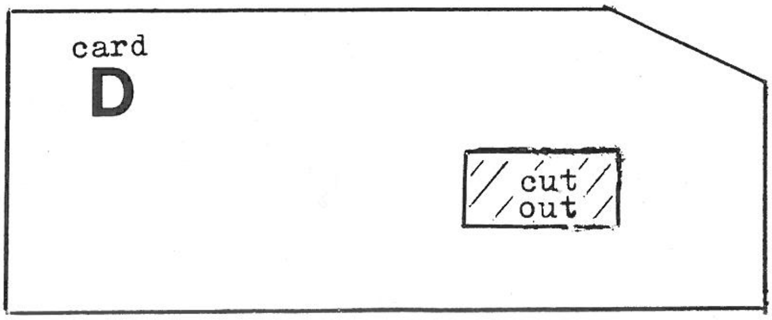

card
K


card
A
cut
out


card
B
cut out


card
C
cut out


S


card
D
cut
out

Smile **1736**

ALGEBRA PAIRS

This booklet will help you to understand the meaning of some algebraic expressions.

You will probably want to substitute numbers into the expressions. Drawing graphs might also be helpful if you want to understand the work fully.

For each pair find a value of the letter which makes the expressions equal.

There may be several answers possible . . . or just one answer . . . or no answers at all.

Write down as much as you can find out.

$q + 7$	$q + 8$
$x + 4$	$4x$
x^2	$3x$
$2y$	$y + 20$
$4(p+2)$	$4p+2$
$3(p+1)$	$3p+3$
$b-3$	$b+3$
$2q$	$3q$

Find another pair of expressions which are equal for only one value of x.

Find another pair which are equal for two values of x.

Find another pair which are never equal.

Find a pair which are probably equal for all values of x.

For each pair decide whether they are

Always Equal

Sometimes Equal

Never Equal

Find another pair which are always equal.

Find another pair which are sometimes equal.

$2x$	x^2
$b+b+b$	$3b$
$3x$	$x+3$
$k+10k$	$11k$
$k+10$	$k+k$
$5p-p$	$5(p-1)$
$2(x+3)$	$2x+6$
$3y+1$	$4y$
$3p+q$	$3pq$

Which of these expressions are always equal?
Explain.

$$\frac{1}{2}a + b$$

$$\frac{b}{2} + \frac{a}{2}$$

$$\frac{1}{2}(a+b)$$

$$\frac{a+b}{2}$$

$$a + \frac{1}{2}b$$

$$\frac{1}{2}a + \frac{1}{2}b$$

$$\frac{a}{2} + b$$

$$\frac{b}{2} + a$$

SMILE **1682**

Number Jumble

Each of the following groups of statements describes a number less than 25.
In fact, all the letters represent an integer less than 25.

J

e is a prime whose square number has digits which include those of e.

E

I

$c = S^2 + 1$
$c = \frac{100}{r}$
$c = 2y - 17$

C

$3 \leqslant d^2 < 25$
$d = 2t + 1$

D

f is an integer which is one more than a square number.
(f + 1) is not a multiple of 3.

F

$a = 2^x$
$a = 4p$
$ax = 24$

A

G

g is a two-digit number whose square root is a power of 2

$b = 3m$
$b = 1 + 2 + 3 + 4 + \ldots + n$
$b = k^2 + 5$

B

H

Identify the integers represented by a, b, c, d, e, f and g.
Write a description for h, i and j.
Test them on a friend.

Two-digit sums

Choose any 3 digits.

List all the 2-digit
numbers that can be
made with these.

Find x *the total of all these 2-digit numbers.*

y *the total of the original three digits.*

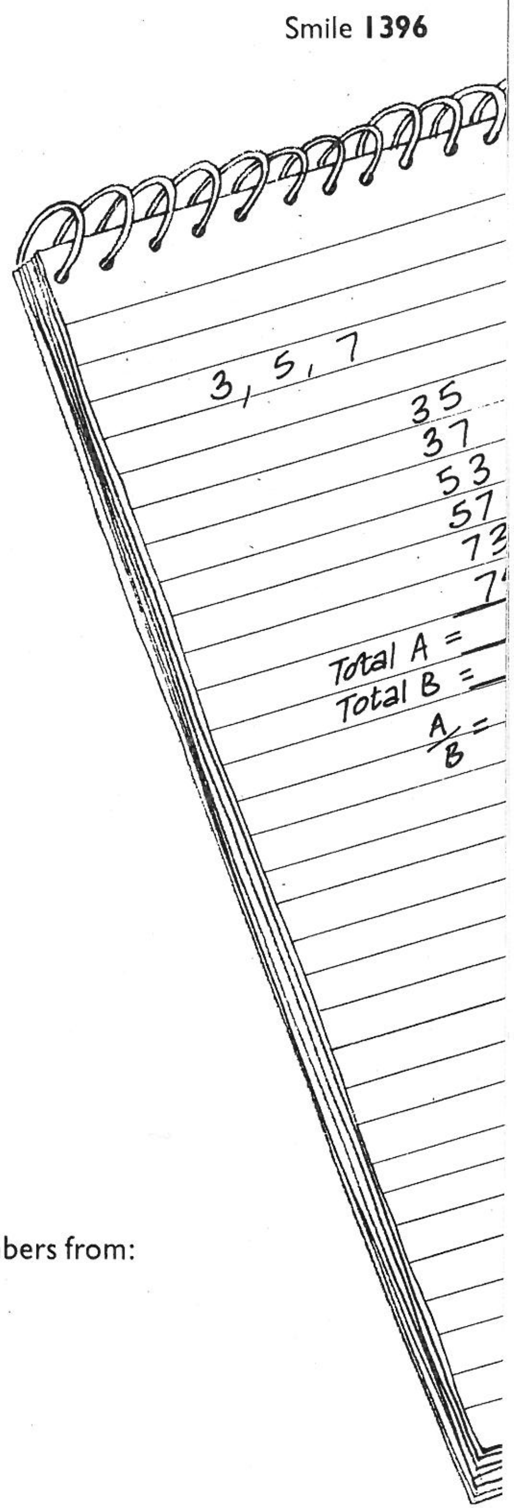
Calculate the ratio $\frac{x}{y}$

Investigate with other sets of 3 digits.

Use the 3 digits a, b, c to prove any
results you have found with 3 digits.

If you want some hints see the back of the card. The notebook shows what happens if you start with two digits instead of three.

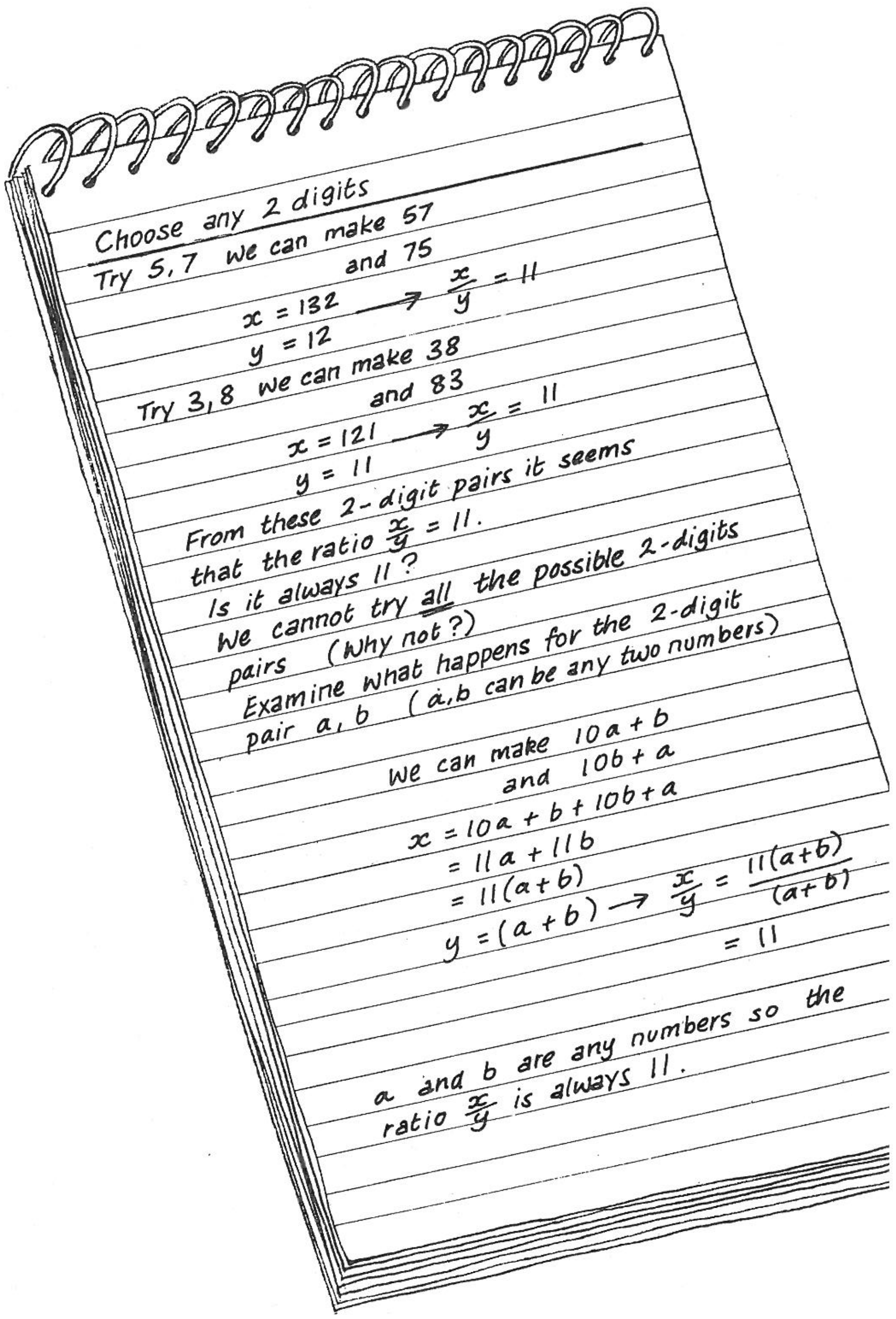
Investigate the results for making 2 digit numbers from:
4 digits
5 digits
n digits



3, 5, 7

35
37
53
57
73
7

Total A =
Total B =
$\frac{A}{B}$ =



Choose any 2 digits

Try 5,7 we can make 57
and 75

$x = 132 \longrightarrow \frac{x}{y} = 11$
$y = 12$

Try 3,8 we can make 38
and 83

$x = 121 \longrightarrow \frac{x}{y} = 11$
$y = 11$

From these 2-digit pairs it seems
that the ratio $\frac{x}{y} = 11$.

Is it always 11?

We cannot try all the possible 2-digits
pairs (Why not?)

Examine what happens for the 2-digit
pair a, b (a,b can be any two numbers)

We can make $10a + b$
and $10b + a$

$x = 10a + b + 10b + a$
$= 11a + 11b$
$= 11(a+b)$
$y = (a+b) \rightarrow \frac{x}{y} = \frac{11(a+b)}{(a+b)}$
$= 11$

a and b are any numbers so the
ratio $\frac{x}{y}$ is always 11.

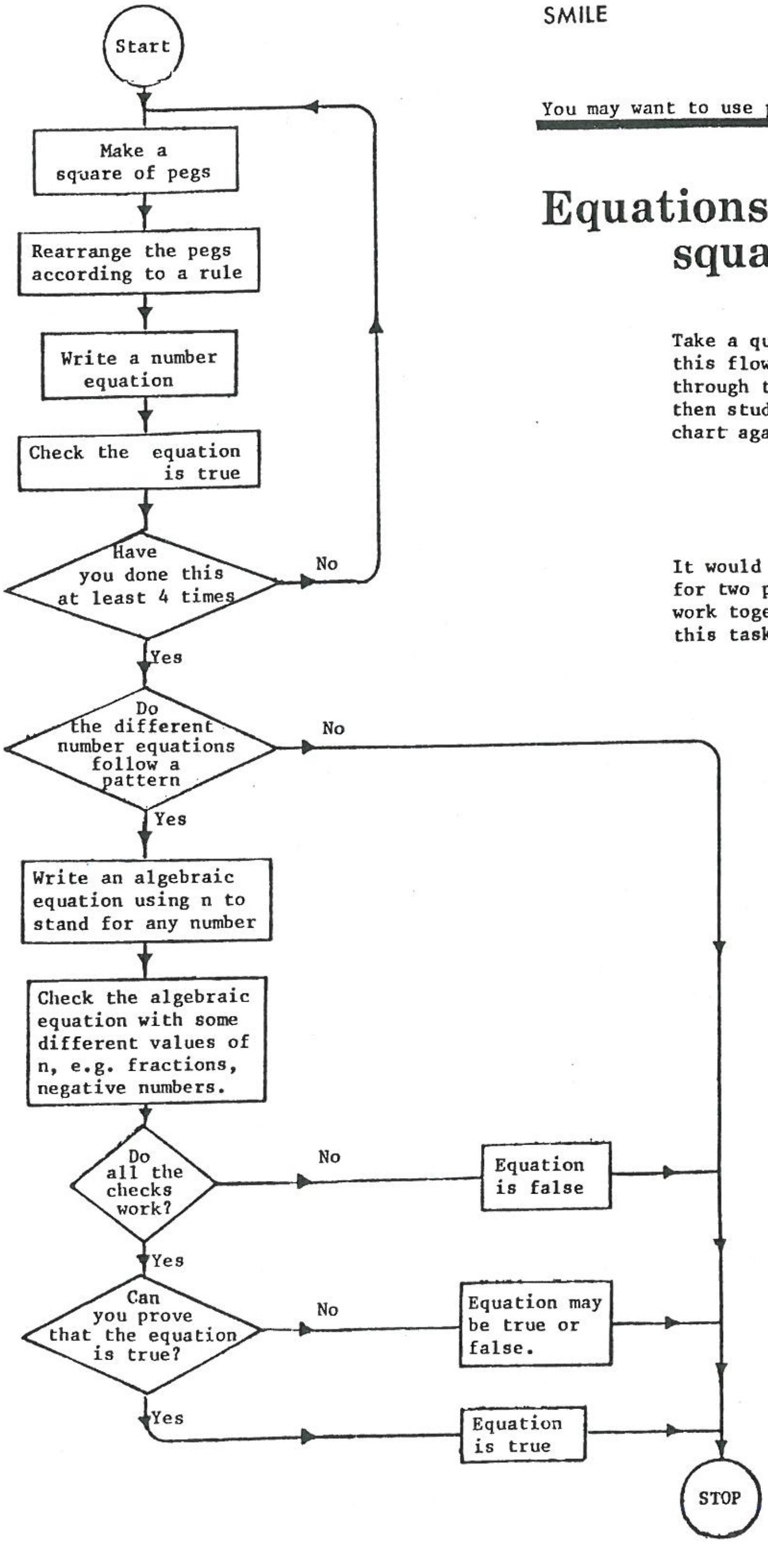
SMILE 0820

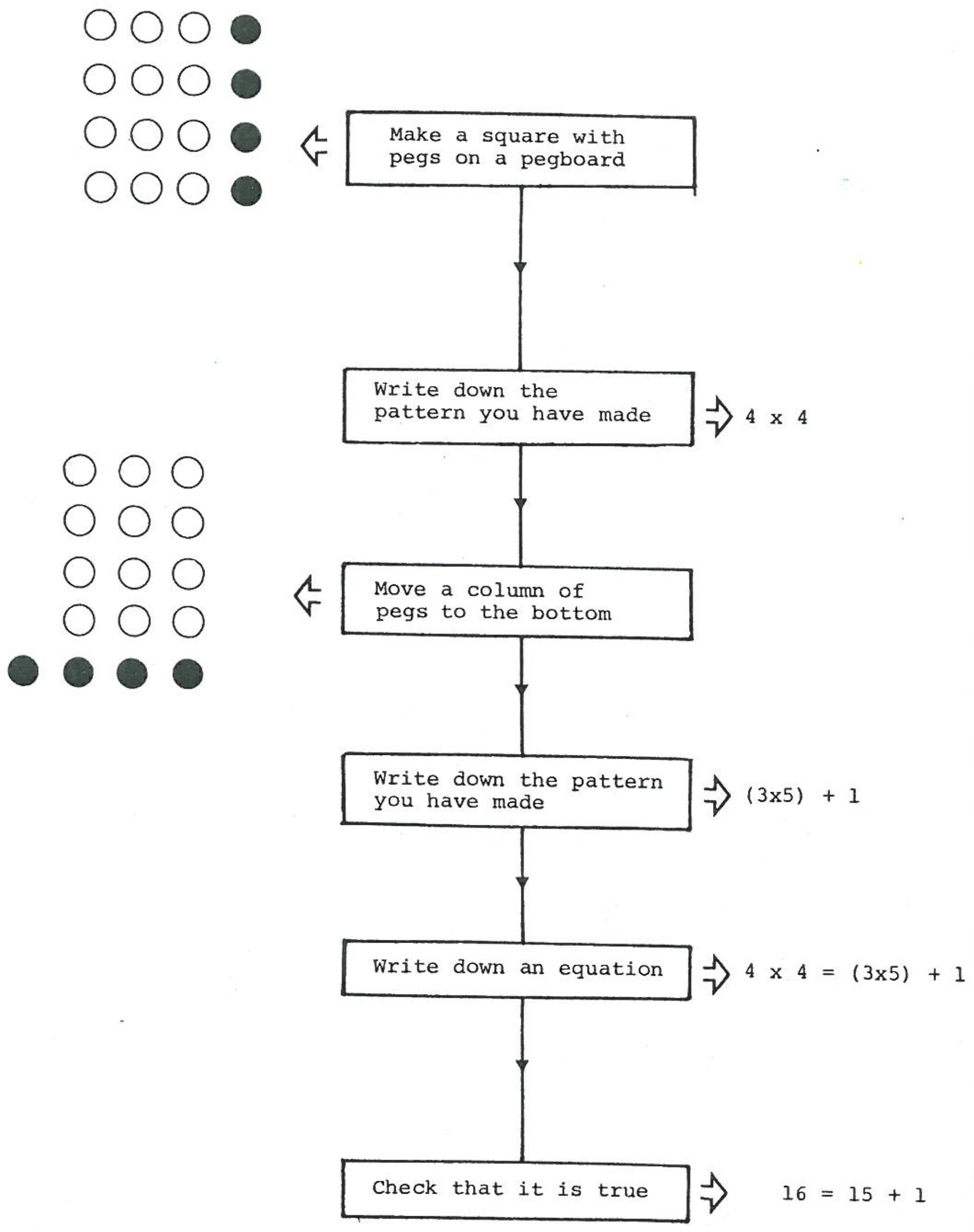
You may want to use pegboard and pegs

Equations from squares

Take a quick look at this flow-chart, work through the booklet and then study the flow chart again.

It would be helpful for two people to work together on this task.





(1) Follow the flow-chart opposite for these squares:

(a) 3 x 3 (b) 8 x 8 (c) 6 x 6

(2) Complete the equations for each one:

(a) 3 x 3 = (■ x ■) + 1 (b) 8 x 8 = (■ x ■) + 1
(c) 6 x 6 =

(3) Complete the equation for a 20 x 20 square:

20 x 20 = (■ x ■) + 1

(4) Complete the equation for an n x n square

n x n =

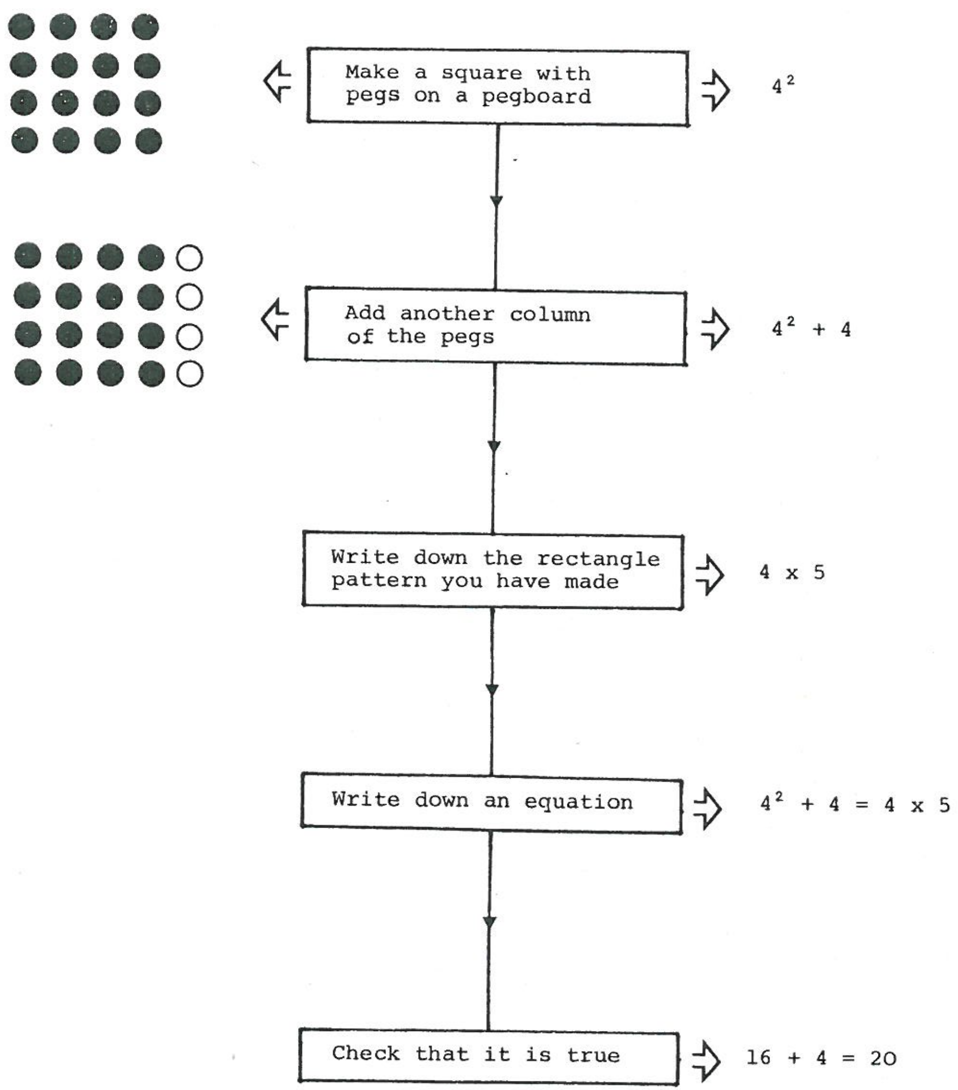
(5) If n = 7 then,

n x n
= 7 x 7
= 49

(n+1)(n-1) + 1
= (8 x 6) + 1
= 48 + 1
= 49

So the equation is true when n = 7
Check the equation for two other positive integers.

(6) Check the equation when (a) n = 1½
(b) n = 0
(c) n = 8.7
(d) n = -4



(1) Follow the flow-chart opposite for these squares

(a) 5 x 5 (b) 3 x 3 (c) 7 x 7

(2) Complete the equations for each one:

(a) $5^2 + 5 = $ ■ x ■ (b) $3^2 + 3 = $ ■ x ■ (c) $7^2 + 7 =$

(3) Complete the equation for a 20 x 20 square:

$20^2 + 20 = $ ■ x ■

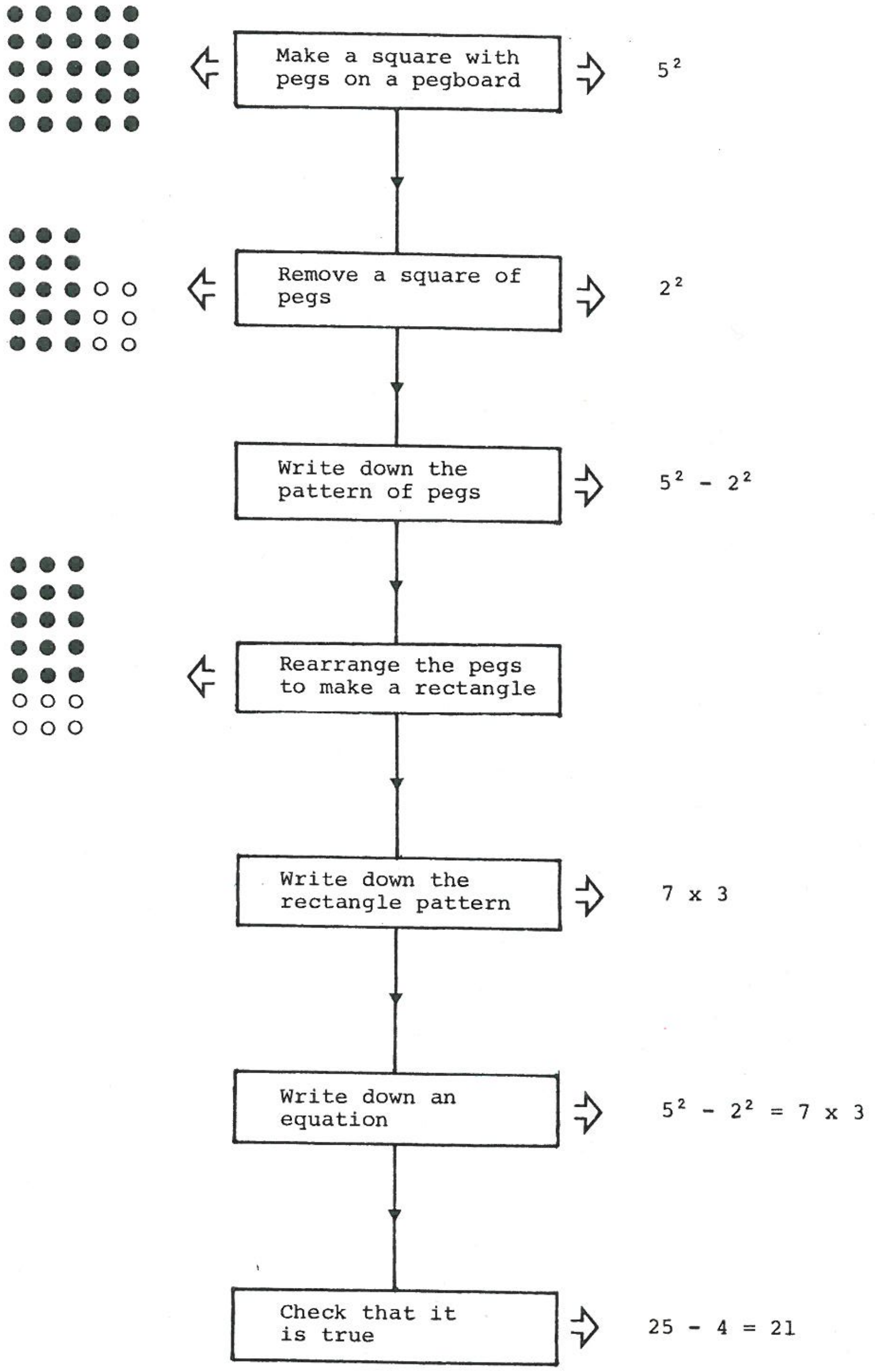
(4) Complete the equation for an n x n square:

$n^2 + n =$

(5) Is the equation true for <u>any</u> value of n ?

Check the equation .... (a) when n is an integer
(b) when n is a fraction
(c) when n is negative
and then discuss your answer.

(6) Look back at the front cover and study the flow-chart.



(1) Follow the flow-chart opposite starting with a 7 x 7 square and removing a 3 x 3 square:

Complete the equation $7^2 - 3^2 = ■ \times ■$

(2) Using the pegboard if necessary, copy and complete :

(a) $5^2 - 3^2 = ■ \times ■$ (b) $7^2 - 2^2 = ■ \times ■$

(c) $6^2 - 2^2 = ■ \times ■$ (d) $8^2 - 5^2 = ■ \times ■$

(3) Complete the equation for starting with a 20 x 20 square and removing a 3 x 3 square:

$$20^2 - 3^2 = ■ \times ■$$

(4) Complete the equation for starting with an $x \times x$ square and removing a y x y square

$$x^2 - y^2 = ■ \times ■$$

(5) The equation

$$x^2 - y^2 = (x + y)(x - y)$$

is true for <u>all</u> values of x and y

Can you follow this working?

$$\begin{aligned} \text{L.H.S.} &= x^2 - y^2 \\ &= x^2 + xy - xy - y^2 \\ &= (x^2 + xy) - (xy + y^2) \\ &= x(x + y) - y(x + y) \\ &= (x - y)(x + y) \\ &= \text{R.H.S.} \end{aligned}$$

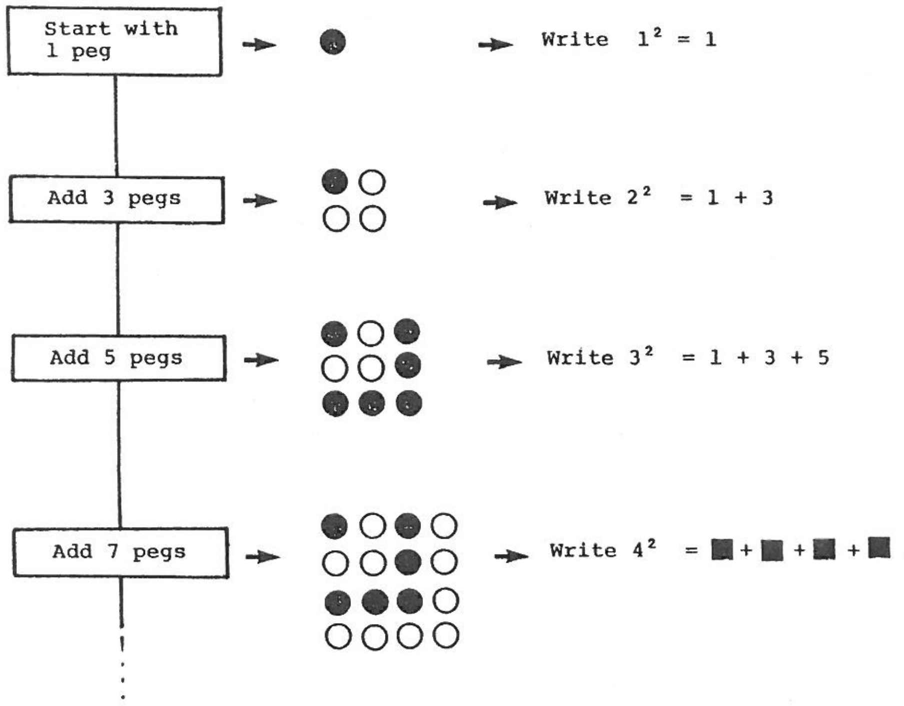
The proof works both ways. Copy and complete:

$$\begin{aligned} \text{R.H.S.} &= (x + y)(x - y) \\ &= x\,(x - y) + ■(x - y) \\ &= x^2 - xy + ■ - ■ \\ &= x^2 - ■ \\ &= \text{L.H.S.} \end{aligned}$$

(6) Use the equation to evaluate

(a) $51^2 - 49^2$

(b) $77^2 - 67^2$



(1) Complete the equations:

i) $5^2 = ■ + ■ + ■ + ■ + ■$

ii) $10^2 =$

(2) Complete the equation for a 20 x 20 square:

$20^2 = 1 + 3 + \cdots \qquad \cdots + ■$

(3) Write the equation for an n x n square:

$n^2 = 1 + 3 + \cdots \qquad \cdots + ■$

(4) Check if the equation is true for the following values of n:

i) $n = 6$ ii) $n = 15$ iii) $n = \frac{1}{2}$ iv) $n = -2$

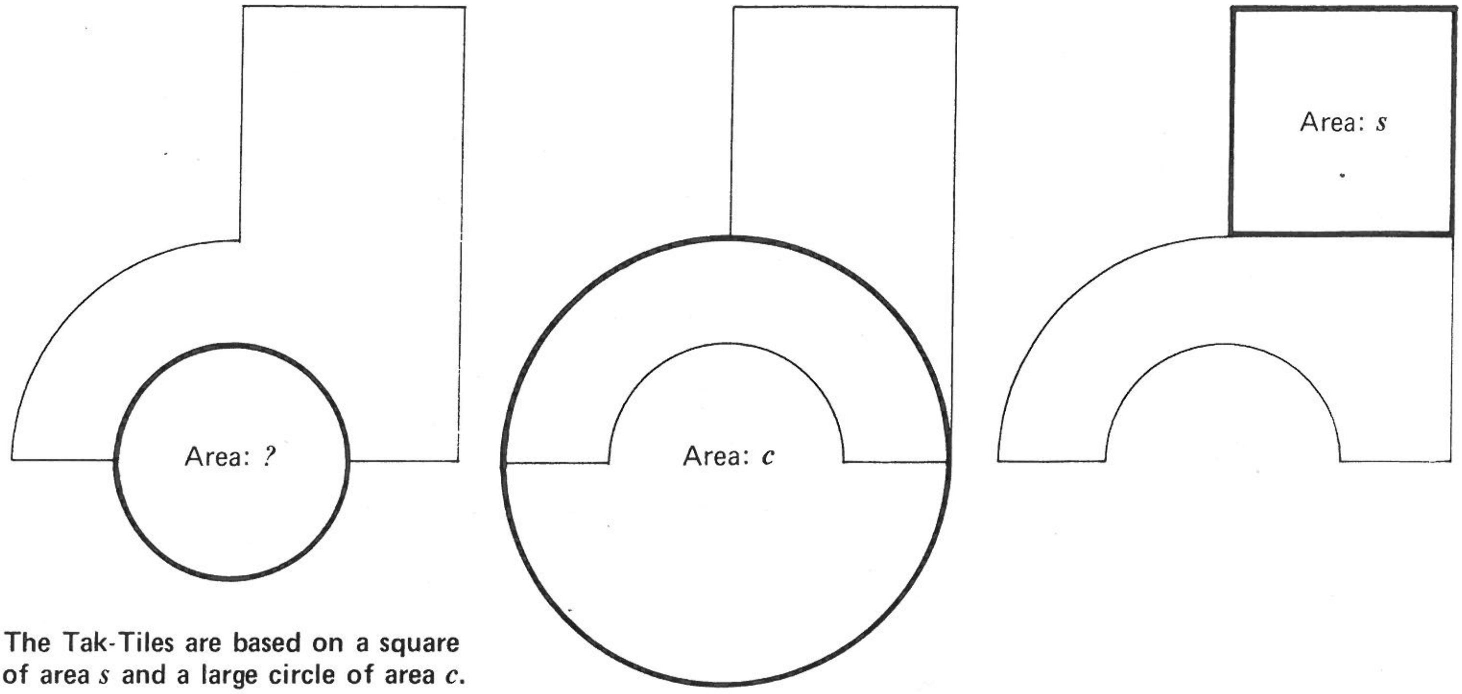
The equation is not true for all values of n.

For which values of n do you think it is true. Discuss this and try to give reasons.

You will need a set of Tak-Tiles

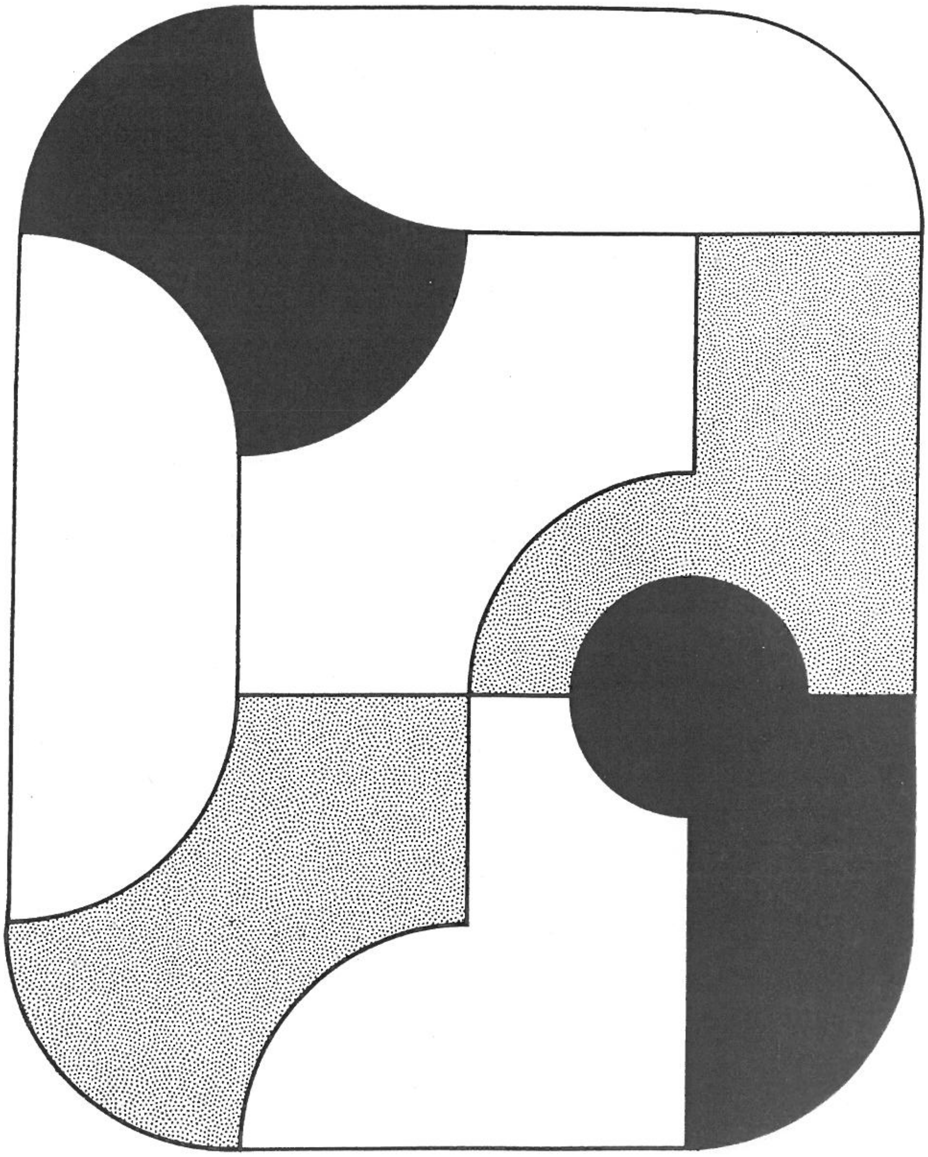
SMILE 1323

Tak-Tile Areas



The Tak-Tiles are based on a square of area s and a large circle of area c.

1. The diameter of the small circle is half the diameter of the large circle.
   What is the area of the small circle in terms of c ?
2. Find the area of each Tak-Tile in terms of s and c.
3. What is the total area of the eight Tak-Tiles?
4. You might have found the total area by adding the areas of the separate tiles or by looking at the complete shape.
   Use the other method to check your answer to question 3.
5. If the radius of the large circle is r, express the total area in terms of r.
6. Express s in terms of c.



0819

You may want to use pegboard and pegs

SMILE

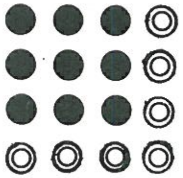
PROVE YOUR IDENTITY

(1) Copy and complete:

$$\longrightarrow 1^2 = 0^2 + 1$$

$$\longrightarrow 2^2 = 1^2 + 3$$

$$\longrightarrow 3^2 = 2^2 + \blacksquare$$



$$\longrightarrow \blacksquare = \blacksquare + 7$$

$$10^2 = \blacksquare + \blacksquare$$

$$\blacksquare = \blacksquare + 39$$

In general, $n^2 = \blacksquare + \blacksquare$

The general equation from page 2 is

$$n^2 = (n-1)^2 + (2n-1)$$

We know that this equation is true for some values of n (the ones on page 2). We could easily find out if the equation is true for other particular values of n by substitution. But is the equation an identity? (That means it is always true?)

To find out, start with the right-hand side of the equation,

$$\text{i.e. } (n-1)^2 + (2n-1)$$

Simplify this expression - start by rewriting $(n-1)^2$ as n^2-2n+1

You should get n^2. This proves that the equation is true for every value of n. It is an identity.

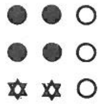
N.B. The proof might be set out like this :

$$\begin{aligned} RHS &= (n-1)^2 + (2n-1) \\ &= (n^2-2n+1) + (2n-1) \\ &= n^2 - 2n + 1 + 2n - 1 \\ &= n^2 \\ \therefore\ n^2 &= (n-1)^2 + (2n-1) \end{aligned}$$

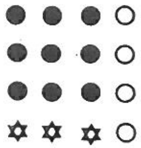
(2) Copy and complete:

$1^2 = 0^2 + 1$

$2^2 = 1^2 + 2 + 1$



$3^2 = \blacksquare + 3 + 2$



$4^2 = \blacksquare + \blacksquare + 3$

$10^2 = \blacksquare + \blacksquare + \blacksquare$

$\blacksquare = \blacksquare + 20 + \blacksquare$

In general, $n^2 = \blacksquare + \blacksquare + \blacksquare$

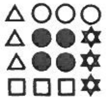
Can you prove that this is an identity?

---

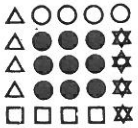
(3) Copy and complete:

$2^2 = 0^2 + 4x1$

$3^2 = 1^2 + 4x2$



$4^2 = \blacksquare + 4x3$



$5^2 = \blacksquare + \blacksquare$

$10^2 = \blacksquare + \blacksquare$

$\blacksquare = \blacksquare + 4x20$

In general $n^2 = \blacksquare + \blacksquare$

Can you prove that this is an identity?

(4) Copy and complete:

× ×
× ×

$2^2 = 0^2 + 2x2 + 2x0$

× ○ ×
× ● ×
× ○ ×

$3^2 = 1^2 + 2x3 + 2x1$

× ○ ○ ×
× ● ● ×
× ● ● ×
× ○ ○ ×

4^2 = ■ + 2x4 + 2x2

× ○ ○ ○ ×
× ● ● ● ×
× ● ● ● ×
× ● ● ● ×
× ○ ○ ○ ×

■ = 3^2 + ■ + ■

⋮

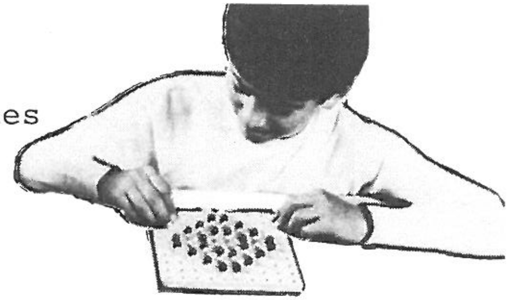
10^2 = ■ + ■ + ■

⋮

In general, n^2 = ■ + ■ + ■

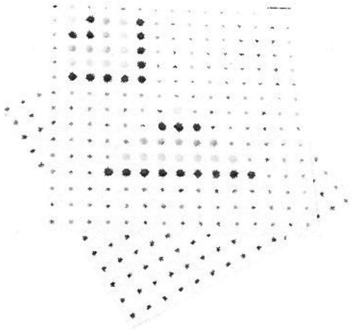
Can you prove that this is an identity?

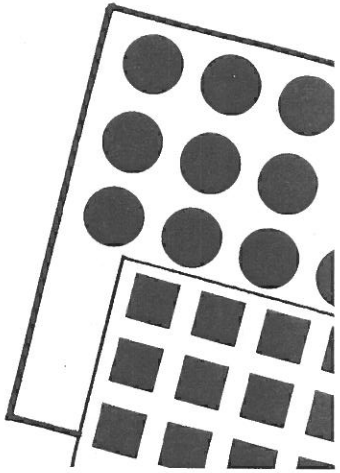
(5) Construct another series according to a rule.



(6) Try to write an equation for the general case.

(7) Try to prove your equation is an identity.





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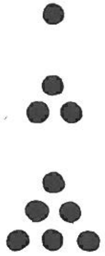
(1)



$T_1 = 1$

$T_2 = 3$

$T_3 = 6$

and so on...

Make a table of
n and T_n for
n = 1,2,3,...6

Draw a suitable
grid and plot T_n
against n.

Why would it not
be sensible to join
the points on this
graph?

(2)

$T_1 + T_2 = 2^2$

$T_2 + T_3 = 3^2$

and so on...

Generalise this
pattern for $T_{n-1} + T_n =$ ■

(3)

$T_1 + 2 = T_2$

$T_2 + 3 = T_3$

and so on...

Generalise this pattern
for $T_{n-1} +$ ■ $=$ ■

(4) Your answers to questions (2) and (3) both use T_{n-1} and T_n.

Rearrange the first so that it becomes $T_{n-1} =$ ■

Rearrange the second so that it too, becomes $T_{n-1} =$ ■

You now have two statements for T_{n-1} so ■ = ■

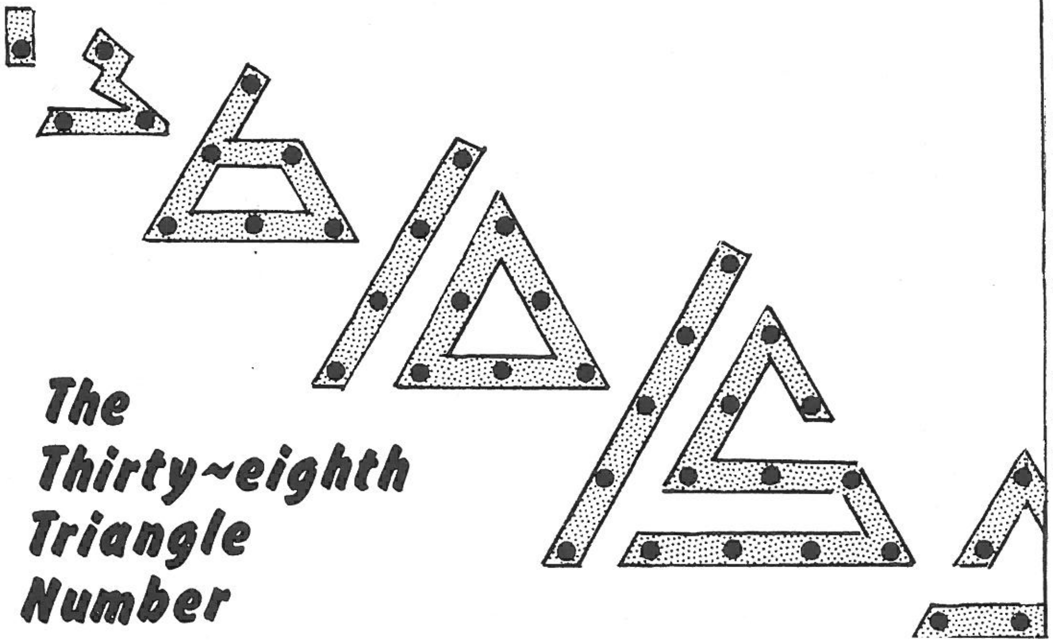
Rearrange this and write an expression for T_n in terms of n.
Test your result.

Look at the number pattern on card 0221.

Does your result agree with this?

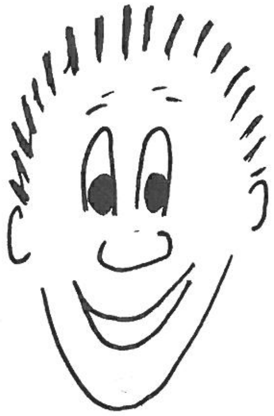
How does the expression for T_n explain the shape of the graph you plotted in question (1)?

Use your equation to find
a) the 100th triangle number.
b) the 38th triangle number.



The Thirty~eighth Triangle Number

PROVE IT !



Think of a whole number.
Square it.
Divide by 4.

Try several numbers.
What do you notice ?

O.K. Dennis, I've found a result but does it always work ?

Of course it does.

Prove it !

Well, an even number must be
twice some other number; call
it 2n.

An odd number is alw
one less than an even
I'll call the odd
2n−1.

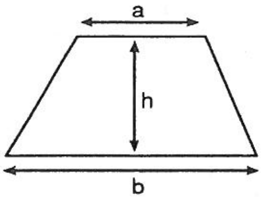
If you squar
you would
and th
then
w

WHAT IS THE RULE AND HOW DID DENNIS PROVE IT ?

Subject of a Formula

$A = \frac{1}{2}(a + b)h$ is the formula used to calculate the area of a trapezium where:

A = area of a trapezium
a = length of one of the parallel sides
b = length of other parallel side
h = height



A is the **subject** of this formula $A = \frac{1}{2}(a + b)h$

$A = \frac{1}{2}(a + b)h$ can be rearranged to make **b** the subject of the formula $b = \frac{2A}{h} - a$

Here are two methods to show how to make **b** the subject of the formula:

Using rearrangement

$$A = \frac{1}{2}(a + b)h$$

multiply both sides by 2 $2A = (a + b)h$

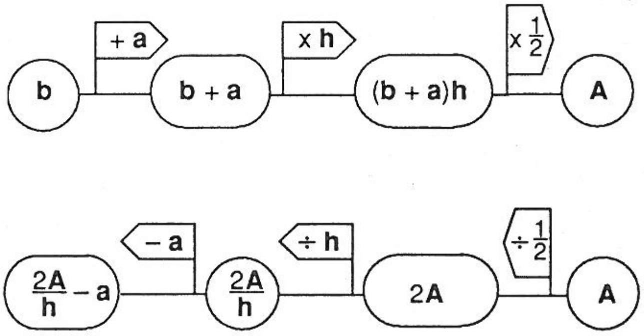
divide both sides by **h** $\frac{2A}{h} = a + b$

subtract **a** from both sides $\frac{2A}{h} - a = b$

$$\boxed{b = \frac{2A}{h} - a}$$

Using flag diagrams

$$A = \frac{1}{2}(a + b)h$$



$$\boxed{b = \frac{2A}{h} - a}$$

- Choose either method to check that: the formula is $\boxed{a = \frac{2A}{h} - b}$ when **a** is the subject.

  the formula is $\boxed{h = \frac{2A}{a + b}}$ when **h** is the subject.

- Substitute the values **a** = 4, **b** = 6 and **h** = 7 into the formula where **A** is the subject, to find a value for **A**.

- Substitute the values for **a**, **h** and **A** into the formula where **b** is the subject, to check that **b** = 6.
- Substitute the values for **b**, **h** and **A** into the formula where **a** is the subject, to check that **a** = 4.
- Substitute the values for **a**, **b** and **A** into the formula where **h** is the subject, to check that **h** = 7.

$T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$ is the formula for calculating the **time** of a swinging pendulum where:

T = time
L = length of the string
g = acceleration of gravity.

T is the subject of this formula. $T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$

$T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$ can be rearranged to make **L** the subject of the formula $L = g\left(\frac{T}{2\pi}\right)^2$

Using rearrangement

$$T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$$

divide both sides by 2π $\frac{T}{2\pi} = \sqrt{\left(\frac{L}{g}\right)}$

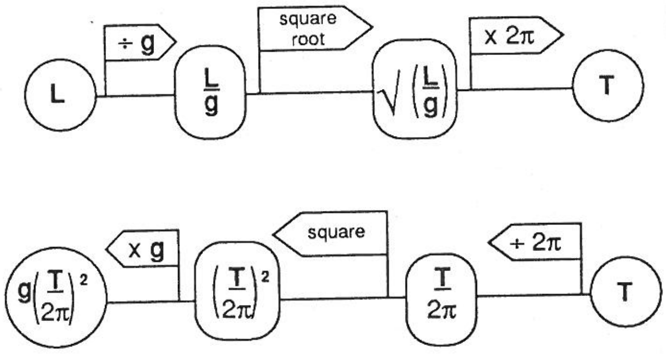
square both sides $\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g}$

multiply both sides by **g** $g\left(\frac{T}{2\pi}\right)^2 = L$

$$\boxed{L = g\left(\frac{T}{2\pi}\right)^2}$$

Using flag diagrams

$$T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$$



$$\boxed{L = g\left(\frac{T}{2\pi}\right)^2}$$

- Choose either method to check the formula is $\boxed{g = L\left(\frac{2\pi}{T}\right)^2}$ when **g** is the subject.
- Substitute values for **T**, **L** and **g** to check that the rearrangements are correct.
- Rearrange each formula to make the letter in brackets the subject of the formula.

1. $A = 3b$ **(b)**
2. $v = lk$ **(l)**
3. $v = u + at$ **(t)**
4. $m = \frac{1}{2}(x + y)$ **(x)**
5. $F = \frac{mv^2}{r}$ **(v)**
6. $F = \frac{mv^2}{r}$ **(r)**
7. $d = \sqrt{(11.5h)}$ **(h)**
8. $A = 3(p + 5)$ **(p)**
9. $v^2 = u^2 + 2as$ **(s)**
10. $v^2 = u^2 + 2as$ **(u)**
11. $a = 6 - \frac{12}{r}$ **(r)**
12. $s = \frac{(u + v)t}{2}$ **(v)**

13. Make **c** the subject of the formula $T = \frac{12}{\sqrt{c}}$

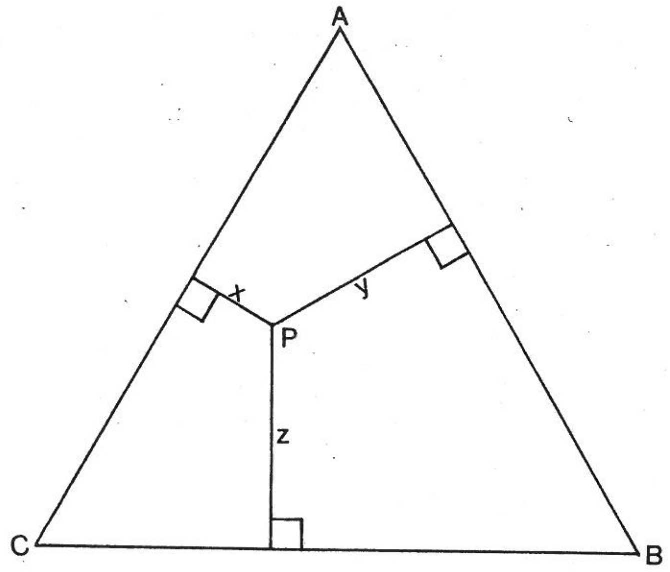
    Use your formula to find **c** when $T = 3$
    $T = 2.4$

14. The formula for the perimeter **P** of a rectangle is **P = 2L + 2W** where **L** = length and **W** = width. Make **L** the subject of this formula and find the length of a rectangle whose perimeter is 36cm and whose width is 7.3cm.

15. The formula for the surface area of a cylinder is $S = 2\pi rh + 2\pi r^2$ where **r** = radius and **h** = height. Make **h** the subject of the formula and find the height of a cylinder with surface area of 84cm^2 and radius 2cm.

You may want to use isometric paper.

Perpendicular Proof



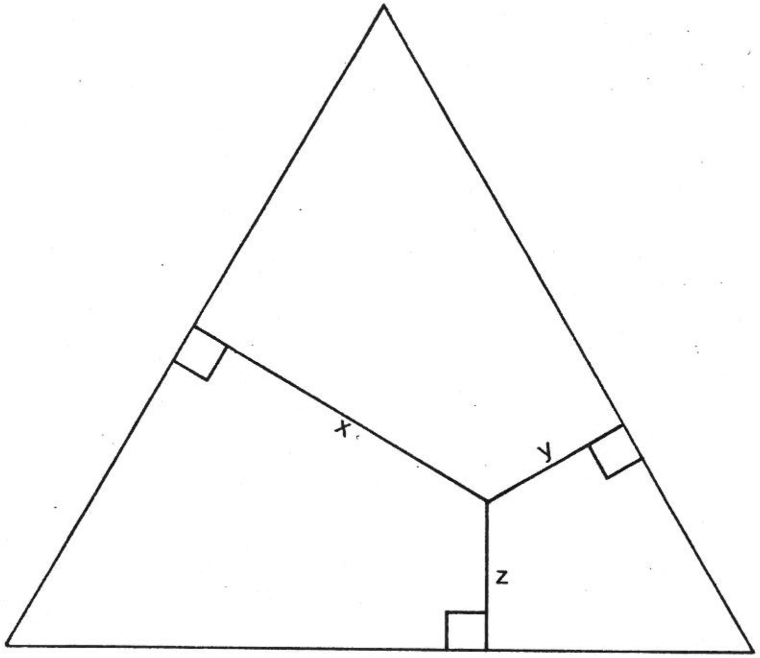
Draw a large equilateral triangle.

Choose a point, P, somewhere inside the triangle and measure x, y and z, the perpendicular distances from P to the three sides.

Record your results.

Draw several more equilateral triangles all the same size. Choose P in a different position inside each triangle and measure x, y and z.

What do you notice about your results?



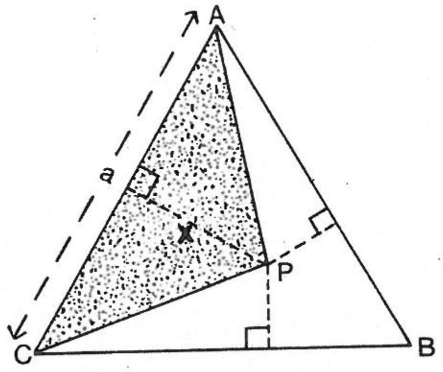
Did you find that x + y + z is constant?

Follow the same procedure for a different size triangle.

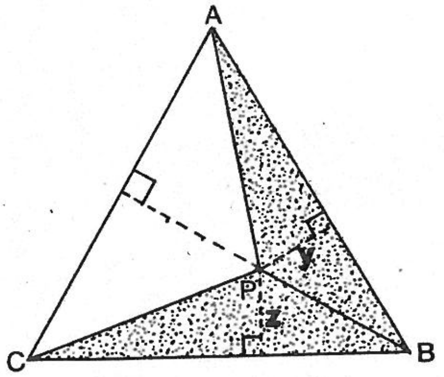
Is x + y + z always constant for this triangle?

In fact this result is true for ***any*** size of equilateral triangle. It is impossible to show this by drawing and measuring (why?) but we can still be absolutely sure that it is true.

Work through the rest of this card to ***prove*** that in ***any*** equilateral triangle, x + y + z is constant.



If the length of each side of the equilateral triangle is a, find the area of triangle APC.



Find also the areas of triangles APB and BPC.

What is the area of the equilateral triangle ABC?

Use this result to ***prove*** that in any equilateral triangle, x + y + z is constant, whatever the position of P inside the triangle.

Partners

You wouldn't expect to find that the product of two numbers is the same as their sum. But it's true for these pairs . . .

$$4\tfrac{1}{2} \xleftarrow{+} \left(\tfrac{3}{2}, 3\right) \xrightarrow{\text{x}} 4\tfrac{1}{2}$$

$$5\tfrac{1}{3} \xleftarrow{+} \left(\tfrac{4}{3}, 4\right) \xrightarrow{\text{x}} 5\tfrac{1}{3}$$

$$6\tfrac{1}{4} \xleftarrow{+} \left(\tfrac{5}{4}, 5\right) \xrightarrow{\text{x}} 6\tfrac{1}{4}$$

Find some more pairs.

Can you find a partner for $1\frac{5}{6}$? Do all numbers have a partner?
Can you explain?

Changing the Subject

In the equation $\mathbf{x = ab + 2b}$ **'x'** is the **subject** of the equation, because the equation is in the form x = . . . and 'x' only occurs once.

The equation can be rearranged to make '**b**' the subject. 'b' appears twice in the equation. To make 'b' occur once a factor of b has to be taken.

$$x = ab + 2b$$

Take a factor of b $\quad x = b(a + 2)$

Divide both sides by (a + 2) $\quad \frac{x}{a + 2} = b$

or $\boxed{\mathbf{b = \frac{x}{a + 2}}}$

- Rearrange the equation to check that with '**a**' the subject, the equation is $\mathbf{a = \frac{x - 2b}{b}}$

- Make the letter in brackets the subject of each of these equations.

1. $Z = 3p + pq$ (**p**)
2. $s = 2ac + 4ab$ (**a**)
3. $h = d^2 - 3hR$ (**h**)

In the equation $\mathbf{x = \frac{3y + 4}{2 - y}}$ 'x' is the subject. It can be rearranged to make '**y**' the subject.

$$x = \frac{3y + 4}{2 - y}$$

Multiply both sides by (2 – y) $\quad x(2 - y) = 3y + 4$

Expand the brackets $\quad 2x - xy = 3y + 4$

Add xy to both sides $\quad 2x = 3y + xy + 4$

Subtract 4 from both sides $\quad 2x - 4 = 3y + xy$

Take a factor of y $\quad 2x - 4 = y(3 + x)$

Divide both sides by (3 + x) $\quad \frac{2x - 4}{3 + x} = y$

or $\boxed{\mathbf{y = \frac{2x - 4}{3 + x}}}$

- Make the letter in brackets the subject of each of these equations.

4. $y = \frac{x + 5}{x}$ (**x**)
5. $p = \frac{1}{2}mv^2u^2$ (**m**)
6. $A = p + \frac{prt}{100}$ (**p**)
7. $Z = \frac{x}{x + 2}$ (**x**)
8. $R = \frac{s - 5}{s - 2}$ (**s**)
9. $v = \frac{uf}{u - f}$ (**f**)
10. $W = \frac{2t - 3}{3t - 2}$ (**t**)

Smile **1437**

Four Consecutive Numbers

Investigate what happens when four consecutive numbers are multiplied together and 1 is added to the product.

Try to prove your result.

$$9 \times 10 \times 11 \times 12 + 1 =$$