SMILE WORKCARDS

Algebraic Structure Pack Two

Contents

Who's Rule Okay?

1. Four students are working on problems where they have to find rules. They each have found a rule connecting the *number of rails* and the *number of posts* in this sequence.

3 6 9

Karen's rule: *The number of rails is the number of posts minus 1 then times by 3.*

Joe's rule: *The number of rails is the number of posts take away 3 then multiply by 3.*

Rajan's rule: *The number of rails is the number of posts times 3 then take away 3.*

Nikki's rule: *The number of rails is the number of posts multiplied by 2 then take away 3, then add the number of posts.*

If **r** = *number of rails* and **p** = *number of posts*

Karen's rule can be written in algebra as **r = 3(p – 1)**

- Write the other three rules in algebra.
- One of the student's rules is incorrect. Which one?
- Which rule looks the simplest?

2. They have each found a rule to connect the *number of matches* with the *number of rectangles* in this sequence.

8 15 22

Karen's rule: *To get the number of matches add 1 to the number of rectangles, then multiply by 7 then subtract 6.*

Joe's rule: *Matches equal the number of rectangles times 8, then take away 1 less than the number of rectangles.*

Rajan's rule: *The number of matches is the number of rectangles times by 7 then add 1.*

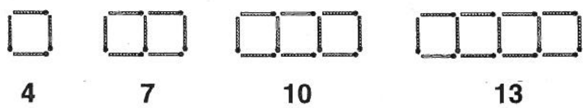
Nikki's rule: *Multiply the number of rectangles by 8, then take away the number of rectangles, then take away 1 to get the number of matches.*

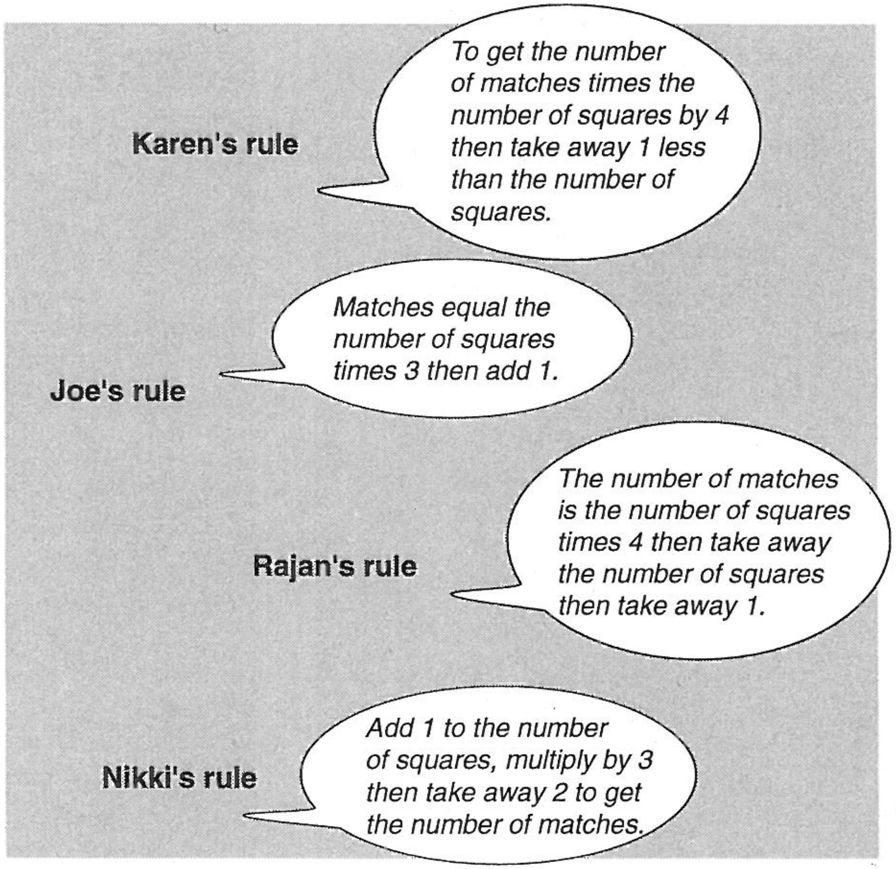
If **m** = *number of matches* and **r** = *number of rectangles*

Karen's rule can be written in algebra as **m = 7(r + 1) – 6**

- Write the other three students' rules in algebra.
- One of the student's rules is incorrect. Which one?
- Which rule looks the simplest?

3. They each have found a rule to connect the *number of matches* with the *number of squares* in this sequence.



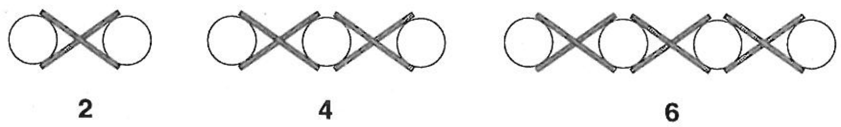


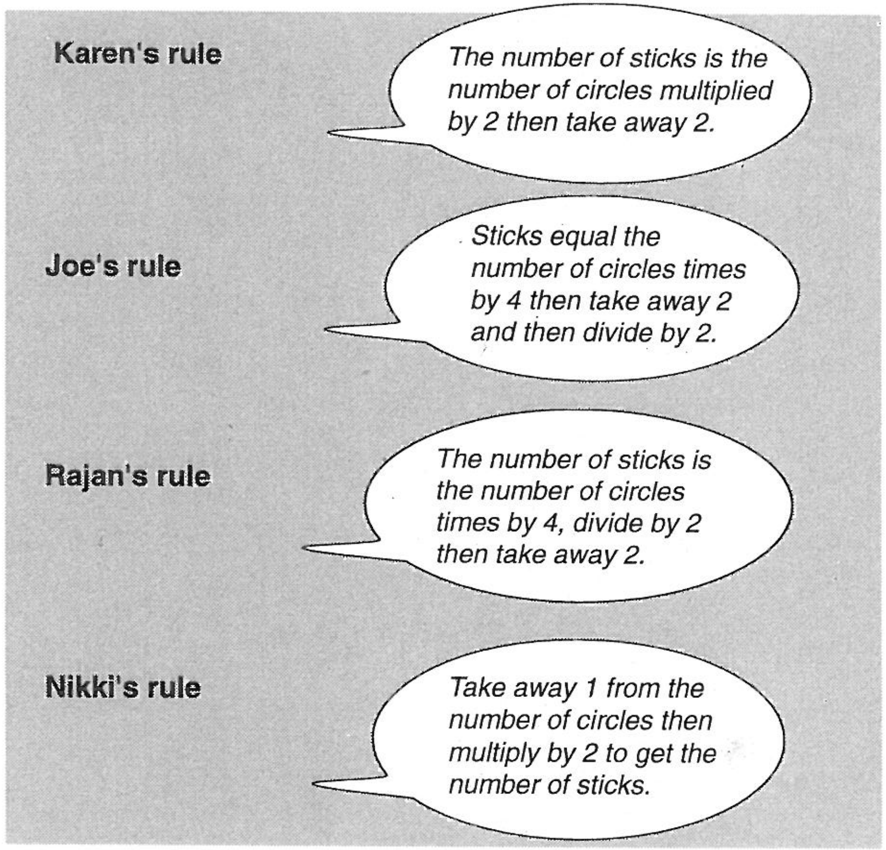
If **m** = *number of matches* and **s** = *number of squares*

Karen's rule can be written in algebra as $\mathbf{m = 4s - (s - 1)}$

- Write the other three students' rules in algebra.
- One of the student's rules is incorrect. Which one?
- Which rule looks the simplest?

4. They each have found a rule to connect the *number of sticks* to the *number of circles*.





- Write all the rules in algebra.
- One of the student's rules is incorrect. Which one?
- Which rule looks the simplest?

Look at some other sequences and try to write the same rule in different ways.

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ALGEBRA MATCH

- Match each algebraic expression from the first column to one in the second column.
- Check your answers by substituting values for **a**, **b** and **c**.

first column

$(a + b) + c$	
$(a + b) - c$	
$(a + b) \times c$	
$(a + b) \div c$	
$(a - b) + c$	
$(a - b) - c$	✓
$(a - b) \times c$	
$(a - b) \div c$	
$a + (b + c)$	
$a + (b - c)$	
$a + (b \times c)$	
$a + (b \div c)$	
$a - (b + c)$	✓
$a - (b \times c)$	
$a - (b \div c)$	
$a - (b - c)$	
$(a \times b) + c$	
$(a \times b) - c$	
$(a \times b) \times c$	
$(a \times b) \div c$	
$a \times (b + c)$	✓
$a \times (b - c)$	
$a \times (b \times c)$	
$a \times (b \div c)$	
$a \div (b + c)$	
$a \div (b - c)$	
$a \div (b \times c)$	
$a \div (b \div c)$	
$(a \div b) + c$	
$(a \div b) - c$	
$(a \div b) \times c$	
$(a \div b) \div c$	

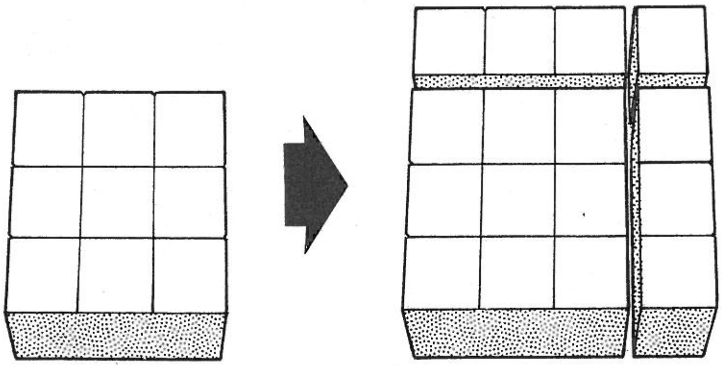
second column

$a - b - c$	$= (a - b) - c = a - (b + c)$
$a + b + c$	
abc	
$a - b + c$	
$a + b - c$	
$ab + ac$	$= a \times (b + c)$
$ab - ac$	
$ac - bc$	
$ac + bc$	
$a - bc$	
$a + bc$	
$ab + c$	
$ab - c$	
$\frac{a}{bc}$	
$\frac{ab}{c}$	
$\frac{ac}{b}$	
$a - \frac{b}{c}$	
$a + \frac{b}{c}$	
$\frac{a}{b} - c$	
$\frac{a}{b} + c$	
$\frac{a}{b + c}$	
$\frac{a}{b - c}$	
$\frac{a + b}{c}$	
$\frac{a - b}{c}$	

$(x+1)^2$

A base three flat can be enlarged to a base four flat by adding 2 longs and a unit.

Base three Flat + 2 Base three Longs + 1 Unit = Base four Flat

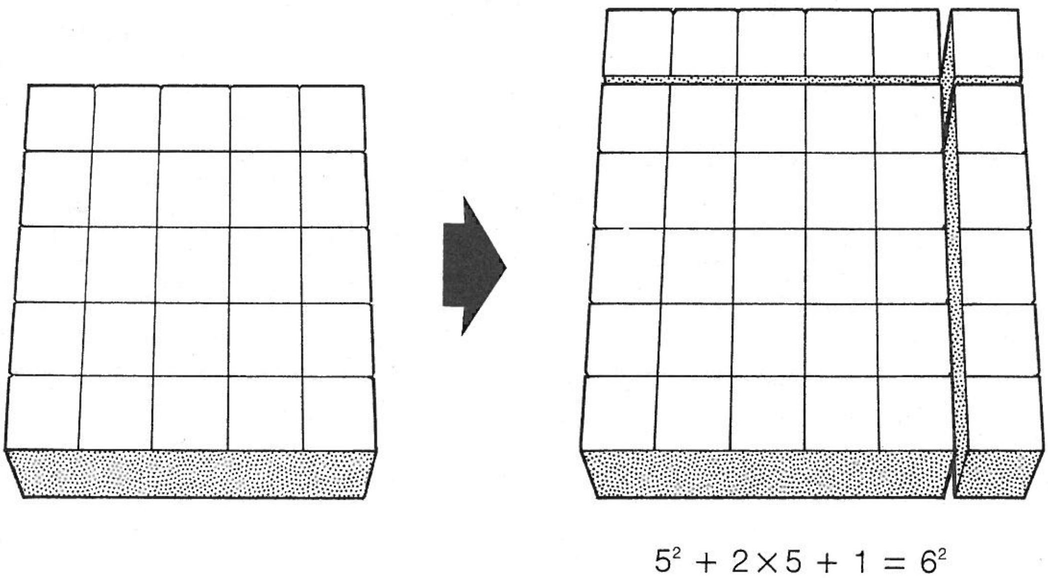


This shows that $3^2 + 2\times 3 + 1 = 4^2$

Check:

$$3^2 + 2\times 3 + 1 = 4^2$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$9 + 6 + 1 = 16$$

A base five flat can be enlarged to a base six flat by adding 2 longs and a unit:



$5^2 + 2\times 5 + 1 = 6^2$

1. Try this for 4 other pairs of flats.
   Write an equation for each pair and check the equations.

Turn over

2. Using your results from question 1 complete this table:

$3^2 + (2 \times 3) + 1$	4^2
$5^2 + (2 \times 5) + 1$	6^2

3.

$20^2 + (2 \times 20) + 1$	

4.

	55^2

5. For any size flat?

$x^2 +$	

6. Test your answer for question 5 by putting x = 3 or x = 5.

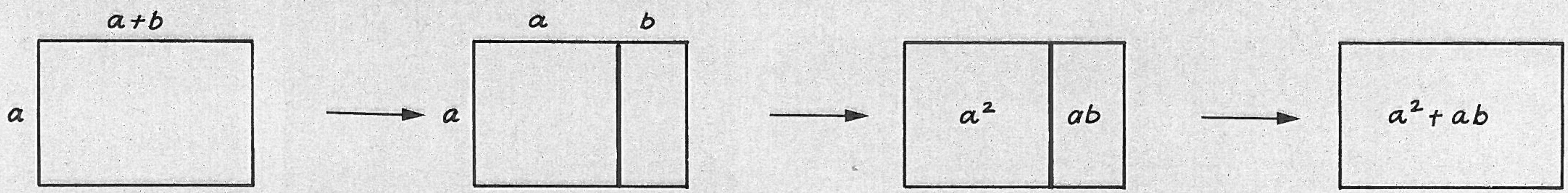
7. The base-apparatus which leads you to the identity $(x + 1)^2 = x^2 + 2x + 1$ obviously only used whole numbers.

   Does the identity work for decimals or negative numbers?

   Choose some simple examples and test them.

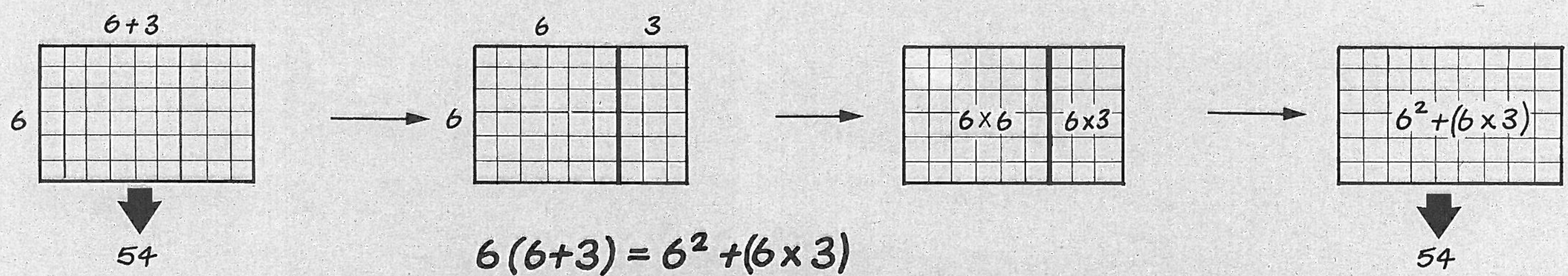
   Will it *always* be true?

IDENTITIES



$a(a+b) = a^2 + ab$

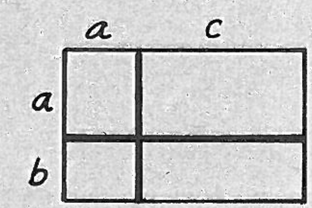
Check this identity for $a = 6$, $b = 3$:



$6(6+3) = 6^2 + (6 \times 3)$

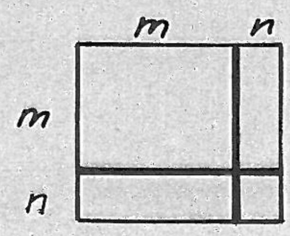
1) Check the identity above for $a = 12$, $b = 3$.

2) Find an identity for $(a+b)(a+c)$



3) Check your identity

4) Find an identity for $(m+n)^2$



5) Check your identity

FIND IDENTITIES FOR

6) $(x+y+z)z$

7) $(a+2b)(2a+b)$

8) Check each of these identities

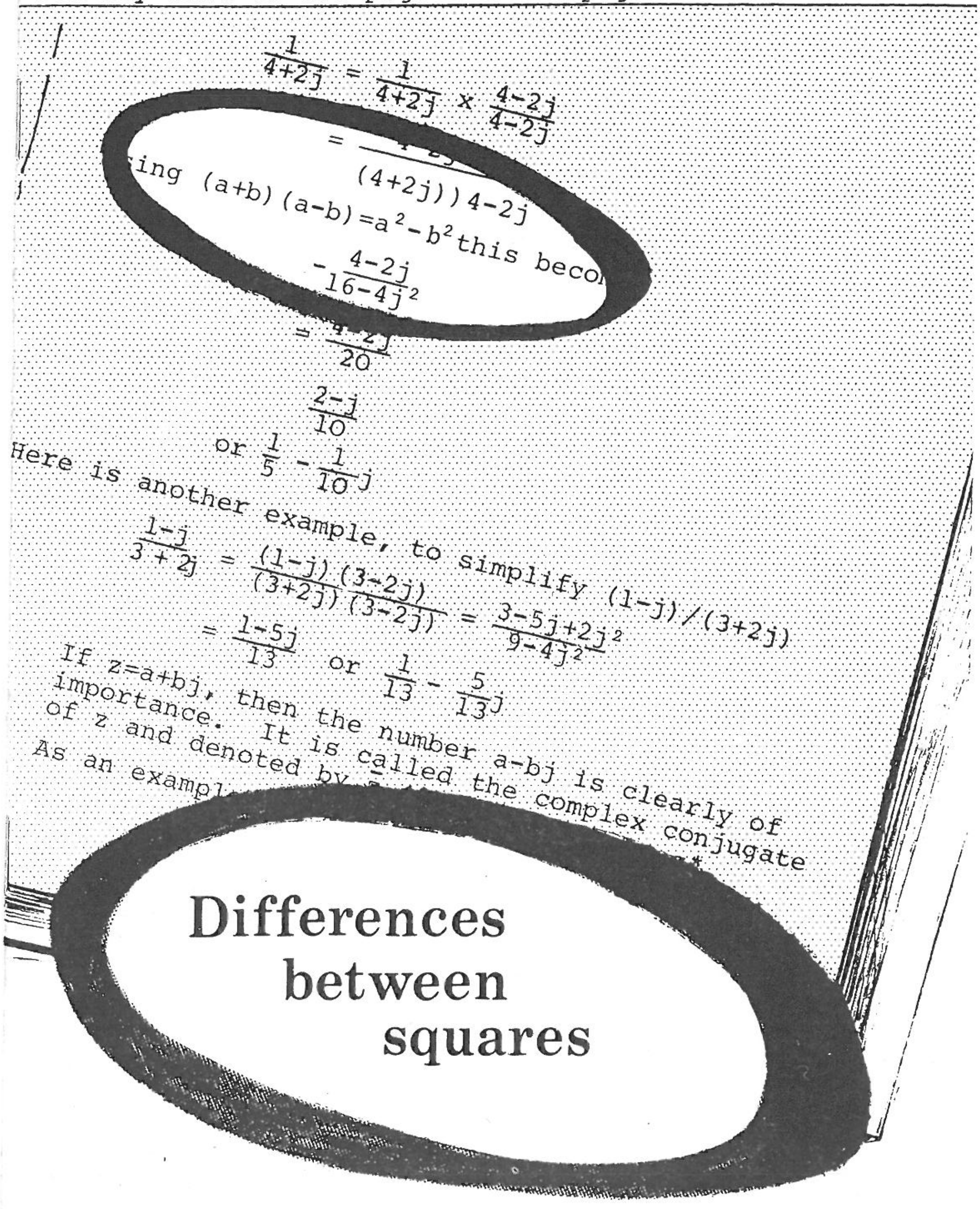
a) with fractions/decimals

b) with negative numbers

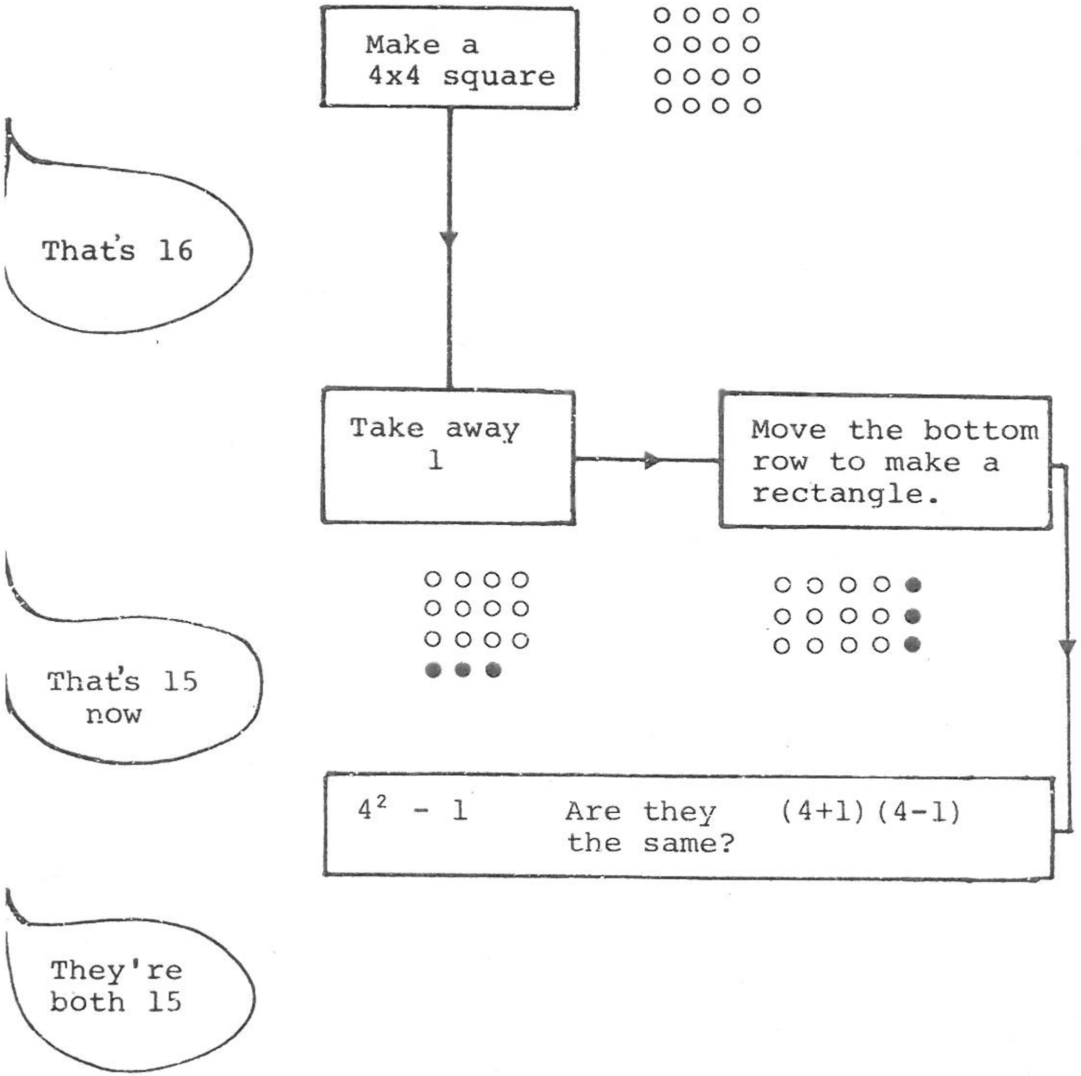
0818

You may want to use pegboard and pegs

SMILE



Differences between squares

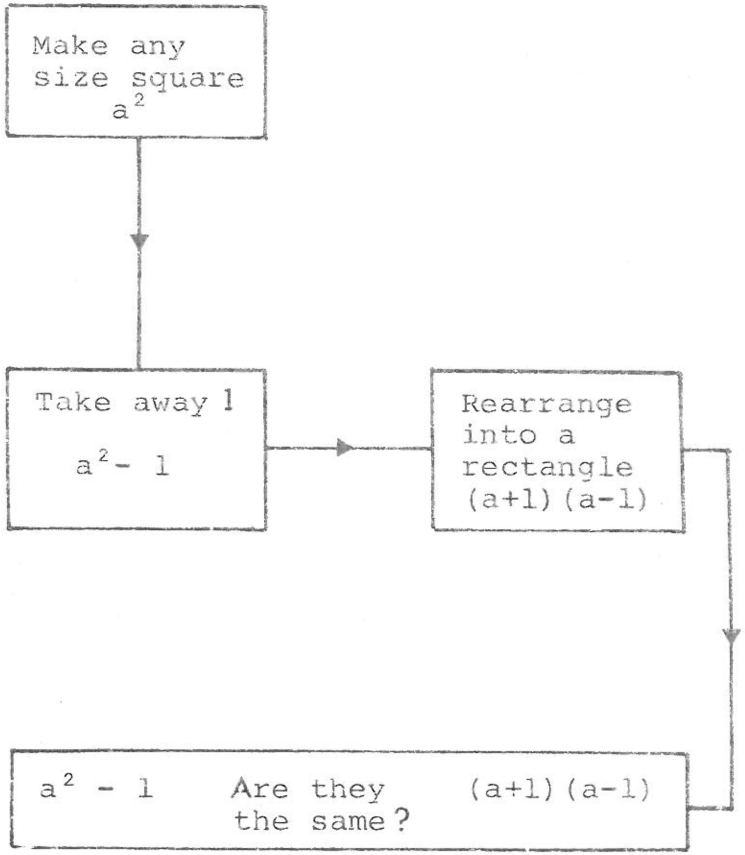


Try other size squares, taking away 1 each time.

Record your results:

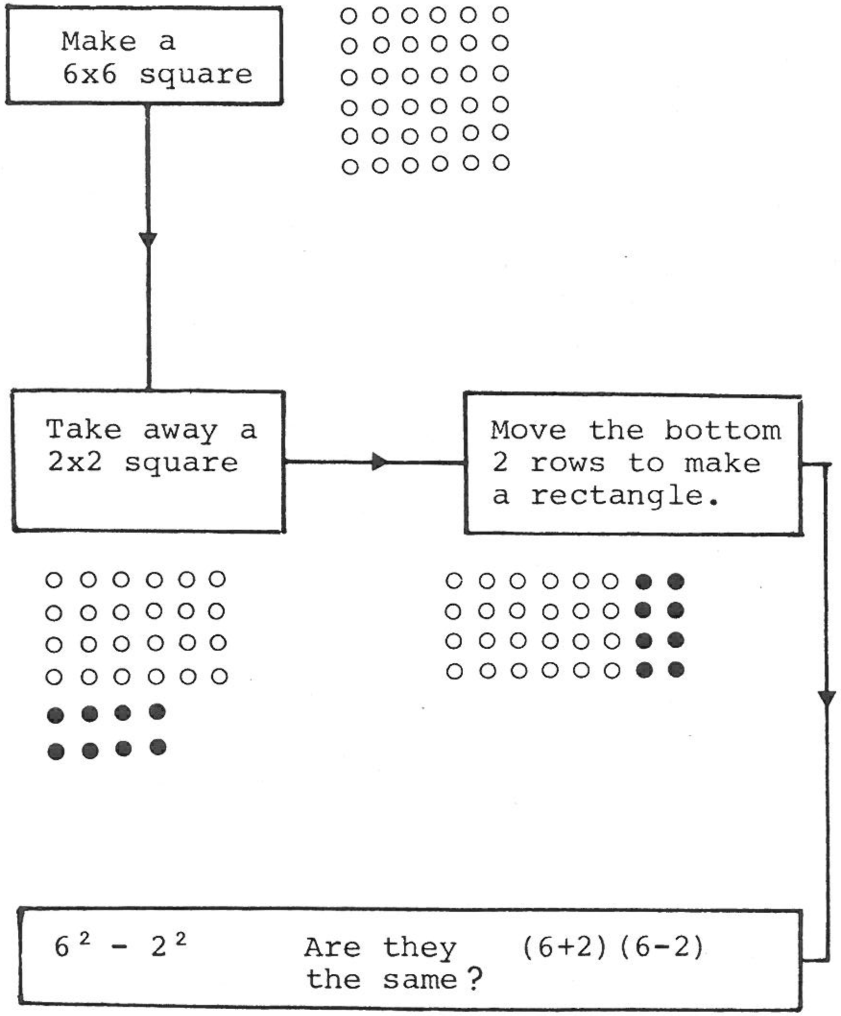
$4^2 - 1 = (4+1)(4-1)$
$5^2 - 1 = (5+1)(5-1)$
$6^2 - 1 = (6+1)(6-1)$

This is a summary of what you have done on page 2:



Substitute $a = 10$ in $a^2 - 1 = (a+1)(a-1)$ to check that the identity works.

Use the identity to evaluate 21×19
$= (20+1)(20-1)$
$= \dots$



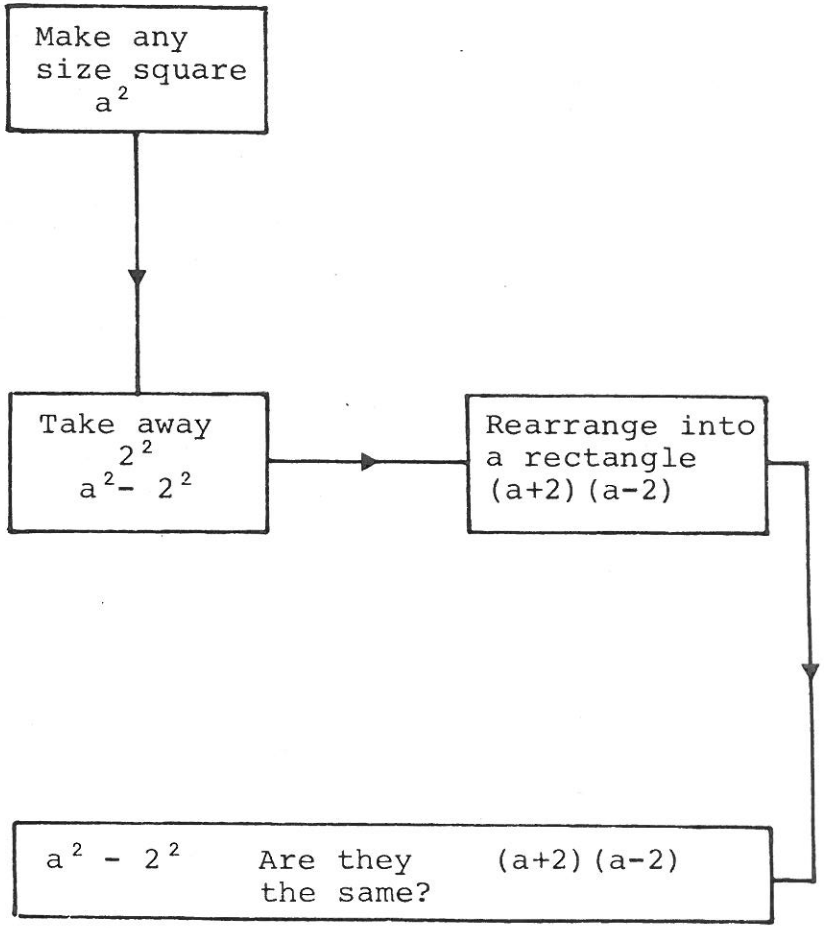
Try other size squares, taking away 2^2 each time.

Record your results $6^2 - 2^2 = (6+2)(6-2)$

$3^2 - 2^2 =$

⋮

This is the summary of your work on page 4:

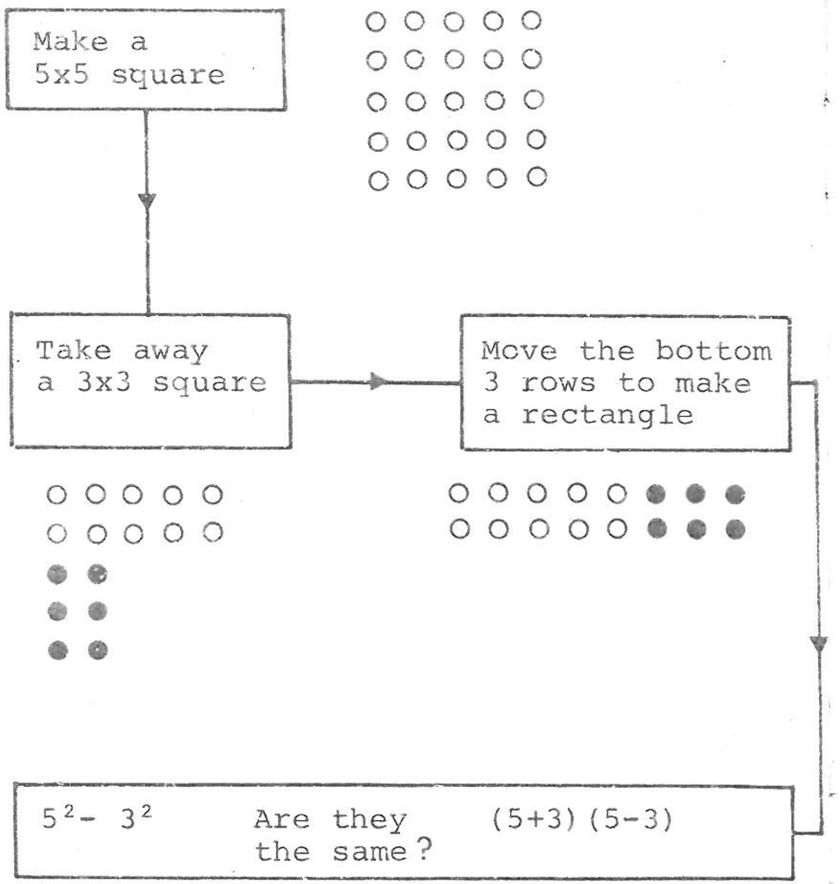


Substitute $a = 2\frac{1}{2}$ in $a^2 - 2^2 = (a+2)(a-2)$ to check if the identity works.

Use the identity to evaluate 32×28

$= (30+2)(\ldots)$

$= \ldots.$



Try other size squares, taking away 3^2 each time.

Record your results $5^3 - 3^3 = (5+3)(5-3)$

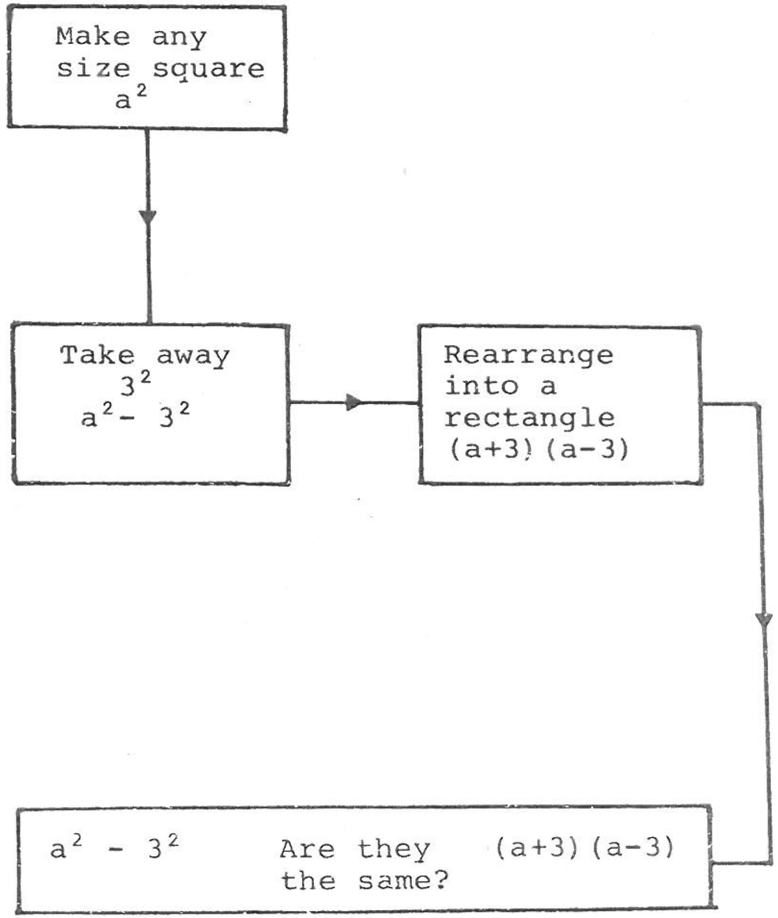
.

.

.

.

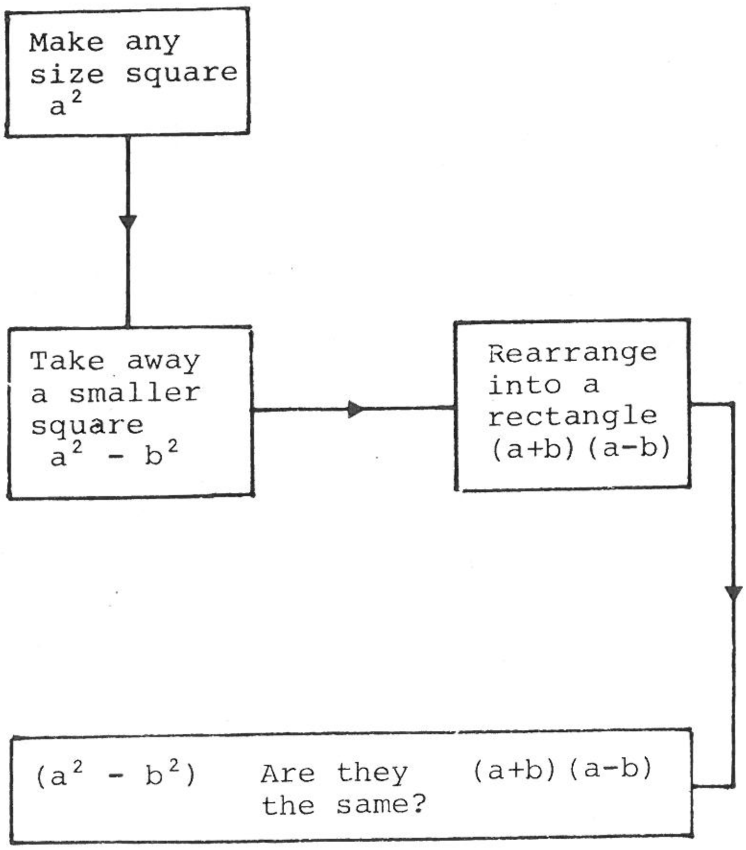
Here is the summary of the work on page 6:



What identity can you write?
Check that the identity works ..... for integers
..... for fractions.

Does it work for $a = 3$?

This is a summary of all the work on pages 2, 4, 6:



Choose any pair of numbers for a and b (choose $b<a$) to see whether

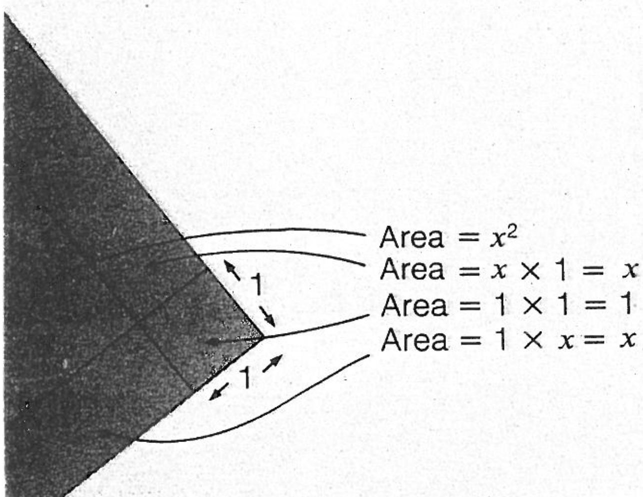
$$a^2 - b^2 = (a+b)(a-b)$$

If you are satisfied that the identity works, write the identity in your own words.

he unknown square

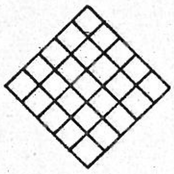
Area = x^2

The sides of this unidentified square are increased by 1 unit

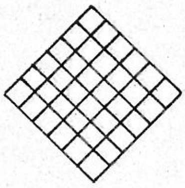


New enlarged area = $x^2 + 2x + 1$

If $x = 5$,
then the smaller square is 5×5 ....



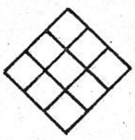
...and the larger square $(x + 1)^2$ is 6×6...



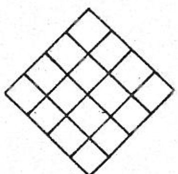
and $6^2 = 5^2 + (2 \times 5) + 1$

Check $6^2 = 5^2 + (2 \times 5) + 1$
↓ 36 ↓ 25+ ↓ 10 ↓ + 1

If $x = 3$,
then the smaller square is 3×3...



...and the larger square $(x+1)^2$ is 4×4...



Complete this identity:

$4^2 =$ ■$^{■}$ + (2 × ■) + 1

and check that your answer is correct.

TURN OVER

$4^2 = 3^2 + (2 \times 3) + 1$

1. Write a similar identity to describe the areas if $(x + 1) = 8$. . . and check it.

2. Investigate other values for $(x + 1)$ and list your results in a table:

4^2	$(3 + 1)^2$	$3^2 + (2 \times 3) + 1^2$	$9 + 6 + 1$	16
5^2	$(4 + 1)^2$			
6^2	$(5 + 1)^2$			
	$(6 + 1)^2$			
	$(7 + 1)^2$			
	$(8 + 1)^2$			

3. Compare and check the first and last columns

4. Writing $(x + 1)^2$ as $(x^2 + 2x + 1)$ makes calculations like 101^2 much easier.

For example, $101^2 = (100 + 1)^2$
= 100^2 + (2 × ■) + ■
= 10000 + ■ + ■
= ■

5. Use this method to calculate
(a) 21^2
(b) 51^2
(c) 301^2

6. Find the value of $(x + 1)^2$ if $x = ½$

Equivalent Expressions

These three expressions are equivalent.

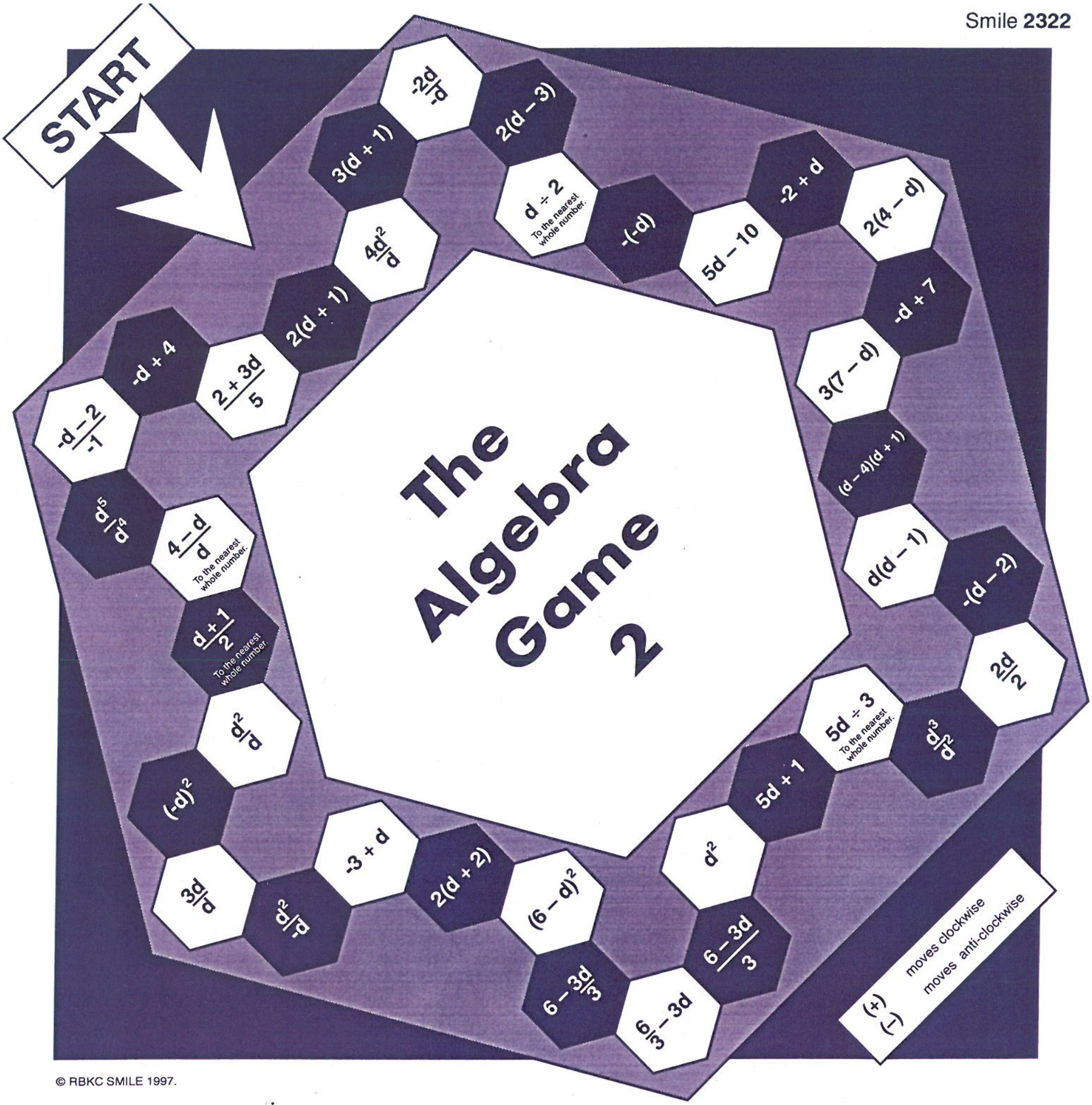
$6a^2 + 3$ = $3(2a^2 + 1)$ = $4a^2 + 3 + 3a + 2a^2 - 3a$

Check that they are equivalent by substituting a value for **a**. e.g. let **a** = **7**

(6 x **49**) + 3	3[(2 x **49**) + 1]	(4 x **49**) + 3 + (3 x **7**) + (2 x **49**) − **21**
= 294 + 3	= 3(98 + 1)	= 196 + 3 + 21 + 98 − 21
= 297	= 297	= 297

- Cut out these pieces and match them in groups of three equivalent expressions.
- Check your groups of three by substituting any value for **a**.

$2a - a^2$	$\frac{1}{2}(10a + 10)$	$3a^2 - 2$
$a(a + 1) + a$	$3(a - 8) + 3$	$0.5(a^2 + 6a)$
$\frac{1}{2}(6a^2 - 4)$	$-a(a - 2)$	$2(2a^2 - a)$
$5(a + 1)$	$5a + 5$	$3(2a + a^2)$
$4a^2 - 2a$	$a^2 + 2a - 2a^2$	$\frac{a(a + 3)}{2}$
$3a^2 + 2a - 2 - 2a$	$0.5a^2 + 3a$	$3a - 21$
$6a + 3a^2$	$\frac{1}{2}(a + 4)$	$a^2 + 2a$
$a(a + 2)$	$2a + 6$	$3(a - 7)$
$2(a + 3)$	$\frac{a}{2} + 2$	$2(a + 2) + 2$
$a - 0.5a + 2$	$2a(2a - 1)$	$3a(2 + a)$



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The Algebra Game 2

A game for 2 to 4 players.

You will need a dice and each person will need one counter and one copy of Smile Worksheet 2321a.

Rules

Take it in turns to play.

- Throw the dice.
- Substitute the number on the dice for the '**d**' in the expression that your counter is on.
- Move this number of hexagons.

  (+) moves clockwise
  (−) moves anti-clockwise
- Record your moves on the worksheet.

Choose a target score of 200, 300, . . .
The first player to reach it is the winner.

The Algebra Game

Number on the dice.

This is the start of a game using the board for Smile 2321.

	Algebra Rule	d	Working		Move	Total Score
START	d + 2	4	d + 2	= 4 + 2 = 6	6	6
	2d	5	2d	= 2 x 5 = 10	10	16
	12 - 2d	2	12 - 2d	= 12 - (2 x 2) = 21 - 4 = 8	8	24
	5 - d	6	5 - d	= 5 - 6 = -1	-1	23
	$\frac{2d}{d}$	3	$\frac{2d}{d}$	= (2 x 3) 3 = 6 3 = 2	2	25

Now start a game of your own. Record your moves below.

	Algebra Rule	d	Working	Move	Total Score
START					

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Find the operation

In this operation table

a $*$ b means **treble a** then **add b**

so **a $*$ b** = $3a + b$

$*$	0	1	2	3	4	5
0			2			
1				6		
2	6					
3				12		
4		13				
5					19	20

(columns: b; rows: a)

1. Complete the table.

What number patterns do you notice?

2. Find the operation in these tables.

$*$	0	1	2	3
0	0	0	0	0
1	0	2	4	6
2	0	4	8	12
3	0	6	12	18

a $*$ b = ____________

$*$	0	1	2	3
0	0	−1	−2	−3
1	1	0	−1	−2
2	4	3	2	1
3	9	8	7	6

a $*$ b = ____________

$*$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

a $*$ b = ____________

Use the sheet below to make a puzzle for a friend.

- -

Find the operation

Designed by ____________________

for ____________________

Solution

a $*$ b = ____________________

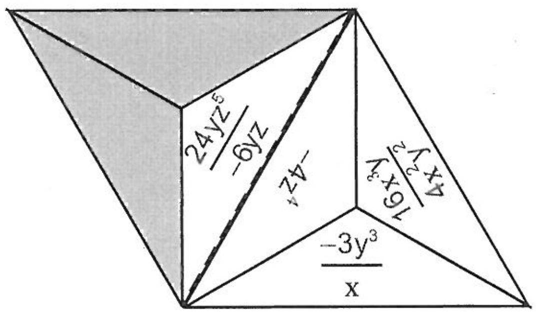
$*$	0	1	2	3	4
0					
1					
2					
3					
4					

Matching Algebraic Expressions

1. Cut out the 9 equilateral triangles along the dotted lines.
2. Match the equivalent algebraic expressions:

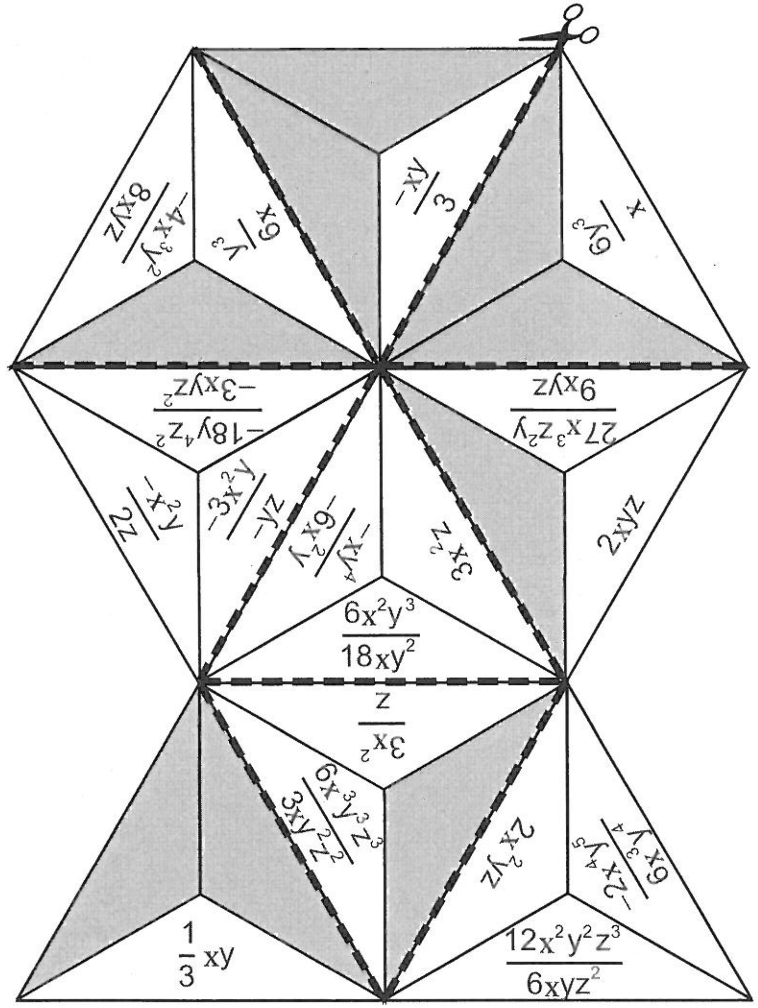
Example: $\frac{24yz^5}{-6yz} = \frac{24 \times y \times z \times z \times z \times z \times z}{-6 \times y \times z}$

$= -4z^4$



3. Record your working out in your book.

4. Fit the equilateral triangles together to make one large triangle. The shaded sections mark the edges of the triangle.



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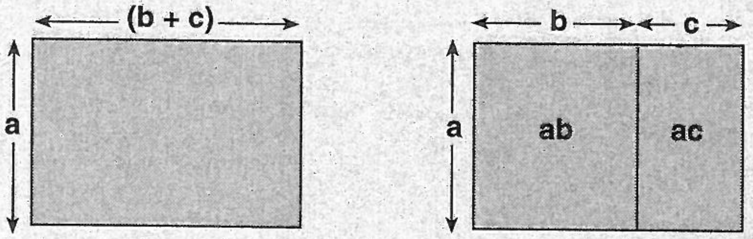
Brackets

Expanding brackets

The expression **a(b + c)** means multiply **a** by **(b + c)**.

These rectangles



show that **a(b + c)** = **ab + ac**

Both sides of the equation represent the area of the rectangle. Expanding the brackets in the expression **a(b + c)** gives the equivalent expression **ab + ac**.

Check that the expressions are equivalent by substituting values for **a**, **b** and **c**.

e.g If **a** = -7, **b** = 1·2 and **c** = 4

a (b + c)	**ab + ac**
= -7 (1·2 + 4)	= (-7 x 1·2) + (-7 x 4)
= -7 x 5·2	= -8·4 + -28
= **-36·4**	= **-36·4**

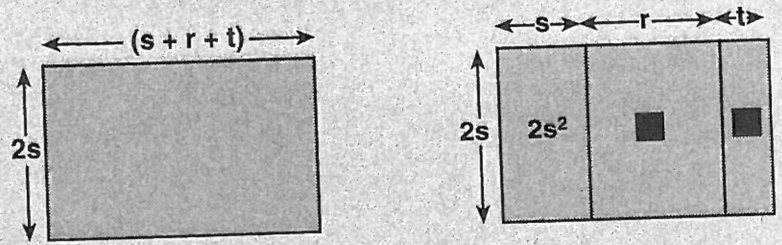
Both of the expressions equal **-36·4**, so **a(b + c) = ab + ac** is true for these values.

1. Substitute other values for **a**, **b** and **c** into **a(b + c)** and **ab + ac** to check that **a(b + c) = ab + ac**.

Include negatives and decimals in your checking.

2. Expand the brackets to give an equivalent expression for **2s(s + r + t)**.

These rectangles may help.



2s(s + r + t) = $2s^2$ + ■ + ■

- Check that the expressions **2s(s + r + t)** and $2s^2$ + ■ + ■ are equivalent by substituting values for **s**, **r** and **t**.

  Include negatives and decimals in your checking.

3. Expand the brackets in these expressions to find equivalent expressions.

a) **3(p + q)** b) **5(a + b + c)**

c) **x(x + y + z)** d) **2j(k + m – n)**

e) **s(1 + 2t)** f) **d(d – 1)**

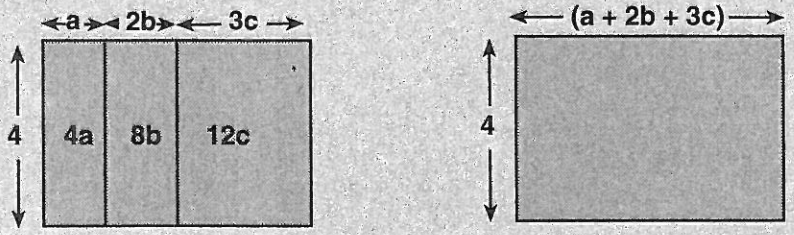
g) **2e(e + 2)** h) **2g(f – g)**

- For each pair of expressions substitute values for the letters to check that both expressions are equivalent.

Factorising

4. The expression **4a + 8b + 12c** can be factorised to give an equivalent expression.

4 is the common factor of each term of the expression.



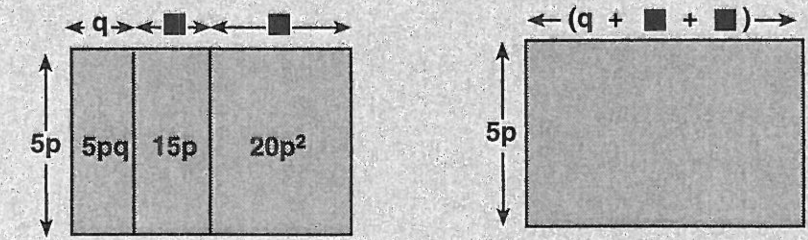
4a + 8b + 12c = **4(a + 2b + 3c)**

Both sides of the equation represent the area of the rectangle. Factorising the expression **4a + 8b + 12c** gives the equivalent expression **4(a + 2b + 3c)**.

- Check that both expressions are equivalent by substituting values for **a, b** and **c**.

5. Factorise **5pq + 15p + 20p²** to find an equivalent expression.

These rectangles may help.



5pq + 15p + 20p² = **5p(q + ■ + ■)**

- Check that both expressions are equivalent by substituting values for **p** and **q**.

6. Factorise these expressions to find equivalent expressions.

a) **5p + 5q + 5r**

b) **3m + 3n – 3p**

c) **4s – 8**

d) **16t – 12s**

e) **2p + 3pq – pr**

f) **fg – 3f + 4f²**

g) **4x + 8xy + 12x²**

h) **f + ef + 4f²**

- Substitute values for the letters in each pair of expressions to check that both expressions are equivalent.

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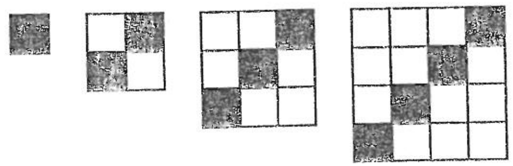
Quadratic Rules

Three students Jo, Pretty and Molly, are working on problems where they have to find rules.

1. They have each found a rule connecting the number of black squares to green squares.



Jo's rule: The number of green squares is the number of black squares, squared, minus the number of black squares.

Pretty's rule: The number of green squares is the number of black squares minus 2, multiplied by the number of black squares, then add the number of black squares.

Molly's rule: The number of green squares is equal to the number of black squares, mutiplied by the number of black squares minus 1.

Jo's rule can also be written algebraically:

Let g = number of green squares.
and b = number of black squares.

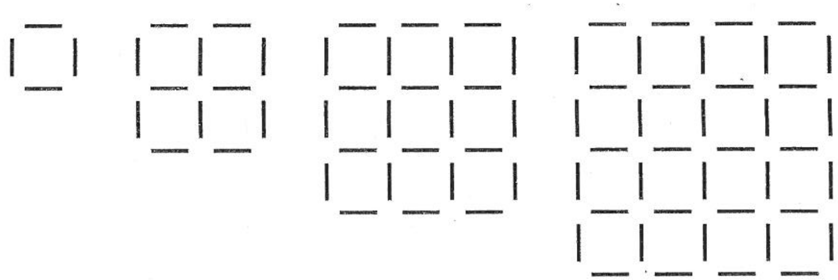
The number of green squares... The number of black squares, squared... minus... The number of black squares...

$$g = b^2 - b$$

(i) Write the other two rules algebraically.

(ii) Rearrange the rules to show that they are all equivalent.

2. The students have found rules connecting the side of the square to the number of matches in this sequence.



Jo's rule: The number of matches equals the side of the square squared plus the side of the square, all mutiplied by 2.

Pretty's rule: The number of matches equals 2 times the side length squared, plus 2 times the side of the square.

Molly's rule: $m = 2s(s + 1)$

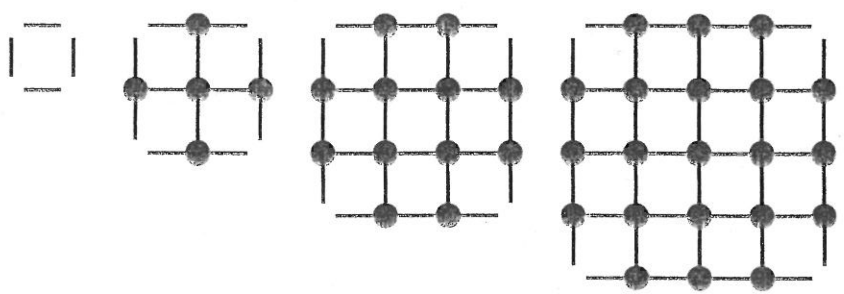
Let s = side of square.
and m= number of matches.

(i) Write Jo's and Pretty's rules algebraically.

(ii) Molly has already written the rule algebraically. Write it in words.

(iii) Rearrange the rules to show that they are all equivalent.

3. The students have found rules connecting the side of the square to the number of dots in this sequence.



(i) Copy Jo's and Molly's rules and fill in the missing numbers to make them correct.

Jo's rule: The number of dots equals the side of the square add one, all squared, then minus ___.

Pretty's rule: $d = s^2 + 2s - 3$

Molly's rule: The number of dots is equal to the side of the square plus 2, mutiplied by the side of the square then minus ___.

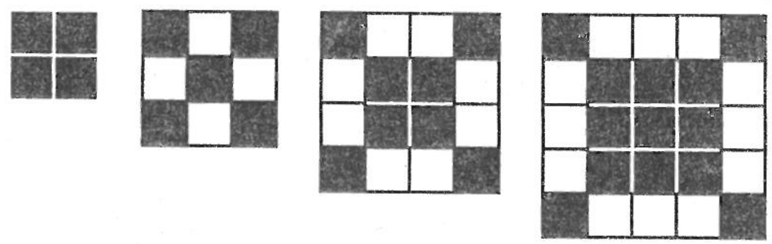
Let s = side of square, and d = number of dots.

(ii) Write Jo's and Molly,s rules algebraically.

(iii) Pretty has already written the rule algebraically. Write it in words.

(iv) Rearrange the rules to show that they are all equivalent.

4. The students have found rules connecting the number of black squares to the side of the square in this sequence.



(i) Copy Pretty's and Molly's rules and fill in the missing numbers to make them correct.

Jo's rule: $b = (s - 2)^2 + 4$

Pretty's rule: The number of black squares equals the side length squared, minus 4 times the side length, then add ___.

Molly's rule: The number of black squares equals the side length squared minus ___ times the side length minus two.

Let s = side of square, and b = number of black squares.

(ii) Write Pretty's and Molly's rules algebraically.

(iii) Jo has already written the rule algebraically. Write it in words.

(iv) Rearrange the rules to show that they are all equivalent.

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