Research and analysis

Coordinating mathematical success: the mathematics subject report

Published 13 July 2023

Applies to England

Contents

Secondary

Appendix

This publication is available at https://www.gov.uk/government/publications/subject-report-series-maths/coordinating-mathematical-success-the-mathematics-subject-report

Context

Mathematics is a universal language that helps us to understand the world, and it is a core part of the curriculum. As well as teaching about numbers, shapes, statistics and patterns, it provides important tools for work in areas such as physics, architecture, medicine and business. It helps learners to develop logical and methodical thinking, to focus and to solve a wide range of mathematical problems. Success can be ‘coordinated’ when leaders ensure all the elements of pupils’ mathematics education are supporting each pupil’s progress in the subject. Success in mathematics leads to many opportunities for further study and employment. Mathematics is a core subject of the national curriculum (https://www.gov.uk/government/collections/national-curriculum). It is an important entitlement for all pupils in England’s schools: academies, free schools and maintained schools.

At GCSE, with the exception of results in 2020 and 2021, the proportion of pupils attaining a grade 4 or above has steadily increased over time. In 2019, 71.5% of pupils obtained a grade 4 or above, and 20.4% of pupils obtained a grade 7 or higher. These are increases of 9.1 and 3.3 percentage points since 2013 (https://analytics.ofqual.gov.uk/apps/GCSE/Outcomes_Link1/) The results of national reference tests (https://www.gov.uk/government/publications/the-national-reference-test-in-2022/national-reference-test-results-digest-2022#performance-in-maths-in-2022), introduced by Ofqual in 2017, also reflect a steady improvement in pupil performance between 2017 and 2020. Data from 2021 does not follow this trend, which is likely due to the impact of pandemic restrictions. However, national reference test results in 2022 suggest that attainment is improving again, although it has not yet reached the levels recorded in the 2020 test.

Mathematics is increasingly the most popular choice at A level. Within the top 15 subjects chosen by students, mathematics and further mathematics are the subjects that have the highest proportion of A* and A grades awarded (https://analytics.ofqual.gov.uk/apps/Alevel/Outcomes/). In addition, level 3 core mathematics is an increasingly popular post-16 course taken by pupils on top of 3 A levels, or equivalents.

Although pupils in England, on average, perform better in mathematics than pupils in many other countries, there is a large gap between the lowest and highest achievers, and between disadvantaged and advantaged pupils. (https://www.gov.uk/government/publications/research-review-series-mathematics/research-review-series-mathematics#fn:8) Studies and media reports continue to show that there is a shortage of specialist mathematics teachers (https://www.gov.uk/government/publications/research-review-series-mathematics/research-review-series-mathematics#fn:7) and that this is a longstanding issue found in other countries.[footnote 1]

This report evaluates the common strengths and weaknesses of mathematics in the schools inspected and considers the challenges that mathematics education faces. The evidence was gathered by His Majesty's Inspectors as part of routine inspections. The report follows our mathematics research review (https://www.gov.uk/government/publications/research-review-series-mathematics) published in 2021, which sets out our idea of a high-quality mathematics education. The report is split into findings in primary schools and those in secondary schools, and includes evidence from Reception classes and sixth forms. Within each of these sections, we talk about:

- aspects of the curriculum
- pedagogy
- assessment
- the way schools are organised
- the impact of this on what pupils learn

It is important to note that we evaluate schools against the criteria in the school inspection handbooks. Findings from this report will not be used as a 'tick list' by inspectors when they are inspecting schools: we know that there are many different ways that schools can put together and teach a high-quality mathematics curriculum.

During the period of evidence-gathering, schools were facing many challenges because of COVID-19. As other studies have shown, pupils have been affected by remote learning, lockdowns and national restrictions. Therefore, some of the evidence gathered for this report may not represent 'business as usual'. However, by focusing on the curriculum and its implementation over time, we hope that this risk has been reduced.

Key terms used in this report

Knowledge in mathematics

Throughout this report, we use the same terminology for mathematics knowledge as we used in our mathematics research review (https://www.gov.uk/government/publications/research-review-series-mathematics/research-review-series-mathematics#fn:8). These are not necessarily terms that Ofsted would expect pupils or teachers to use:

- declarative knowledge: facts, concepts, formulae
- procedural knowledge: methods, procedures, algorithms
- conditional knowledge: strategies formed from the combinations of facts and methods to reason and problem-solve

Main findings

Primary schools

In the last few years, a resounding, positive shift in mathematics education has taken place in primary schools. Curriculum is now at the heart of leaders' decisions and actions. Generic approaches, such as the expectation that all teaching should always be differentiated, have dissipated. We now see high quality curriculums, collaborative support for teachers and a focus on mathematics teaching. Leaders intend that pupils 'keep up, not catch up'. These approaches set out a better path to proficiency for pupils.

Teachers help pupils to understand new concepts. Networks of support, such as the Maths Hubs, provide regular and highly useful training. This helps teachers to adopt new and improved ways of explaining and modelling concepts. Often, teachers use physical resources and pictorial representations to help pupils see underlying mathematical structures. They also teach and model new vocabulary, regularly check pupils' understanding and swiftly pick up misconceptions.

There are some deficiencies in the quality and quantity of practice that pupils undertake. Even when teachers teach with clarity and precision, it is likely that these deficiencies undermine pupils' ability to remember important knowledge. For older pupils, these deficiencies affect their ability to attain procedural fluency (speed and accuracy).

Pupils’ gaps in knowledge tend to be centred around, but not limited to, addition facts in younger year groups. This was for some, but not all pupils. These early gaps in knowledge may not become apparent until a significant amount of time has elapsed. This is because it is possible, in the medium term, for pupils to understand what is being taught and then keep up with extra classroom support and slower calculation. However, this is at the expense of later ability to access the curriculum.

Accountability measures and wide spreads of attainment tend to influence leaders’ decision making and resource allocation for Year 6 cohorts. Allocating additional resources to year 6 leaves leaders with fewer resources to invest in pupils’ earlier education. Further, a goal of true proficiency is superseded by ‘age related expectations’ which roughly equates to 50% accuracy in end of key stage tests. As a result, many pupils aren’t as prepared for the rigours of secondary education as they could be.

Secondary schools

Notable improvements have taken place in mathematics education in recent years. Widespread weaknesses identified at the time of Ofsted’s last mathematics subject report, around deficiencies in curriculum guidance and weaknesses in ongoing professional development for staff, are now much less likely to be evident in schools. However, some weaknesses identified in that report persist and continue to limit pupils’ learning of mathematics. The teaching of disparate skills to enable pupils to pass examinations but not equip them for the next stage of education, work and life, and weaknesses in the teaching of mathematical problem solving, remain areas of weakness across many schools. These weaknesses are disproportionately likely to be evident in schools that struggle to recruit and retain specialist mathematics teachers.

Recruiting and retaining high-quality, specialist, maths teachers is a challenge for many schools. Leaders’ curriculum decisions are increasingly influenced by the need to cope with these difficulties. Some schools that identify this problem do not take steps to develop the subject knowledge and subject specific pedagogical knowledge of less experienced and non-specialist teachers, usually provide a weaker mathematics education to their pupils.

A high quality of education leads to strong pupil outcomes, but this is not necessarily true in reverse. Strong exam outcomes do not, necessarily, indicate a high-quality mathematics education because, in some schools, pupils are taught a narrowed curriculum that allows them to be successful in exams without securing the mathematical knowledge they need to be successful later. These decisions are made because leaders and teachers are acutely aware of the impact of pupils achieving certain threshold grades in terms of post-16 opportunities, and implications for school accountability.

Curriculum planning around the teaching of mathematical facts and methods is usually strong. Where it is weaker, however, it tends to take less account of what pupils have learned previously and what they will study later. In these schools, pupils often are taught a series of disconnected mathematical methods and 'tricks' that apply only in specific circumstances.

Long term curriculum planning to develop pupils' ability to use the facts and methods they have been taught to solve familiar, and unfamiliar, problems is uncommon. Curriculum decisions about problem solving are often left to individual class teachers. The quality of these decisions is variable. As a result, some pupils, particularly those who find learning mathematics more challenging and those taught by non-specialist teachers, are not effectively taught how to solve problems mathematically.

Maths leaders, and teachers, consistently emphasise the importance of clarity and technical accuracy in written, and spoken, mathematics. Where teachers model this accurately and explicitly, it is more likely to be present in pupils' communication.

In most schools, exercises and activities are used by teachers, but in some schools, pupils are asked to undertake exercises and activities that are not carefully designed, or some pupils are moved on without having had sufficient practice to consolidate new learning.

Pupils who are learning mathematics more slowly than their peers frequently receive a mathematics education that does not meet their needs.

They are often rushed through the study of new content, in order to 'complete the course', without securely learning what they are studying. This frequently results in pupils repeating content, in key stage 4 that they have already studied, but not learned, in key stage 3 (and 2). Often the curriculum for these pupils is narrowed with little teaching of how the facts and methods learned can be used to solve problems mathematically. Many of these pupils develop a negative view of mathematics.

Leaders' application of GCSE grade thresholds to internal assessments gives false assurance about pupils' learning. As a result of teachers and leaders accepting this level of achievement in internal assessments, some pupils are progressing through the mathematics curriculum with significant, and growing, gaps of knowledge.

In the vast majority of secondary schools, department meeting time is allocated to improving the quality of provision in mathematics as opposed to undertaking administrative tasks. Many leaders and teachers note that this had been a significant and positive change over the recent years.

Discussion of the findings

The overall picture of mathematics education in England is broadly healthy. This positive picture did not arise through chance but through the commitment of school leaders, teachers and members of the mathematics subject community.

Leaders prioritise creating or adopting a high-quality mathematics curriculum. They give careful and ongoing consideration to the effective teaching of that curriculum. Leaders make good use of support and resources from Maths Hubs, the National Centre for Excellence in the Teaching of Mathematics (NCTEM) and from commercial providers. As a result, many teachers receive high quality, subject-specific continual professional development (CPD) and there are flourishing formal and informal networks of teaching professionals.

This is a significant shift compared to when Ofsted's last mathematics subject report, Made to Measure (https://www.gov.uk/government/publications/mathematics-made-to-measure), noted that 'very few schools provided curricular guidance for staff, underpinned by professional development that focused on enhancing subject knowledge and expertise in the teaching of mathematics, to ensure consistent implementation of approaches and policies’.

Given this very positive picture, this report explores the features of this widespread effective practice to support its replication. This report also considers the factors which might explain the continued weaknesses in outcomes for some children.

The phrase ‘coordinating mathematical success’ describes how effective schools make sure that curriculum plans, teaching approaches, pupil tasks, assessments and mechanisms for evolving these align well. When successful, each individual element is of high quality, and the elements work in harmony, together supporting pupils to learn effectively. It means setting out a path to proficiency in the subject, checking pupils are on that path and helping them to stay on that path.

Many of the features of the conception of quality, as outlined in our [research review (https://www.gov.uk/government/publications/research-review-series-mathematics/research-review-series-mathematics)](https://www.gov.uk/government/publications/research-review-series-mathematics/research-review-series-mathematics), are prevalent in the schools visited. For example, curriculum sequencing that includes ‘small steps’ approaches towards increasing mathematical proficiency, teaching that helps pupils to understand, and carefully curated opportunities to practise. In most schools, teachers routinely assess whether pupils have the necessary prior knowledge to undertake new learning. Primary school teachers quickly identify misconceptions, including through computerised tests.

However, it was when each element of mathematics education was of high quality, and those elements worked together, that pupils learnt most effectively. A centralised approach, with a carefully sequenced curriculum at its core, also supports schools faced with higher teacher turnover and/or teachers with less experience and subject knowledge. Even in schools without these challenges, teachers’ shared understanding of curriculum progression and high-quality teaching contributes to high quality mathematics education.

It is now common for teachers, in both primary and secondary schools, to receive regular subject-specific professional development. In primary schools, this support is often provided through Maths Hubs and is put in place by leaders who have a clear focus on developing teachers’ subject-specific teaching knowledge. Leaders’ strong understanding of high-quality mathematics education tends to be reflected in their monitoring foci. In secondary schools, it is much more common for professional development to take the form of departmental meetings that focus on curriculum design and effective curriculum practice.

In schools with less experienced or non-specialist teachers, there is a need to develop a shared understanding of curriculum progression and features of effective practice. Schools with more experienced subject-specialist staff engage in professional debates about how to teach aspects of the mathematics curriculum most effectively. Schools where mathematics provision is strongest ensure that other adults working with pupils, including teaching assistants and tutors, understand the curriculum and its implementation. Where these other adults do not have this shared understanding, the effectiveness of their support is limited.

There are examples in some schools of less successful practices. At primary level inspectors encountered curriculums that lack specified detail in the Reception Year, that allocate geometry to the summer term only or do not provide for enough learning of conditional knowledge. Sometimes questioning causes pupils to guess rather than recall. In other cases, multiple representations cause confusion rather than clarity.

In some schools, towards the end of primary and secondary phases, the focus of assessment shifts away from identifying pupils' needs and moved towards exam or test preparation. This phenomenon is often coupled with an increase in resources to provide for interventions and, in primary schools, reduced class sizes. The need for significant 'last-minute' intervention in some schools suggests deficiencies in the curriculum, teaching or rehearsal earlier on in pupils' mathematical education.

In most secondary schools visited, decisions about GCSE entry tiers are, in practice, taken at the end of Year 9, based on levels of attainment at that time rather than, more appropriately in Year 11. These early decisions about tiers of entry determine the curriculum pathway for these pupils in key stage 4 and this limits the mathematics that some pupils learn.

In some schools, the choice of mathematical methods taught is left to individual teachers' preferences rather than taken as a broader curriculum decision. This means that the approaches that are taught may only be useful to answer questions of the precise type identified in the curriculum for a particular year or key stage. The opportunity to engineer success over time is lost and pupils in these schools typically develop what can be characterised as 'disconnected pieces of knowledge'.

Pupil practice is sometimes limited in quality and quantity in both primary and secondary schools. This happens when leaders see practice as an activity, rather than focusing on its outcomes – whether pupils have practised until they have learned, to automaticity, the intended mathematical knowledge. There is often no consensus among leaders about benchmarks for optimal quality and quantity of practice that gives assurance that pupils have learned what is intended.

In some secondary schools, lack of revisiting conditional knowledge and, in primary schools, less emphasis on procedural automaticity (which develops through practice) is likely to compound this situation. Leaders are increasingly identifying the need for more practice, but not the reasons for this need: deficiencies that might be occurring in curriculum quality, teaching, or in the quality and quantity of practice in pupils' main lessons, for example.

An ambitious curriculum is one that maximises the mathematics that pupils learn. In some schools, teachers move on before ensuring pupils have learned important knowledge and committed that knowledge to long term memory. In schools where this is common, leaders focus on what pupils study, rather than on what pupils learn. Moving on when pupils are not mathematically ready gives the illusion of progress but creates ever greater gaps that will take more time to address in the future. Some leaders gain false assurance about the effectiveness of their curriculum design and practice through internal assessments that closely align with 'expected performance thresholds' of external assessments. This approach often leads to an acceptance of pupils moving through the mathematics curriculum with significant gaps in their knowledge and leaders failing to make necessary adjustments to their curriculums. In these schools, some pupils would be better served by studying less, but securely learning more.

Curriculum thinking about problem-solving and reasoning differs between primary and secondary schools. Pupils need to learn strategies and the most useful combinations of facts and methods to solve types of problem. Since it is not possible for pupils to encounter every possible problem, a suitable curriculum can identify strategies to solve an identified range of problem types. In some primary schools, pupils self-selected the problems they will solve and in many secondary schools there is a lack of curriculum planning to ensure all pupils encounter a range of problem types and have practice solving these problems. In some cases, pupils have little or no opportunity to use the mathematical knowledge they have to reason mathematically or solve problems. This is especially an issue for pupils who find learning mathematics more challenging.

Recommendations

Curriculum

All schools should make sure that:

- curriculums emphasise secure learning of, rather than encountering, mathematical knowledge.
- curriculum sequencing prepares pupils for transitions between key stages and phases

Primary schools should make sure that:

- they identify and sequence small steps in the Reception Year curriculum
- all pupils learn to apply facts and methods to wider problem-solving
- geometry knowledge is sequenced throughout, rather than at the end of, each year's curriculum

Secondary schools should make sure that:

- the curriculum specifies the mathematical methods that leaders want all pupils to learn, and that these form a coherent, 'forward-facing', base of mathematical knowledge rather than a collection of disconnected algorithms and tricks
- key stage 4 curriculums focus on maximising pupils' learning, and that decisions on GCSE tier of entry are based on what pupils know and can do towards the end of key stage 4 rather than what pupils knew and could do at the end of key stage 3, or earlier
- the curriculum plans how pupils will learn conditional (problem-solving) knowledge over time, for example by:
  - clearly identifying the range of problems to which pupils should be able to apply their new declarative (facts) and procedural (methods) knowledge
  - teaching pupils how their new facts and methods can be used to solve a range of problems
  - providing all pupils with sufficient practice in solving problems using newly learned facts and methods
  - ensuring that all pupils have enough opportunities to practise solving problems, after they have first being taught, and that these opportunities require pupils to make decisions about how best to solve these unfamiliar problems

Pedagogy and assessment

All schools should:

- make certain that teachers routinely check whether pupils have secure knowledge and understanding of prerequisite mathematics and address any gaps identified, before moving on to the next stage of learning
- make sure that teachers regularly connect new learning to what pupils have learned before, including showing pupils how it connects with learning in other subjects
- make sure that all pupils practise and consolidate new learning through well-designed exercises and activities, including sequences of problem-solving
- check that pupils are developing ‘procedural fluency’ (speed and accuracy of recall of methods) and address gaps in pupils’ procedural knowledge at the earliest possible opportunity

Primary schools should:

- consider using routines, keeping noise levels low and making sure that pupils are facing the teacher is explaining new content and giving instructions, to help them focus on what is being taught
- help younger pupils to learn their addition facts by heart and regularly check their recall of this knowledge
- reflect on the extent to which additional afternoon practice is due to deficiencies in the early curriculum and its implementation
- aim for pupils to become proficient and ready for Year 7, rather than just meet age related expectations for end of key stage tests
- make sure that questioning helps all pupils to recall and make connections, rather than allowing pupils to guess
- provide pre-teaching, additional teaching and extra practice for most pupils with special educational needs and/or disabilities (SEND)

Secondary schools should:

- make sure that pupils have sufficient opportunities to practise reasoning, explaining and problem-solving using the facts and methods they have been taught
- make sure that assessment information is used to reflect on the effectiveness of the curriculum and the way it is implemented, to guide future improvements

Systems at subject and school level

All schools should:

- provide continuing professional development for teaching assistants, and other adults working with pupils, to help them to understand the intended school mathematics curriculum and the way it is put into practice

Primary schools should:

- make sure that discussions with leaders about progress specifically address the needs of the lowest attaining younger pupils
- aim to prioritise resourcing for younger year groups, to better engineer success from the start of a pupil's mathematics journey
- when leaders observe lessons, focus on pupils' thinking and the quality and quantity of practice they undertake

Secondary schools should:

- make sure that non-specialist teachers receive the necessary professional development, including subject knowledge and subject specific pedagogical knowledge, to teach mathematics effectively

Other organisations

Other organisations should:

- provide sufficient quality and quantity of practice within scheme resources
- develop and offer computerised tests of facts and methods that share information about progress and attainment with pupils, teachers and leaders. Benchmarks for attainment should be based on proficiency
- extend the remit of Maths Hubs to make sure that most schools benefit from them
- offer the Mastering Number programme (https://www.ncetm.org.uk/maths-hubs-projects/mastering-number-at-reception-and-ks1) to all schools as an example of good practice in early mathematics

Those responsible for recruiting teachers should:

- make sure that all pupils are taught by teachers with appropriate levels of subject knowledge and subject teaching knowledge

The Department for Education, Ofqual and Awarding bodies should:

- explore whether the current design of the mathematics GCSE, including the tiers of entry offered and current typical grade thresholds, contribute to practices in schools that are not in pupils' best interests

Primary

Curriculum intent: identifying what pupils need to know and do

Summary of the research review

> The curriculum should identify and sequence, in small steps, declarative, procedural and conditional knowledge, and plan for pupils to learn this in small steps. This will make sure pupils' so that pupils' knowledge builds steadily over time. Linked facts and methods should be sequenced to take advantage of the ways that knowing facts helps pupils to learn methods, and knowing methods helps them to learn facts. Declarative and procedural knowledge can be combined and taught as strategies for problem-solving. A well-sequenced curriculum, and systematic teaching and opportunities for practice help pupils to become proficient in mathematics. This leads to success and motivation in the subject.

1. Whether adopted or 'home-grown', most primary schools' mathematics curriculums covered the national curriculum and were carefully sequenced. Leaders and teachers were aware of the importance of setting out a coherent path of progression for pupils. Often, planning included 'small steps' for each lesson, revisiting of previous learning and scheduled 'buffer zones' to allow teachers to respond to pupils' needs.

2. Occasionally, subject areas such as geometry were moved to later in the academic year. This was so that pupils could focus on foundational number and arithmetic at the start of the academic year. A small proportion of pupils were relatively insecure in their knowledge of geometry facts. This was possibly because geometry topics had been allocated to the summer term, leading to long gaps before pupils revisited them. In contrast, some schools had organised their curriculums so that a day a week was devoted to geometry, to make sure that pupils were regularly revisiting this topic.

3. Leaders' reasons for adopting commercial schemes included a desire to improve sequencing and resources. These schemes often included planned sequences of recapping and revisiting previously learned content. Leaders realised that this reduced the planning workload for teachers. This is a positive development.

4. 'Home-grown' curriculum plans were sometimes the result of collaboration across a multi-academy trust. In many schools, the NCTEM's 'ready-to-progress' criteria provided a useful framework for content. This helped leaders and teachers to see how concepts were interconnected and built over time. Additional resources from commercial schemes, to provide much-needed extra practice for pupils, were common.

5. Leaders wanted staff to follow the curriculum plans closely. This made sure that pupils experienced a smoother path to proficiency, supported with consistent language and routines, rather than a series of disjointed lessons. However, leaders also encouraged teachers to adapt lessons in response to the needs of their classes. The NCETM's 'ready-to-progress' criteria helped many teachers to prioritise key content.

6. In some schools, curriculum sequencing in the Reception Year was less detailed. Occasionally, leaders used early learning goals for curriculum planning. Potentially, this approach is problematic because the early learning goals are not frameworks to be used in this way. In contrast, the most effective Reception Year curriculum planning was as detailed as the planning that teachers of older pupils had access to. In many schools, staff had worked together to make sure that the curriculum prepared children for Year 1.

7. In some schools, leaders made sure that parents and pupils could find out what pupils would learn. One way of doing this was through sharing 'knowledge organisers'. These were used as well as, rather than instead of, curriculum planning.

Declarative knowledge

Summary of the research review relevant to declarative knowledge

> The curriculum should identify and sequence key facts, formulae, concepts and vocabulary. This helps pupils to avoid relying on derivation, guesswork or looking for clues.

8. In many schools, staff wanted pupils to learn key mathematics facts by heart. Pupils knew that this knowledge was important, too. The introduction of the times tables check in Year 4 has undoubtedly raised the profile of this type of knowledge. Curriculums generally emphasised mathematics facts, such as times tables, alongside helping pupils to understand the connections within ‘families’ of numbers. It is likely that these factors are the reason why it was rare to see pupils at key stage 2 having to rely on times tables grids.

9. Pupils in key stage 1 were often expected to develop flexibility and ‘deep understanding’ when thinking about number. However, there appeared to be less emphasis on learning addition and subtraction tables (number bonds) by heart. This is potentially problematic, as pupils need to be able to recall this type of knowledge quickly in order to access more complex mathematics in key stage 2.

10. In some schools, the curriculum in the Reception Year and key stage 1 emphasised both understanding and quick recall of addition facts. The NCETM’s Mastering Number programme was particularly helpful. In these schools, pupils were successful and received lots of praise. They were learning how to subitise (recognise a number of objects without having to count), understand numerical concepts and recall addition facts. Key features of this programme, in addition to a carefully sequenced curriculum, include:

- whole-class teaching
- use of a rekenrek (a type of counting frame)
- low-distraction dice patterns for subitising
- clear diagrams and representations
- regular questioning
- videos for staff training

11. Many schools’ curriculums identified and sequenced mathematical vocabulary, sentence stems and speaking frames. It was rare to see pupils relying on a ‘working wall’ for such vocabulary. Teachers explained and

modelled important, age appropriate, mathematical vocabulary, such as 'part – whole', 'lowest common multiples', 'array' and 'integer'. In Reception Year, songs and rhymes to helped with early language learning. Older pupils used technical vocabulary in their discussions and their writing. Occasionally, pupils had gaps in their vocabulary that affected their ability to reason and problem-solve.

12. In most schools, teachers quickly identified rare misconceptions in declarative knowledge. Older pupils were often accurate in their recall of declarative knowledge. However, in younger year groups, some pupils were unable to subitise or easily recall addition facts. Staff noted the impact of the pandemic restrictions on pupils' ability to recall this knowledge. In response, leaders adapted planning to give them additional opportunities to revisit it. Occasionally, pupils' books showed that they inverted numbers. This phenomenon may be linked to a lack of coherent sequencing, or insufficient modelling and practice in the Reception Year.

13. Younger pupils' inability to subitise or easily recall addition facts hampers their progress. They may be able to understand a teachers' instruction in, for example, Year 2 or 3, but they struggle to complete tasks with the speed and accuracy of their peers. They eventually obtain the correct answers ('getting by'), thus demonstrating their understanding, but are less likely to remember this new knowledge. This cycle continues, but with pupils increasingly unable to understand, let alone apply, new knowledge. This is likely to be one of the reasons why interventions are so ubiquitous in Year 6: pupils' internal struggles manifest after a significant amount of time has elapsed.

An example of stronger practice, in which pupils' success with fractions was underpinned by strong knowledge of mathematics facts:

In one school, a joined-up approach to curriculum, teaching and pupil practice had helped older pupils to gain knowledge and confidence in working with fractions. Among many other positive features, the mathematics lead was helping to steer teachers towards consistency in carrying out the school's chosen scheme of learning. Teachers were encouraged to slow the pace of learning, if necessary, to make sure that pupils mastered this knowledge before moving on. Learning key mathematics facts in class, for homework and through competitions between classes was part of a recently revised strategy to help pupils learn mathematics facts to automaticity (the point at which they could use them automatically). The mathematics lead ran a mathematics club for pupils who were less likely to be able to do homework at home. A Year 6 book scrutiny that focused on fractions showed that the sequence of lessons on fractions built in complexity in a logical order. Pupils successfully used different pictorial and concrete representations to help them to understand fractions. They were getting enough practice in calculating fractions. Pupils' highly accurate work with equivalent fractions showed the impact of leaders' work to make sure that pupils learned their mathematics facts to automaticity.

Procedural knowledge

Summary of the research review relevant to procedural knowledge

> There is a difference between methods that help pupils to understand concepts and perform mental calculations and methods that are efficient and useful now and in the next stage of learning. The curriculum needs to carefully sequence the teaching of mathematical methods. It should allow for some early methods, such as one-to-one counting, parsing, derivation and complex diagrams, to fade over time ('designed obsolescence'). Pupils should learn the most efficient, systematic and accurate mathematical methods, so that they can use them for more complex calculations and in their next stage of learning

14. Many primary schools' policies for calculation set out how pupils will learn procedural knowledge in a logical way. This made sure that language and models were consistent. Sequences tended to move from expanded and informal methods to compact and formal methods. This approach prioritises pupils developing an understanding of mathematical ideas. For example, using the 'grid method' of multiplication helps pupils to understand place value and the concepts that underpin multiplication. However, this can be at the expense of developing automaticity in using efficient and formal methods. The expectation that pupils should develop significant automaticity in these procedures appeared to be less of a priority. This is problematic because efficient and formal methods, once learned, enable pupils to engage with problems using larger numbers, for example.

15. In some schools, leaders encouraged pupils to learn and then choose from a wide range of methods. The goal was to ensure that pupils had different methods at their disposal to solve a range of problems, and to practise selecting from them. However, such an approach could lead to pupils choosing 'easier' methods and not getting enough practice in using the methods they will need most in the future. This would make it more difficult for them to progress

through the curriculum step by step

16. In some schools, younger pupils could use calculation tools quickly and accurately. For example, they used number lines or a rekenrek to help with addition and subtraction. This is likely to be the result of careful teaching of how to use these tools and of pupils' secure knowledge of 'one-to-one correspondence' (the ability to match an object to the corresponding number and recognise that numbers represent a quantity), subitisation and 'addition facts'.

17. Knowing how to set out written work is another form of procedural knowledge. In the best examples, leaders saw presentation as a part of the mathematics curriculum. Teachers carefully modelled and taught this form of knowledge. Textbooks and worksheets also helped to guide and support pupils' presentation, which gave them a sense of pride. Careful presentation is also likely to help pupils spot patterns and identify their own mistakes.

18. When leaders talked to us about gaps in pupils' knowledge, they often mentioned fractions. This was interesting, as this topic was generally sequenced well. Pupils grasped the conceptual knowledge underpinning this topic. Some leaders referred to the difficulties in teaching this topic online. However, working with fractions involves knowing and using procedural knowledge. Pupils can encounter difficulties when teachers have not prioritised procedural automaticity enough.

19. The bar model (https://www.ncetm.org.uk/classroom-resources/ca-the-bar-model/) had been integrated into mathematics curriculums, which was a positive development. In addition to helping pupils understand proportion, for example, it also helped older pupils to solve word problems and algebraic equations.

20. In some schools, some older pupils could not remember methods of multiplication and division. Pupils need procedural fluency to be able to solve a range of problems and to then learn which types of problems a method is useful for. Both of these develop with practice. Lack of procedural fluency is likely to be one of the reasons why pupils eventually need interventions in Year 6. Their lack of procedural fluency may not be apparent until they encounter a sample test paper that requires them to choose which method to use.

Conditional knowledge

Summary of the research review relevant to conditional knowledge

> Pupils should be able to recall facts and methods to some level of automaticity before using them for wider problem-solving. The curriculum should reflect this optimal ordering.
>
> 'Problem-solving' is not a generic skill, and pupils cannot become problem-solvers by imitating the activities of experts. Pupils need to learn strategies and the most useful combinations of facts and methods to solve types of problem. Since it is not possible for pupils to encounter every possible problem, a suitable curriculum identifies strategies to solve a range of problem types (topic-specific). For younger pupils, these include how to interpret word problems that are more common at their stage of learning. This approach helps pupils to know what to do, without relying on guesswork.

21. Leaders in all schools wanted pupils to be able to reason and solve a wide range of problems. However, curriculum approaches differed. In many schools, curriculum plans specified the models, explanations and sentence stems that would teach pupils strategies for wider problem-solving. Often, teachers dedicated a section of the lesson to this form of knowledge. This was a positive approach. One pupil said to us, 'They do it in a structured way by looking at a problem and then we do a similar one.'

22. In some schools, however, reasoning and problem-solving were an activity or task, and something that pupils could choose. This approach may result in some pupils skipping ahead of vital practice of facts and methods, or sticking with repeated practice of already-secure knowledge. This is problematic because pupils are entitled to learn all types of knowledge.

23. Some leaders had identified lack of fluency in procedures and lack of language and comprehension as barriers to reasoning and wider problem-solving. This shows that leaders were increasingly aware of the forms of knowledge that pupils need, to be able to reason and problem-solve.

24. A lack of conditional knowledge ultimately leaves pupils unable to choose the best method when completing a mixed set of questions, for example during a test. Multiple factors contribute to this. For example, pupils may lack automaticity in using declarative knowledge and procedural fluency. If pupils' declarative and procedural knowledge is secure, it is likely that they do not know the ranges and boundaries of the problems to which the methods apply. This comes from teaching and practising when to use methods, based on identifying underlying mathematical structures. In schools where pupils choose problem-solving challenges, some pupils will discover this form of knowledge, but many will not. This contributes to differences in pupils' attainment and a

need for more intervention in Year 6.

Meeting the needs of pupils

Summary of the research review relevant to meeting pupils' needs

> A well-sequenced path to proficiency, with the small steps identified, is important for all pupils and crucial for pupils with SEND. This helps pupils to keep up and reduces the need for catch-up support. Many pupils with SEND benefit from explicit, systematic instruction and from practice in using declarative and procedural knowledge. They may also need more time to complete tasks and opportunities to practise, rather than different tasks or curriculums. The value of knowing crucial facts and methods to automaticity is even more important for some pupils, such as those with autism spectrum disorder. This is because it frees up working memory for listening, learning and thinking about new knowledge.

25. The 'keep up, not catch up' approach, often directly referred to by leaders, made sure that pupils really understood and remembered what was being taught before moving on. This is an inclusive approach, particularly for pupils with SEND, provided there is enough support in place to help them keep up. Many pupils with SEND were following the same curriculum, with support and adaptations in class. For example, they received the same teaching as the main class, and then the teaching assistant supported them during their independent practice. However, this support may circumvent, rather than close gaps in knowledge.

26. Some of the more effective examples of additional help included pre-teaching and same-day interventions. This support gave pupils, including those with SEND, vital additional opportunities to review and practise core knowledge. Some SEND coordinators had introduced an additional 'precision teaching' programme for pupils who were working well below age-related expectations.

27. In some schools, leaders tried to maintain the ideal of all pupils moving on together when this was not a successful approach for pupils who were working at significantly below age-related expectations. Pupils in some schools, particularly those with SEND, were less likely to be 'keeping up and catching up' in lower year groups. Further, some pupils were less secure in the basic facts than their peers. Teachers' instructions and explanations of relatively

advanced mathematical concepts were beyond their comprehension, even if the teacher explained them well. In these situations, the ‘appearance’ of inclusivity may be taking the place of real inclusivity. These pupils’ needs might be better met if they were to learn different content practised using different tasks. This could be in groups of pupils with a similar level of attainment. This approach would be similar to how schools manage the teaching of early reading for older pupils who have yet to master basic reading skills.

28. Interventions were common. They were more likely to happen in Year 6 and, to a lesser extent, Year 2. In some schools, leaders had reduced class sizes and introduced setting in older year groups. As one school leader reported, the ‘gaps just got bigger and bigger’. These year groups are associated with external accountability measures, in the form of end of key stage tests. The focus on these year groups shows that leaders were having to balance the needs of individual pupils with the pressures of accountability. But leaders knew that some pupils needed to spend more time developing fluency and confidence in ‘the basics’ in earlier years. We observed differences in the quality and quantity of practice and handwriting proficiency in pupils’ books. These are likely to be contributing to holding back pupils with and without SEND.

Pedagogy: teaching the curriculum

Summary of the research review relevant to teaching

> The novice, whether they are starting school or starting a new topic, needs more instruction rather than less. Teaching should help them on the journey to expertise. Ideally, teaching should be systematic, follow the curriculum sequence and help pupils to understand. A systematic approach works well for pupils of all ages and stages.

29. In many schools, a consistent approach to designing and implementing the curriculum, with an emphasis on content and ‘small steps’ sequencing, involved a shift of responsibility for the curriculum from the individual teacher to the school’s leadership. This approach assured leaders that pupils’ progression through the curriculum was joined up and balanced and that teachers were using mathematical language and representations consistently. Additional whiteboard resources, often associated with commercial schemes, included representations that were clear and consistent. This helped pupils to

understand underlying mathematical structures better.

30. In the Reception Year, mathematics teaching sessions happened daily and tended to be quite short. This is to be expected, given the shorter attention spans of children in this age group. Many practitioners planned to use fun, interesting books, songs and rhymes to help children learn the language and concepts of early mathematics. This is positive, considering leaders' observations that speech and language delays were increasingly common.

31. Teachers often showed strong subject teaching knowledge in the classroom. They used careful explanations and demonstrations. They were able to break learning down into small steps for pupils. This shows the effect of strong networks of support, such as from the Maths Hubs. Where available, accompanying progression documents were also useful. By setting out what pupils had learned and what they would learn in the future, teachers could understand how each lesson fitted into the bigger picture of mathematics progression. Teachers adapted plans to make sure that pupils were confident before moving on to new learning.

32. Where the curriculum was chosen well and sequenced, and adapted to meet pupils' needs, this was reflected in the activities in pupils' books. These showed how learning was built from small component tasks to more complex mathematical processes.

33. Leaders often viewed teaching and use of mathematical vocabulary as 'non-negotiable' in lessons. Teachers introduced new vocabulary at the start of lessons and used it throughout. They frequently gave pupils opportunities to revisit this form of knowledge. This approach helped pupils to understand and remember more.

34. Teachers often used the concrete-pictorial-abstract approach[footnote 2] to teach new ideas and methods. Pupils were often able to use the objects used in demonstrations themselves. This is helpful for pupils, and leaders were keen for this to happen. However, occasionally, there was a tendency for teachers to use too many concrete and pictorial demonstrations. When this happened, pupils ended up confused from multiple representations.

35. In many schools, leaders expected teachers to use frequent questioning in lessons. They understood that a lack of back-and-forth interactions had been one of the limitations of online learning during the pandemic. Questioning tended to be used well. Familiar sets of questions were almost routine: 'What do you notice?' 'What's the same and what's different?' and 'Convince me'. Many teachers used questioning deftly throughout lessons to check whether pupils were ready to learn the material, to check their understanding and to encourage their reasoning. In some schools, teachers targeted questioning at

pupils who they knew needed extra practice. This made sure that opportunities to take part in the lesson were distributed fairly. However, in a minority of schools, questioning was less effective. Pupils were expected to second-guess what was going to be taught. More confident pupils would be able to volunteer, while others would stop concentrating. It is likely that, in these situations, pupils who need systematic, explicit instruction were missing out on opportunities to understand and learn new knowledge.

36. In many schools, lessons were efficient because of the use of routines. For example, in one school, pupils recited times tables during the transition time between instruction and going to their tables. Many teachers used routines and hand signals that younger pupils were familiar with from their phonics lessons. Pupils knew what was expected of them and what was going to happen next. This meant they could focus on new learning. The use of routines was associated with pupils being on task.

37. In some schools, pupils did not always listen or take part. This included, but was not limited to, pupils with SEND. Some pupils with challenging behaviour had gaps in their workbooks where there should have been evidence of practice. These pupils were practising much less than their peers. This may lead to a vicious circle where pupils with poor behaviour increasingly do not understand what is happening in the classroom, fall behind and are ultimately labelled as having SEND.

38. In some schools, the classroom layout affected pupils' learning opportunities. In classrooms where pupils faced the teacher, pupils engaged more and were better able to listen and pay attention. This makes sense, as teachers can better gauge pupils' reactions and know whether they need another explanation or worked example. In contrast, pupils found it difficult to concentrate in classrooms where they had been split into multiple groups for teaching and practice, mainly because of the noise.

Pedagogy: pupils' practice

Summary of the research review relevant to pupils' practice

> Practice helps pupils to understand and remember mathematical knowledge. There are broadly 2 types of practice. Type 1 involves retrieving and rehearsing facts, methods and strategies to the point of familiarity, speed and accuracy. Type 2 is more exploratory. It requires pupils to explain

> relationships, prove that they understand them and describe their reasoning. Both types are important. Pupils need quantity and quality of practice to help them understand and commit knowledge to long-term memory. This practice does not always involve textbooks and worksheets. It can include songs, games and rhymes.
>
> Tasks should help pupils to focus on the mathematics to be learned. They should provide for overlearning and, ideally, include variation. Support for learning and understanding should be gradually withdrawn over time. Tasks should give pupils opportunities to be successful, rather than having to rely on guesswork or unstructured trial and error.

39. In most schools, staff and pupils knew that practice, including the need for overlearning, was important. This is very positive. However, practice was not always accompanied by checks to ensure that all pupils were learning the intended declarative, procedural and conditional knowledge, to automaticity, before moving on.

40. Leaders had moved away from an assumption that correct answers showed that work was 'too easy'. When choosing schemes of work, some leaders looked for opportunities for retrieval, including quizzes. Worksheets accompanying schemes of learning were generally well designed. They included worked examples to help pupils understand, and did not contain distracting pictures. This was very positive for pupils.

41. Teachers consistently built in opportunities for pupils to rehearse knowledge. For example, they would start lessons with a 'fluent in 5' approach. Schools were increasingly adding a short, discrete session of extra practice. This usually took place in the afternoon and included mental arithmetic. In one notable example, leaders had built 'fast fractions' into this extra session, and pupils had opportunities to practise finding fractions of amounts and counting in fractions. Some schools used the early morning registration period to provide extra opportunities for pupils to receive focused support or practise content they had been taught. These additional sessions helped pupils. However, the need for them may indicate deficiencies in the design and implementation of the curriculum, including opportunities for practice, in pupils' main lessons.

42. Opportunities for practice sometimes skipped plainer, 'type 1' practice and moved too quickly to wider problem-solving. Teachers provided additional workbooks when they thought pupils needed more practice of the basics. However, if this happens frequently it may indicate that, despite being carefully presented, worksheets and tasks do not always provide enough practice for pupils.

43. Linked to the issue of limited practice on worksheets, there was often no consensus among leaders on the amount of quality and quantity of practice that gives assurance that pupils have learned what was intended. Leaders and teachers often needed a better understanding of what an adequate amount of practice is.

44. Staff in the Reception Year frequently made sure that children had repeated exposure to mathematical language and concepts. They planned mathematics activities carefully. Simple, rather than complex, activities helped children to think about mathematical concepts. However, when they were not working in a group with an adult, children tended to choose opportunities for practice from what was laid out in their classroom. Schools need to have a system for monitoring children's access to mathematics-related tasks from these resources. Otherwise, some children will get more practice than others, and some children will make less progress.

45. Most pupils' workbooks showed some quality and quantity of practice. However, there was some variation in pupils' completion rates and access to wider problem-solving. Older pupils' confidence with recall was associated with high-quality book work. In addition to pupils' relative proficiency, quality and quantity of practice is likely to be linked to how well they focus on the work, and the amount of effort they make. Pupils who were confident working with fractions had also experienced plenty of practice in calculating with fractions and knew important mathematics facts to automaticity. However, where pupils' work in key stage 1 was less thorough, they struggled to recall number bonds, for example.

46. In many schools, songs and rhymes gave pupils low-stakes opportunities (where pupils can make mistakes without penalty) to practise counting, shapes and vocabulary. Times table starters (such as 'rolling your numbers') and songs in lessons helped pupils to remember important knowledge. This is positive, as it helps all pupils and creates a culture in which mathematics is celebrated. Pupils, too, when asked about what helped them remember, talked about the usefulness of songs.

47. Frequent use of choral response (responding in unison) for low-stakes practice of concepts, vocabulary and mathematical sentences was a positive theme. In one example, leaders and teachers informed us that this was similar to their phonics approach. They noted that the approach was more inclusive, helping pupils with SEND and reducing anxiety about mathematics. This 'choral response' is a key feature of the NCETM's Mastering Number programme for younger pupils. A 'my turn your turn' approach to modelling and reciting stem sentences helps pupils to learn new mathematical language and understand important concepts.

Example of stronger practice, incorporating choral response:

In one school, leaders and teachers informed us how the use of choral response was similar to their phonics approach. The approach was more inclusive, helping pupils with SEND and reducing mathematics anxiety:

'Lessons [include] lots and lots of practice, a "ping-pong" approach between teacher and children, partner work, small steps of progress with teachers circulating… There is a lot of whole-class choral response with the teachers… You can hear who has got it slightly wrong without having to draw attention to the child.'

'It does feel "repetitive", but this is necessary.'

48. In many schools, homework consisted of arithmetic practice and access to online platforms to rehearse key mathematics facts. The latter was very popular with pupils. Homework tended to require pupils to practise something they had recently learned. This was because leaders wanted homework to require minimal parental support. In some schools, Year 6 pupils were set questions similar to those in national curriculum tests as homework. This is potentially problematic, as some questions require pupils to know relatively complex strategies and have high levels of understanding. Pupils who are working below age-related expectations become confused or resort to guessing. Leaders were often careful to make sure that test questions were related to content that pupils had already been taught. Some leaders laid on an extra homework club for pupils who were unable to complete homework at home.

49. There has been a cultural shift away from the previously ubiquitous '3 levels of differentiation'. This was where pupils were assigned or chose tasks set at different levels. In practice, this reinforced different declarative, procedural and conditional knowledge. In schools where pupils chose their own level of task, some pupils found this choice motivational, as it was a source of pride to take on the difficult 'challenge'. However, only a subset of pupils would access the reasoning or problem-solving version of whatever was being learned. Further, some pupils would proceed to the 'extra challenge' that teachers had prepared for early finishers, while others might not finish basic tasks. This approach may cater for a wider range of proficiency in the moment, but it does not allow each pupil to reinforce the knowledge they need to learn. This reinforces diverging rates of progress and attainment. In some schools, leaders and teachers overcame this by setting a minimum expectation that all pupils should at least do the basic tasks to consolidate their knowledge of facts and methods. Pupils experienced independent practice that was largely similar, at least at the beginning of sequences of learning. Leaders were aware that doing the same

thing can, in some circumstances, be 'OK', and that every pupil needed to be familiar with the basics.

Assessment

Summary of the research review relevant to assessment

> Frequent low-stakes testing (that is, without risk of failure), with an element of timing, is useful for checking pupils' knowledge of key facts and methods. This helps pupils to remember and gives leaders an insight into gaps in pupils' knowledge. Assessment at the end of a year or phase should assess pupils on what they have learned and rehearsed, rather than on what they do not know and cannot do.

50. Many teachers used live marking in lessons, as well as whole-class feedback. They swiftly noticed pupils' successes, misconceptions and errors. They could direct pupils to revisit knowledge at the start of the lesson, adjust the next lesson for all or focus on pupils who needed additional support. In some schools, same-day marking fed into short interventions in the afternoon. The speed and responsiveness of this approach minimised workload for teachers, as it was less likely that pupils would carry forward their errors and misconceptions.

51. Most leaders checked children's knowledge on entry to Reception Year. Practitioners were responsive to children's needs, but there was little evidence that they were systematically addressing gaps in the children's mathematical knowledge. It is possible that, at this stage of learning, gaps in learning slip through the net. More positively, in some schools, leaders were assessing pupils' 'addition facts'. This reflected leaders' awareness of the importance of declarative knowledge.

52. Most schools used regular end-of-unit tests aligned with the school's curriculum. Pupils would be tested on what they had learned and practised. This highlighted to leaders which forms of knowledge pupils needed to revisit. Intervention plans and resources were available for teachers to use with some commercial test schemes. Occasionally, assessments did not include geometry, data and work with coordinates. Some leaders had developed their own assessments to test and identify gaps in knowledge. Some used multiple-choice questions to expose misconceptions. Both approaches are positive, because

they increase the level of detail of diagnostics. In some cases, benchmarks for proficiency had been set at an expectation of 80% accuracy. This is a more appropriate benchmark than the benchmark for ‘meeting age-related expectations’ at the end-of-key-stage tests (currently only around 50% accuracy). However, there was less testing for, or understanding of, benchmarks for ‘procedural fluency’. This indicated a lack of knowledge in this area.

53. Some leaders had used technology for testing. For example, pupils took end-of-unit quizzes on laptops, then received their overall score and information about which questions were correct. Pupils were keen to take a test again, motivated to do even better. Although these quizzes cannot check pupils’ reasoning or wider problem-solving, they provide low-stakes practice and precise information about pupils’ component knowledge and readiness to move to the next topic. This kind of system gives pupils and teachers instant and accurate feedback, while almost eliminating teachers’ workload of marking and analysis.

54. Pupils liked frequent, low-stakes (and timed) testing because they could achieve personal bests. In some schools, pupils talked about testing being fun, with friendly class-against-class competition, the use of music and even a big voiceover. Given the frequency of low-stakes testing and pupils’ positive view, it is unsurprising that no pupils talked about fear of national curriculum tests. These pupils were well prepared, due to high-quality curriculums, teaching and practice. There was no notion of teachers putting pupils under undue pressure or expecting them to compare themselves with others. Pupils knew that speed and accuracy of recall (of component knowledge) were important to aim for. They were proud of the amount of work they could complete, the proportion of answers that were correct and being seen to access the difficult task. This was set against a backdrop of a positive culture where mistakes were ‘OK’.

55. Leaders often used summative tests to compare pupils with the national average and to carry out question-level analysis. Where this was due to a focus on national curriculum tests in Year 6, for example, leaders wanted to make sure pupils were going to be successful. However, there was little consensus about whether leaders had identified mechanisms for understanding the cause of pupils’ lack of progress. Most schools used end of key stage tests, or similar papers. The most positive approaches kept things ‘low key’ and avoided repeated summative testing to ‘show progress’. However, there are still limitations to this approach. Question-level analysis of summative tests does not accurately pinpoint gaps in pupils’ component knowledge. Further, meeting age-related expectations (around 50% accuracy) does not assure leaders that pupils are ready for key stage 3. This is because the subject is hierarchical: new and increasingly abstract concepts integrate, and therefore depend on, mastery

of foundational concepts. It is also possible that, because resources are limited, prioritising ‘cusp’ groups comes at the expense of pupils who need support the most.

Systems at the school level

Summary of the research review relevant to systems

> School-level systems strengthen the consistency of a pupil’s journey to proficiency. They include monitoring approaches, staff training, resource allocation, teaching and learning expectations, ways of raising the subject’s status and ways of sharing information between stakeholders. Professional development should be a planned and purposeful pathway to expertise in teaching and subject leadership.

56. A well-established culture of collaboration has strengthened and extended networks of support. Leaders across different schools and phases often collaborated on curriculum progression, moderation, supporting professional development and sharing good practice. Larger organisational structures, such as the Maths Hubs and academy trusts, helped to ‘cross-pollinate’ information about high-quality mathematics education. Collaboration and information-sharing extended into staff meetings, leaders’ work to support children’s transition into Year 1, team planning and team teaching. Teachers appreciated opportunities to observe each other, and gained confidence in teaching outside their ‘comfort zone’. This collaboration is positive for pupils’ education and, to use a common safeguarding phrase, results in a large and interconnected ‘team around the child’.

57. Professional development provided by the Maths Hubs and information produced by the NCETM have informed leaders about high-quality mathematics teaching. This knowledge could reduce their vulnerability to quick-fix approaches or unevidenced ‘fads’. This, in addition to choosing or creating high-quality curriculums, appeared to be the key driver of improvement in mathematics education. This is very different from the situation we identified a decade ago, when the ‘Mathematics: made to measure’ report stated, ‘Very few schools provided curriculum guidance for staff, underpinned by professional development that focused on enhancing subject knowledge and expertise in the teaching of mathematics, to ensure consistent implementation of approaches and policies.’

58. These combined aspects were highly positive, signalling a shift away from a culture of high-stakes lesson observations (used, in isolation, for accountability purposes) and planning scrutiny. Senior leaders made sure that subject leads had time to look at books, visit lessons and speak with pupils. Lesson observations were typically frequent, collaborative and low stakes. As teachers at one school told us, 'Nothing is sprung on us'. Leaders used their knowledge of high-quality mathematics education, rather than the teachers' standards, to inform observations. They would cross-check lessons with curriculum plans and check for consistency. Rather than generic approaches, they generally looked for features such as modelling vocabulary, using resources, recapping knowledge and supporting pupils with SEND. Feedback to individual staff and at staff meetings gave small, sequential development points to gradually improve teaching and teachers' understanding of progression.

59. The balance of observations, however, appeared to be weighted more towards what teachers were doing, than pupils' understanding, focus and practice. This may be because leaders relied on a checklist of observable 'features' of teaching[footnote 3]. It is possible that a stronger focus on pupils would give leaders more information about pupils' understanding, and the variation in type and amount of pupils' practice.

60. Most primary schools are small, which means that classes are typically of mixed attainment. Occasionally, some year groups appeared to have higher-than-average proportions of pupils with SEND. This presented resourcing and teaching challenges to leaders and teachers. In some schools, leaders allocated experienced and knowledgeable staff to the pupils and cohorts with the greatest needs. In some schools, children with significant SEND would receive their mathematics instruction as a separate group.

61. In Year 6 and, to a lesser extent, Year 2, setting and smaller class sizes were relatively common. This indicates that, despite strong curriculums and teaching, not all pupils are making good progress. It was rare to hear of this approach being used with other year groups. It is likely that allocating more resources to the final year groups leaves little or no capacity to take the most proactive approach possible: to significantly intervene with the younger pupils, closing gaps right from the start.

62. In many schools, pupils spoke highly of the way that their school celebrated and encouraged mathematics. Pupils were immersed in a supportive culture that reduced anxiety and increased their confidence. Hard work, progress and achievement in mathematics were often celebrated in assemblies. Common approaches to encouraging friendly competition and celebrating pupils' successes were set in a context in which teachers helped pupils to understand why they made mistakes and that it was 'OK' to make mistakes. Wherever

competition was mentioned, it was framed positively. For example, schools offered pupils opportunities to compete on online platforms or against other schools within a trust, similar to a sports tournament.

63. In many schools, leaders often shared information with parents. This included facilitating workshops, games sessions and interactive lesson observations. Parents would learn about age-related expectations, end-of-key-stage-2 tests, mathematics teaching, and how they could help their children at home. In some schools, leaders invited parents to share the ways mathematics helps them in their jobs. Leaders often signposted parents to useful websites and apps that would help their children.

64. When school leaders were held to account by governors and senior trust leaders, challenge was often based on important, reportable data. This is understandable, given the high-stakes nature of accountability. Governors and trust leaders also visited schools regularly to learn more about pupils' experiences. However, summative assessment data does not necessarily give senior leaders enough assurance about the level of younger pupils' proficiency. For example, it is possible for younger pupils to obtain correct answers, but this might mask a reliance on using fingers to count. Governors and trust leaders could easily identify gaps in foundational learning through simple checks, similar to checks of pupils' phonics knowledge, and then discuss them with senior leaders.

Secondary

Curriculum intent: identifying what pupils need to know and do

Summary from the research review

> The curriculum should identify and sequence declarative, procedural and conditional knowledge so that pupils' knowledge builds steadily over time. Linked facts and methods should ideally be sequenced to take advantage of the way that knowing facts helps pupils to learn methods and knowing methods helps them to learn facts. Declarative and procedural knowledge can be combined and taught as strategies for problem-solving. A well-

> sequenced curriculum, systematic teaching and opportunities for practice help pupils to become proficient in mathematics. This leads to success and motivation in the subject.

Curriculum design

65. In just over half the schools visited, leaders considered their mathematics curriculum as a 5-year programme. Their curriculum did not distinguish between key stage 3 and key stage 4. This approach ensured that what pupils studied in key stage 4 built on what they had learned in key stage 3.

66. In a small number of schools that had distinct key stages 3 and 4 curriculums, the key stage 4 curriculum for many pupils who were expected to sit the foundation tier GCSE papers repeated all, or most, of what they had learned at key stage 3. This was often because leaders treated examination specifications as the curriculum and 'started at page 1 of the specification'. They did not design a key stage 4 curriculum that built on what pupils already knew and could do at the end of key stage 3.

67. A minority of schools had a 2-year key stage 3. This often led to pupils being rushed to complete the content of the key stage 3 national curriculum by the end of Year 8. As a result, some pupils' learning was insecure. This meant that topics had to be retaught in key stage 4.

68. In a very small number of schools, the key stage 3 curriculum was designed around projects or themes. Typically, the mathematics that pupils learned through each project was not clearly defined. Decisions about what mathematics to focus on in each unit were left to individual teachers. At the end of Year 9, pupils in these schools had very varied mathematical knowledge, depending on the combination of teachers they had been taught by over key stage 3. As a result, key stage 4 was mainly spent 'identifying and filling gaps' in pupils' knowledge.

69. In the majority of schools visited, decisions about GCSE tiers of entry were, in practice, taken at the end of Year 9, based on levels of attainment at that time. These early decisions determined the curriculum that pupils would study in key stage 4, rather than what pupils knew and could do at the appropriate stage of Year 11. This limited the mathematics that some pupils learned.

70. One of 2 scenarios was likely for pupils judged to be on the border between foundation and higher-tier classes. If they were placed in a foundation-tier class, they tended to progress through the foundation-tier content at a slower rate than they were capable of, or they would complete their mathematics learning up to a year before their GCSE examinations. If they were placed in a higher-

tier class, they were expected to rush through the higher-tier curriculum in the hope that enough content would ‘stick’ for them to be able to pick up lots of ‘method marks’ in their GCSE exams. Both of these approaches denied pupils the opportunity to securely learn mathematics that would be useful in their future study and beyond. Mathematics leaders identified that pressure from pupils, parents and senior leaders to ‘finish the course’ was a significant factor in their decision to design their curriculums in this way.

71. A minority of schools, typically those with historically poor outcomes in mathematics, designed their mathematics curriculum to finish at the end of Year 10. In these schools, pupils spent most of Year 11 in a cycle of completing past GCSE papers, often under examination conditions. Teachers marked the papers and identified ‘gaps in knowledge’. Teachers then re-taught the topics in which pupils had performed less well, focusing on question formats likely to be used in examinations. This approach reduces mathematics education to an exercise in preparing pupils to pass an external examination and does not provide pupils with a rich mathematical education. Leaders in these schools rarely considered why pupils had these gaps in knowledge or whether GCSE assessments were the best way of identifying them. They did not consider whether fewer gaps would exist if pupils were not rushed through the curriculum to finish new learning by the end of Year 10.

72. A minority of mathematics leaders designed their key stage 4 curriculum to maximise the amount of mathematics that each pupil would learn securely. They made a clear distinction between curriculum design and examination specifications. Their curriculums focused on ensuring that pupils finished Year 11 with as much secure mathematical knowledge and understanding as possible. Decisions about which tier of examination each pupil would sit were taken as late as possible. In these schools, some pupils sat the foundation-tier papers despite having securely learned some mathematics beyond the foundation specification. Others sat the higher-tier papers despite not having studied the entire higher-tier specification. This approach better prepared pupils for further study and work, as they had a greater breadth of secure mathematical knowledge.

Example of stronger practice:

Leaders in one school had worked with local post-16 colleges to identify the important mathematical knowledge that pupils might need to be successful in a range of post-16 courses. Leaders used this information to inform their curriculum design for ‘intermediate’ pupils whose learning would extend beyond the foundation-tier specification but not complete the higher-tier specification. This supported pupils to be successful in their post-16 study.

73. Almost all schools visited had curriculums that developed learning sequentially. Leaders had ensured that pupils learned the necessary prerequisite knowledge before beginning to study new mathematical content.

74. All schools had some classes that were split between 2 or more teachers. In some, this was due to timetabling constraints. In others, it was to make sure no classes were taught entirely by non-specialist teachers. Year 7 classes and lower sets in other year groups were disproportionately likely to be taught by more than 1 teacher.

75. Most schools visited had thought carefully about pupils' progression through the curriculum when they were taught mathematics by more than one teacher. In some schools, different units or curriculum strands were allocated to different teachers. In others, the teachers taught sequentially, with one teacher 'picking up where the other left off'.

76. Where teachers taught different units or curriculum strands, learning was most successful when leaders had made sure that pupils were not taught new mathematics that required knowledge that their other teacher had not yet taught. When this was not the case, and when teachers did not identify gaps in pupils' knowledge, pupils struggled to learn new mathematics.

77. In schools where teachers taught sequentially, new learning was less successful when teachers and leaders did not make sure accurate information was shared between teachers after each lesson.

78. A minority of mathematics curriculums explicitly took account of what pupils were learning in other subjects. In these schools, mathematics and other subject leaders had worked together to agree, where appropriate, common approaches to teaching and applying mathematics. For example, in one school, the mathematics leader and science leader had worked together to make sure that the approach to rearranging formulae was consistent across both subjects. They made sure that the examples and exercises used in mathematics drew on formulae that pupils would study in science.

Declarative knowledge (facts) and procedural knowledge (methods)

Summary from the research review

> The curriculum should identify and sequence key facts, formulae, concepts and vocabulary. This helps pupils to avoid relying on derivation, guesswork or looking for clues.
>
> There is a difference between methods that help pupils to understand concepts and perform mental calculations and methods that are efficient and useful now and in the next stage of learning.
>
> The curriculum needs to sequence the teaching of mathematical methods carefully. It should allow for some early methods, such as parsing, derivation and complex diagrams, to fade over time (designed obsolescence). Pupils should learn the most efficient, systematic and accurate mathematical methods, so that they can use them for more complex calculations and in their next stage of learning.

79. In the majority of schools visited, leaders explicitly considered the important mathematical vocabulary that pupils were expected to learn at each stage, when designing the curriculum. Making this explicit ensured that teachers developed and checked that pupils were using mathematical vocabulary accurately.

80. In most lessons visited, teachers carefully and consistently modelled the use of correct mathematical terminology and expected pupils to use it when talking or writing mathematically. On the few occasions when teachers did not use vocabulary accurately, for example referring to the ‘bottom of a fraction’ when talking about the denominator, many pupils also failed to use accurate mathematical terminology when talking about mathematics.

81. Teachers consistently emphasised the importance of pupils presenting work carefully so that others could follow lines of reasoning. In some cases, they presented this as important ‘so that you can get method marks in the exam even if you make a mistake’. However, more often, teachers emphasised the fact that mathematics is ‘a communication subject’.

82. When new mathematical procedures were being taught, modelling of high-quality mathematical presentation for pupils was common. Pupils were then expected to present their mathematics similarly. This explicit teaching of accurate mathematical presentation increased the likelihood of pupils communicating their mathematical understanding clearly.

83. In schools where teachers modelled inaccurate mathematics (for example ‘stringing equals signs’: 3/5 of 60 = 60 ÷ 5 = 12 x 3 = 36), similar errors were seen in pupils’ work. Leaders ascribed such errors to gaps in teachers’ subject

knowledge.

84. The majority of schools had identified core mathematical methods and approaches that they wanted pupils to be taught at the various stages of their learning journey. Most focused on selecting methods and approaches that were:

- ‘forward facing’, in that future learning would build on them
- ‘backward facing’, in that new learning was deepened because it built on what pupils already understood

85. In these schools, the teaching of mathematical methods and techniques was carefully sequenced so that all pupils learned more mathematical knowledge, irrespective of the combination of teachers they were taught by. Leaders in these schools saw the range of mathematical methods to be taught, and the order in which they are taught, as a curriculum decision.

Example of stronger practice:

One school had identified that pupils would initially be taught to expand and simplify pairs of brackets using the ‘grid’ approach. This had been selected following extensive discussion in departmental meetings. It was chosen because it:

- built on pupils’ existing knowledge of multiplication of numbers using the grid approach
- was extendable to more complex situations, and would, ultimately, support pupils to expand brackets without the need for drawing out grids

86. In a minority of schools, the choice of mathematical methods taught was seen as a teaching decision. The goal was that pupils would be able to answer questions of a specific type. This sometimes led to ‘cul-de-sac’ approaches being taught, which could be used to solve questions of the precise type identified in the curriculum for this year, but were not extendable as pupils developed mathematically. This risks pupils seeing mathematics as a collection of unconnected algorithms to be memorised and applied in specific situations. In these schools, pupils were often taught a range of methods and approaches, but no links were drawn between them. The range of methods and approaches taught, and the order in which they were taught, were not picked carefully to deepen understanding. Instead, a range of methods were taught in the hope that one or more would ‘stick’. In these schools, leaders and teachers were more likely to talk about choosing approaches that enabled pupils to answer

examination questions or gain method marks, rather than to develop their understanding.

Example of weaker practice:

In one school, the curriculum aims were identified as ‘I can do’ statements, for example: ‘I can expand and simplify a pair of brackets of the form (ax + b)(cx + d)’. Decisions about the mathematical method(s) to be taught were left to individual teachers. Some teachers taught pupils to complete questions of this type using the ‘FOIL’ mnemonic (‘First. Outside. Inside. Last’). This standalone approach to multiplying a pair of brackets did not build on pupils’ existing knowledge of multiplication. It did not support their future learning, for example being able to expand and simplify expressions of the form (ax + by + c)(dx + e).

Example of stronger practice:

Mathematics leaders in a middle school had worked closely with first schools and high schools in their area to ensure that the methods and approaches they used with children were consistent and coherent as pupils made progress in mathematics. In particular, the schools worked together to explore how bar models could be used effectively to teach new concepts throughout pupils’ mathematics education. The curriculum made explicit how bar models should be used. This ‘joined-up thinking’ ensured that pupils’ mathematical progress did not slow at points of transition between schools.

87. When the choice of method was left to individual teachers, inappropriate methods were sometimes taught. This happened when teachers were not aware of ‘what comes next’ (including education beyond the age range of the school) or ‘what came before’ (including education before the age range of the school). This was because they lacked knowledge of the curriculum beyond the current year group or key stage.

88. A few leaders and teachers identified that high-stakes accountability of internal and external assessments led to them teaching ‘tricks’, such as ‘keep, flip, change (KFC)’ for dividing fractions, that taught pupils to answer questions of the specific type likely to come up in assessments but did not prepare them for future learning. These approaches often led to pupils viewing mathematics as a list of unconnected methods to remember and apply.

Conditional knowledge (strategies)

Summary from the research review

> Pupils should be able to recall facts and methods to some level of automaticity before using them for wider problem-solving. The curriculum should reflect this optimal sequencing.
>
> ‘Problem-solving’ is not a generic skill, and pupils cannot become problem-solvers by imitating the activities of experts. Pupils need to learn strategies and the most useful combinations of facts and methods to solve types of problem. Since it is not possible for pupils to encounter every possible problem, a suitable curriculum identifies strategies to solve a range of problem types (topic-specific).

89. All leaders said that their curriculums were designed to match the national curriculum aim that ‘[pupils] can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions’.

90. In the majority of schools, leaders said that problem-solving was integrated within their curriculum and pupils were taught to apply new facts and methods to solve problems within each unit of work. In most of these schools, however, teachers were given little guidance on the types of problems pupils needed to be taught to solve in the unit of study. There was no common understanding of how to teach problem-solving effectively.

91. In most schools, decisions about teaching problem-solving were left to individual teachers. This often led to lack of fairness. Some classes benefited from considerable teaching of how to solve structurally similar problems using recently taught facts and methods. Other classes had very little. This lack of teaching was disproportionately an issue in lower sets, and in classes taught by less experienced or non-specialist staff.

92. In some schools, pupils were not explicitly taught how to apply the mathematics they had recently learned to mathematical problems. Their only exposure to solving mathematical problems was through answering the final few questions of a predominantly procedure-focused exercise. Often, many pupils did not reach this stage of the exercise. These pupils, therefore, had very little experience of applying mathematical methods beyond routine and established applications. Pupils in these schools were notably less confident

when solving mathematical problems.

93. In a minority of schools, problem-solving was explicitly planned into the curriculum. Teachers understood the importance of demonstrating how to apply mathematical methods to problems and giving pupils multiple opportunities to practise applying these methods to structurally similar problems. In the most successful lessons, teachers clearly ‘drew out’ the similarities between problems to help pupils identify the mathematical techniques that might be useful for different types of problem.

> Example of stronger practice:
>
> One department had clearly identified the range of problems that they wanted pupils to be able to solve at various stages of the curriculum. In lessons, teachers modelled how pupils could use new learning to solve mathematical problems. They drew out the mathematical similarities in a range of problems that, on the surface, looked unlinked. They gave pupils opportunities to practise solving problems of a mathematically similar nature.

Meeting the needs of pupils

Summary from the research review

> A well-sequenced path to proficiency, with the small steps identified, is important for all pupils and crucial for pupils with SEND. It helps pupils to keep up, minimising the need for catch-up support. Many pupils with SEND benefit from explicit, systematic instruction and from practice in rehearsal of declarative and procedural knowledge. They may also need more time to complete tasks and opportunities to practise, rather than different tasks or curriculums.

94. In all schools visited, teaching assistants supported classes in which some pupils had SEND. It most cases, they supported individual pupils or small groups of pupils in the classroom. A small number of schools used teaching assistants to support the work of the wider class, potentially following teacher-led whole-class instruction, while the teacher provided additional support to the pupils with SEND.

95. Teaching assistants who were working directly with individual pupils with SEND were most successful in supporting mathematical development when they had a secure knowledge of the mathematics curriculum and of the range of pedagogical approaches being used by the teacher. In these cases, it was common for the teaching assistant to be assigned to work mainly within the mathematics department, effectively as 'a mathematics-specialist TA', or for the teaching assistant to have received mathematics-specific CPD that focused on the curriculum and appropriate choices of teaching method.

96. Teaching assistants who did not have this specialist knowledge were sometimes restricted to supporting pupils in more generic ways. For example, they offered encouragement or repeated the teacher's explanations. In a very small number of cases, the teaching assistant's lack of mathematical knowledge actively hindered pupils' progress. For example, one teaching assistant advised pupils of 'tricks' to follow rather than explaining the mathematical methods the teacher intended.

97. Almost all schools visited structured their classes in sets based on current attainment, at some stage of the pupils' journey through school. Most had school timetables that allowed pupils to move between sets during the school year. A small minority of schools had explicit systems in place to support pupils who moved from one teaching group to another. For example, they provided interventions to close any gaps in knowledge between the pupil who was moving and their new class. This enabled pupils to be more successful in their new group. Most other schools took ad-hoc approaches to supporting pupils who had moved set. These had varying degrees of success.

Example of stronger practice:

One school employed a subject specialist whose role included providing extra support for pupils who needed it. When a pupil moved teaching group, they received additional small-group or one-to-one teaching from the intervention tutor to make sure that they did not have any gaps in necessary prerequisite knowledge. This made it more likely that the pupil would integrate successfully into the new class.

98. In some schools visited, lower-attaining pupils completed a key stage 4 entry-level qualification. This worked most effectively when leaders designed the mathematics curriculum so that pupils could also be entered for GCSE mathematics, where appropriate. Using the entry-level qualification in this way made sure that the curriculum for these pupils was not narrowed and did not limit the qualification that they could achieve.

99. In a small number of schools, high-attaining pupils were given the opportunity to study for additional mathematics qualifications beyond GCSE, including level 2 additional mathematics and level 3 free-standing mathematics qualification (FSMQ) additional mathematics. Typically, pupils took the examinations for these qualifications at the end of Year 11, alongside their GCSE mathematics, rather than taking their GCSE mathematics early. This approach benefited pupils, as it allowed the curriculum to be structured in a coherent manner. It made the links between GCSE mathematics and the additional qualification clear, rather than artificially dividing the mathematical knowledge between GCSE and the additional qualification.

Pedagogy: new learning

Summary from the research review

> The novice, whether they are starting school or starting a new topic, needs more instruction rather than less. Teaching should help them on the journey to expertise. Ideally, teaching should be systematic, follow the curriculum sequence and help pupils to understand. A systematic approach works well for pupils of all ages and stages.

100. In many schools where mathematics provision had historically not been strong, leaders discussed the challenges of staff recruitment and having less-experienced mathematics teams, often with a number of non-specialist teachers. In these schools, it was common to see teachers using plans and resources developed by others – either from commercial schemes, from multi-academy trust subject teams or developed ‘in house’ by more experienced staff. Textbooks were used in very few schools. When they were, they were usually associated with the specific commercial mathematics scheme that the school was using. Requiring teachers to use provided lesson plans and resources gave leaders a level of quality assurance of the mathematics provision that pupils would receive from teachers who lacked some subject knowledge or subject specific pedagogical knowledge.

101. In almost all schools, leaders said that teachers were expected to adapt and adjust provided resources to meet the specific needs of pupils in their classes. It was, however, relatively rare to see adaptations take place to resources usually being used without any adjustments. On occasions, this led to pupils studying new content without being secure with necessary pre-

requisite learning or pupils moving onto new content before they were ready. Some leaders identified that pre-prepared lesson plans and resources were a ‘necessary but insufficient’ step on the department’s journey of improvement. They had correctly identified that some teachers’ subject knowledge and subject teaching knowledge needed to improve to allow them to make informed decisions about appropriate necessary adjustments to meet the needs of pupils in their classes.

102. In schools where mathematics provision had historically been strong, teachers were often given more flexibility to write or source the resources to use with their classes. This was a successful approach in this context. Often, these departments would have physical and electronic stores of shared resources to reduce teachers’ workload. The departments typically had settled staffing, and new staff were inducted relatively informally into the department’s agreed approaches.

103. In some schools, we saw misconceptions being introduced by teachers. These errors usually occurred in lessons taught by less experienced or non-specialist teachers, who lacked the subject knowledge or subject teaching knowledge to go ‘off script’ when responding to pupils’ questions or when they observed misconceptions.

Example of stronger practice:

The mathematics leader in one school had identified that the use of pre-prepared resources and lesson plans was necessary, in their context, to support the number of non-specialist staff in their team but was not sufficient to ensure high-quality provision. They had identified that ‘off-script’ provision (when teachers needed to respond to pupil questions, misconceptions or errors) was sometimes poor. Senior leaders had dedicated significant subject meeting time to providing CPD to teachers in the department. This aimed to improve subject knowledge and subject teaching knowledge.

Example of stronger practice:

One small trust had developed a central ‘resource hub’ of high-quality resources that teachers could use with their classes. These resources, why they were viewed as being high quality and how they aligned with the curriculum were discussed in department meetings. This approach had been introduced because leaders had noticed that the resources being selected by teachers did not always align with the planned curriculum and this was limiting pupils’ learning.

The 'resource hub' was constantly evolving as teachers from across the trust developed their understanding of the curriculum and effective resource design, and added new resources that were quality assured by mathematics leaders. Leaders and teachers identified that this approach:

- provided teachers with the professional autonomy to select resources that met their pupils' needs
- ensured that resources selected aligned with the curriculum
- developed teachers' expertise
- helped teachers to manage their workload while meeting the needs of pupils in their classes

Pedagogy: consolidation of learning

Summary from the research review

> Practice helps pupils to understand and remember mathematical knowledge. There are broadly 2 types of practice. Type 1 involves retrieving and rehearsing facts, methods and strategies to the point of familiarity, speed and accuracy. Type 2 practice is more exploratory. It requires pupils to explain relationships, prove that they understand them and describe their reasoning. Both types are important. Pupils need quantity and quality of practice to help them understand and commit knowledge to long-term memory. This practice does not always involve textbooks and worksheets. It can include songs, games and rhymes.
>
> Tasks should help pupils to focus on the mathematics to be learned. They should provide for overlearning and, ideally, include variation. Support for learning and understanding should be gradually withdrawn over time. Tasks should give pupils opportunities to be successful, rather than having to rely on guesswork or unstructured trial and error.

104. In all schools, pupils were given the opportunity to practise new content in lessons. In most schools, teachers and leaders carefully selected exercises that became more complex over time. This ensured that pupils' knowledge deepened as they worked through the exercises set over a sequence of lessons. In a minority of schools, however, teachers had not planned for exercises to become more complex over time. As a result, pupils remained

unable to apply their knowledge to more complex problems.

105. Most schools gave pupils opportunities to practise mathematics after they had first learned it. For example, homework was routinely used to practise prior learning, rather than focusing on current learning. Teachers also provided frequent low-stakes quizzes on prior learning and lessons in which pupils routinely spent time answering questions on topics they had studied previously. These approaches helped pupils to develop accuracy and speed in their mathematics.

106. There was much planned practice in the schools visited. However, in only a few schools had teachers agreed on how much practice was sufficient. Often, the amount of practice was determined by how long was left in the lesson once the teacher had modelled new content. It was not based on judgements about whether pupils had successfully engaged with sufficient, well-designed practice to learn the intended mathematics to automaticity. Some pupils were moved on to new content before they had had a chance to get to grips with what they had learned. This was most typically an issue for pupils who were grasping new concepts more slowly. These pupils' workbooks showed that, in some cases, they would complete just a fraction of the practice of their classmates. They were not benefiting from the carefully developed exercises, and their practice was restricted to a few routine applications that were not practised enough for security. This left many pupils unable to build successfully on these concepts later.

107. In most schools visited, pupils did not practise problems enough that required them to explain, prove, justify or describe relationships. This runs the risk that pupils have a shallower understanding of mathematics, with their knowledge restricted to routine application of learned algorithms.

Assessment

Summary from the research review

> Frequent, well-timed, low-stakes testing is useful for checking pupils' knowledge of key facts and methods. This helps pupils to remember and gives leaders an insight into the gaps in pupils' knowledge. Summative assessment should assess what pupils have learned and rehearsed, rather than what they do not know and cannot do.

Assessment for learning

108. In all schools, leaders said that teachers would routinely make sure that pupils had securely learned conceptual knowledge before starting new learning that built on it. However, this assertion was consistently accurate in very few of the schools visited.

109. Teachers used a range of approaches to assess pupils' pre-existing knowledge/understanding. These included quick low-stakes quizzes, with pupils' answers displayed on mini-whiteboards or electronic devices. When used with well-considered questions, these approaches gave teachers appropriate assurance that pupils had the knowledge and understanding necessary to move on to the next stage of the curriculum.

110. However, there were weaknesses in how effectively teachers struck a balance between assumption and assessment when checking pupils' pre-existing knowledge. In a minority of lessons, teachers assumed that the majority of pupils had a firm grasp of pre-existing knowledge based on the answers of only 1 or 2 pupils. In others, teachers knew that pupils had been taught prerequisite knowledge and assumed that they had learned it. In these lessons, teachers moved on to new content while significant numbers of pupils had gaps in their knowledge or a lack of automaticity that would limit their chances of successfully learning new mathematics.

111. The mathematics national curriculum states, 'Decisions about progression should be based on the security of pupils' understanding and their readiness to progress to the next stage. Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on'. Despite this, in just under half the schools visited, teachers' implicit, and occasionally explicit, focus was on 'covering the curriculum' rather than on securing learning. In these schools, teachers often said that they felt pressure to 'stick to the timings in the scheme of work' for fear of not completing the scheme by the end of the academic year.

112. In the other schools, teachers' focus was more clearly on securing learning. In these schools, teachers moved on to the next conceptual step only when pupils were ready. Some schools achieved this by making sure that there was sufficient slack in the curriculum's suggested timings. Others did so by ensuring that teachers worked through the curriculum at the pace suitable for pupils' needs, and had secure methods for passing information on to the next teacher at the end of each academic year.

Assessment as learning

113. The majority of schools visited provided frequent opportunities for pupils to

recall and apply mathematics they had learned previously. These were usually brief activities carried out in most, or sometimes all, mathematics lessons.

114. In most of these schools, teachers monitored pupils' success in these recall activities. In the most successful lessons, when teachers identified a lack of secure knowledge, they quickly decided whether the best course of action was to spend a short time addressing the difficulties as a class or to revisit the topic in more detail in a future lesson. In less successful lessons, teachers sometimes tried to address the difficulties in the same lesson, but without careful planning or appropriate resources. This tended to be more time-consuming and was usually less successful.

Example of weaker practice:

In one lesson visited, pupils were adding fractions with different denominators as part of a 'retrieval practice' activity. A significant number of pupils in the class simply added the numerators and the denominators together. The teacher tried to address this common misunderstanding by re-teaching the concept to the class. The teacher selected 5/12 + 3/8 as the example to use to explain it. The teacher changed both fractions into 96ths, rather than 24ths, while talking about 'lowest common multiple'. The lack of planning led to a poor choice of example to re-explain the method and limited the impact of the intervention.

115. In the majority of schools, teachers had a common understanding about the purpose of these opportunities: to practise the most crucial knowledge and skills that pupils had learned previously, but that was not yet learned to automaticity. When pupils practised retrieval for this purpose, they generally had a high success rate, as they were embedding content they already knew. In some schools, teachers did not have a common understanding of the purpose. Some teachers used 'retrieval practice' for practising exam-style questions or for assessing whether pupils were ready to move on to new learning. This lack of common purpose meant that retrieval practice was less successful in achieving its primary aim: to embed previously learned knowledge more deeply in long-term memory.

Example of stronger practice:

In one school, leaders had identified what important mathematics content needed to be learned to automaticity from each stage of their curriculum. They had collectively developed resources to be used for these brief retrieval activities that took place in lessons, and pupils recorded their answers on mini-whiteboards. Teachers had a common understanding of how to follow up these activities. They expected a high success rate. Any minor errors dealt with 'in the moment'. More often, the teacher noted the errors, which fed into their planning for future lessons.

116. In some schools, pupils were practising mathematics that they had not fully understood at the time of first learning. Pupils were repeating this failure including misconceptions. These misconceptions were becoming embedded.

Example of weaker practice:

One department had a policy that every mathematics lesson began with pupils being asked to complete 'fast 5' questions drawn from prior learning. A significant number of pupils in the class incorrectly found 5% by dividing by 5. Following the department policy, the teacher displayed the correct answers to the 'fast 5' questions on the board, and pupils self-corrected any wrong answers. Pupils' workbooks showed that many of them were regularly unable to recall prior learning. They had made this error repeatedly, and dutifully written the correct answer, during spaced practice activities over a number of months. Further investigation of pupils' workbooks showed that many had consistently made this error when they were first taught how to find percentages without a calculator earlier in the year. At no stage had the teacher been concerned that pupils were regularly unable to recall the intended curriculum content or identified that pupils had this particular misconception and taken steps to address it.

117. These approaches were, usually, limited to the recall of a restricted repertoire of mathematical facts and procedures. Where used consistently well, these approaches were highly effective in supporting pupils to develop automaticity in recalling and accurately applying whatever mathematics needed to be learned.

118. In very few schools were pupils required to practise solving problems using the facts and methods they had previously learned. The exception was when they completed exercises when first applying new mathematical techniques to problems. This lack of practice, except at the point of first learning, prevented pupils from becoming fluent at identifying likely appropriate mathematical approaches to use when faced with unfamiliar problems.

Example of stronger practice:

In one school, leaders had identified that, as well as initially completing exercises with structurally similar problems in class, pupils needed to decide on the appropriate approaches to use when faced with an unfamiliar problem. Leaders had designed their curriculum to ensure, through spaced practice activities, that pupils were solving problems after the point of first teaching. These spaced practice activities required pupils to recall prior learning and choose the most appropriate approach to solve familiar and superficially unfamiliar problems. The problems that pupils had to solve varied from short word problems to longer problems, such as the 'Frogs' investigation. These were aligned with pupils' prior learning.

Assessment of learning – have curriculum goals been achieved?

119. Most schools used termly or half-yearly tests to find out whether pupils had successfully learned the taught curriculum. In most of these schools, the tests were carefully designed to assess what pupils had been taught. In a minority of schools visited, pupils were asked to take tests that included topics that they had not studied and questions that were therefore impossible for them to answer. This was particularly the case when the school was using commercial tests or past GCSE examination papers. This is an inefficient use of pupils' time, which could be better spent learning new mathematics. It could also harm pupils' perceptions of their mathematical capability.

120. Teachers routinely followed summative assessments with activities designed to review pupils' performance in the tests. While summative assessments do not lend themselves to formative purposes, they were most helpful when teachers used them to:

- isolate the specific gaps in the building blocks of knowledge that pupils needed to answer each question correctly
- plan new teaching and practice sequences to address those gaps

121. Going over the test by modelling correct answers was not so successful. It did not identify the specific gaps in pupils' knowledge or provide pupils with additional teaching or opportunities to practise the areas of mathematics they had struggled with.

122. In the majority of schools, test outcomes were converted into some form of grade or attainment descriptor. The thresholds for expected performance were most often set in line with those for GCSE exams. In many schools, little attention was paid to identifying gaps in pupil knowledge and effectively addressing them. Accepting this low level of pupil success as 'good enough' and moving on results in many pupils moving to the next stage of their learning with significant gaps in their mathematical knowledge.

123. In some schools, information from internal and external summative assessments was used to systematically provide feedback on the effectiveness of the way the curriculum was designed or implemented. In these schools, leaders used the assessment information to inform the ongoing development of the curriculum. As a result, the design and implementation were increasingly effective in supporting pupils to learn.

124. In a small number of schools, assessments served little purpose other than determining a current or forecast grade to report to parents and deciding whether a pupil should change teaching class.

125. Many schools visited use the outcomes of tests to track pupils' progress against GCSE, or similar, targets. Teachers were sometimes expected to make intervention plans for pupils who were 'behind target'. This resulted in some pupils with identical mathematical needs not receiving the support given to others. Allocating intervention resources in this way inappropriately limits pupil access to support based on the mathematics they knew potentially 5 years previously.

Systems at the school level

Summary from the research review

> School-level systems strengthen the consistency of a pupil's journey to proficiency. They include monitoring approaches, staff training, resource allocation, teaching and learning expectations, and ways of raising the subject's status, and ways of sharing information between stakeholders. Professional development should be a planned and purposeful pathway to expertise in teaching and subject leadership.

126. Schools visited had taken differing approaches to timetabling mathematics lessons. Averaging over 5 years, the majority of schools ensured that pupils had between 3.5 and 4 hours of mathematics lessons per week. In a small number of schools, pupils had only 3 hours.

127. Schools varied in how they split mathematics curriculum time across the week. In most schools, pupils had mathematics lessons on 3 or 4 days per week. However, in some schools, pupils had 2 much longer lessons per week instead. In a small minority of schools, pupils had a mathematics lesson every

day of the week.

128. In schools that varied the number of mathematics lessons per week between year groups, it was usual for pupils in Year 11 to have slightly more mathematics lessons per week than pupils in key stage 3. Often, this additional teaching time in Year 11 was at the expense of non-examined subjects such as personal, social, health and citizenship education (PSHCE), religious education or physical education. In a small number of schools, Year 11 pupils received an extra lesson of mathematics teaching per week in compulsory ‘after-school’ lessons.

129. At some stage, all schools visited arranged teaching groups based on pupils’ current levels of attainment. Usually, this was based on current levels of mathematics attainment (sets), although occasionally grouping was based on attainment across a wider range of subjects (streams). Almost all schools arranged the school timetable so that pupils could change teaching group midway through the school year if this was judged to be necessary to support their mathematical development.

130. When pupils were placed into classes based on current attainment, it was often on the basis of key stage 2 national curriculum test scores, or baseline assessments taken as pupils started Year 7. It was rare for leaders to include information from Year 6 teachers when deciding classes. This sometimes led to pupils being placed in a teaching class that did not meet their needs, when their key stage 2 assessment did not accurately reflect their level of mathematical knowledge.

131. Where mathematics groupings were not based on current attainment, this was usually in Year 7. A number of leaders said that they had introduced mixed-attainment teaching in Year 7 as a temporary response to the disrupted education and lack of key stage 2 national curriculum test results due to COVID-19. Many of these schools were intending to return to ‘sets’ once they were confident that they had sufficient and accurate assessment information to do so.

132. A small number of schools timetabled an additional mathematics lesson per fortnight for Year 7 pupils who had arrived with significant gaps in their mathematical knowledge. These pupils were typically identified through discussions with primary schools and through their key stage 2 scores. These groups tended to study a curriculum with an assumed earlier starting point rather than identifying and addressing specific gaps in knowledge.

133. Several schools identified that external tutors had had limited success with their pupils. This was because the tutors’ work was not always aligned with the school’s curriculum thinking. These schools had had more success through

employing tutors directly or by spending time making sure that external tutors understood and applied the school's mathematics curriculum.

134. Most schools used departmental meeting time to improve the curriculum and the way it was put into practice. Many subject leaders said that, over the last few years, there had been a significant move from using subject meeting time for administration activities to using it as an opportunity to think about effective teaching.

135. There were significant differences in how often subject teams met. In some schools, subject teams met almost weekly. In other schools, meetings were limited to once or twice a half term. Providing limited opportunities for members of the mathematics department to work together increases the risk of incoherence in implementing the mathematics curriculum.

136. There was a noticeable difference in teachers' CPD between schools with historically stronger provision and those where it had been weaker. In schools that had historically stronger provision, CPD was most often department-led, and often focused on effective teaching of specific parts of the mathematics curriculum. In schools with historically weaker provision, CPD was often at whole-school level on more generic themes, such as 'retrieval'. In these schools, each department was sometimes given time to consider how this CPD could improve mathematics provision in the school.

Appendix

Methodological note

This thematic report draws on findings from 50 research visits to schools in England. These visits were carried out between September 2021 and November 2022.

Deep dives into mathematics took place as part of scheduled school inspections under the education inspection framework. They were carried out by inspectors with relevant expertise in mathematics education who had received training for this work. They carried out a deep dive as part of our methodology for evaluating the quality of education. Inspectors gathered a rich range of data from speaking to senior leaders, subject leaders and teachers, visiting mathematics lessons and speaking to pupils. They also reviewed pupils' work in mathematics.

Various criteria were monitored consistently to identify characteristics for the sample that risked being underrepresented. These criteria were: region, inspection outcome, disadvantage quintile, size of school, and a rural or urban location. We made sure that the sample was broadly representative of the national picture and there was some representation from schools with different characteristics. The deep dive evidence collected was split evenly between primary and secondary schools.

Inspectors gathered qualitative evidence about mathematics education in schools they visited. The range of evidence gathered across these visits enabled us to identify common themes in mathematics education which are likely to be relevant in a wide range of schools.

Inspectors focused on gathering evidence which related to the following areas:

- curriculum
- pedagogy
- assessment
- school-level systems and their impact on mathematics education

When analysing this evidence, we drew on the conception of quality in mathematics education which we outlined in our mathematics research review. This enabled us to consider how mathematics education in English schools relates to our best evidence-based understanding of how schools can ensure a high-quality mathematics education for all pupils.

---

1. Maths education in the UK: exploring the shortage of maths teachers (https://uat.ncfe.org.uk/core-maths/maths-education-in-the-uk-exploring-the-shortage-of-maths-teachers/); NCFE.
2. Merttens, R (2012), 'The 'concrete – pictorial – abstract' heuristic', Mathematics Teaching, 228, pages 33 to 38.
3. 'How valid and reliable is the use of lesson observation in supporting judgements on the quality of education?' (https://www.gov.uk/government/publications/inspecting-education-quality-lesson-observation-and-workbook-scrutiny), re 'inattentional blindness' Ofsted, June 2019.

↑ Back to top