

Card Flip Magic—*Error Detection & Correction*

Summary

When data is stored on a disk or transmitted from one computer to another, we usually assume that it doesn't get changed in the process. But sometimes things go wrong and the data is changed accidentally. This activity uses a magic trick to show how to detect when data has been corrupted, and to correct it.

Curriculum Links

- ☑ Mathematics: Number Level 3 and up. Exploring computation and estimation.
- ☑ Algebra Level 3 and up. Exploring patterns and relationships.

Skills

- ☑ Counting
- ☑ Recognition of odd and even numbers

Materials

Pack of playing cards or cards cut from a sheet of paper with a colour on one side

The “Magic Trick”

When the trick is done

You will need a pile of playing cards, interested in the backs or fronts (suits and number are not important) or identical, two-sided cards.

*To make your own cut up a large sheet of card that is coloured on one side only.

1. Get someone to lay out the cards in a 5×5 square, with a random mixture of sides showing.
2. Casually add another row and column, “just to make it a bit harder”.

These cards are the key to the trick. You must choose the extra cards to ensure that there is an even number of coloured cards in each row and column.

3. Get a someone to flip over one card only while you cover your eyes. The row and column containing the changed card will now have an odd number of coloured cards, and this will identify the changed card.

Here’s how it works

1. Lay out their cards 5×5 .
2. How many coloured cards are there in each row and column? Is it an odd or even number? It is important to remember, 0 is an even number.
3. Now add a sixth card to each row, making sure the number of coloured cards is always even e.g. if there are 3 coloured cards add a card coloured side up to make it an even number. If there are 2 coloured cards place the new card coloured side down to make sure there is an even number of coloured cards. *This extra card is called a “parity” card.*
4. Add a sixth row of cards along the bottom, to make the number of cards in each column an even number. Again make sure there are an even number of coloured cards in each column.
5. Now flip a card. What do you notice about the row and column? (They will have an odd number of coloured cards.) Parity cards are used to show you when a mistake has been made.
6. Repeat the trick until you are confident with it and try it with another person.

Extension Activities:

Try using other objects. Anything that has two ‘states’ is suitable. For example, you could use coins (heads or tails) or cards with 0 or 1 printed on them (to relate to the binary system). What happens if two, or more, cards are flipped? (It is not always possible to know exactly which two cards were flipped, although it is possible to tell that something has been changed. You can usually narrow it down to one of two pairs of cards. With 4 flips it is possible that all the parity bits will be correct afterwards, and so the error could go undetected.)

Another interesting exercise is to consider the lower right-hand card. If you choose it to be the correct one for the column above, then will it be correct for the row to its left? (The answer is yes, always.)

4. In this card exercise we have used even parity—using an even number of coloured cards. Can we do it with odd parity? (This is possible, but the lower right-hand card only works out the same for its row and column if the numbers of rows and columns are both even or both odd. For example, a 5×9 layout will work fine, or a 4×6 , but a 3×4 layout won’t.)

A Real-Life Example for Experts!

This same checking technique is used with book codes. Published books have a ten-digit code usually found on the back cover. The tenth digit is a check digit, just like the parity bits in the exercise. This means that if you order a book using its ISBN (International Standard Book Number), the publisher can check that you haven't made a mistake. They simply look at the checksum. That way you don't end up waiting for the wrong book!

Here's how to work out the checksum:

Multiply the first digit by ten, the second by nine, the third by eight, and so on, down to the ninth digit multiplied by two. Each of these values is then added together.

For example, the ISBN 0-13-911991-4 gives a value:

$$(0 \times 10) + (1 \times 9) + (3 \times 8) + (9 \times 7) + (1 \times 6) + (1 \times 5) + (9 \times 4) + (9 \times 3) + (1 \times 2) = 172$$

Then divide your answer by eleven. What is the remainder?

$$172 \div 11 = 15 \text{ remainder } 7$$

If the remainder is zero, then the checksum is zero, otherwise subtract the remainder from 11 to get the checksum.

$$11 - 7 = 4$$

Look back. Is this the last digit of the ISBN? Yes!

If the last digit of the ISBN wasn't a four, then we would know that a mistake had been made.

TASK: TRY with your own book. Can you make it work?

It is possible to come up with a checksum of the value of 10, which would require more than one digit. When this happens, the character X is used.

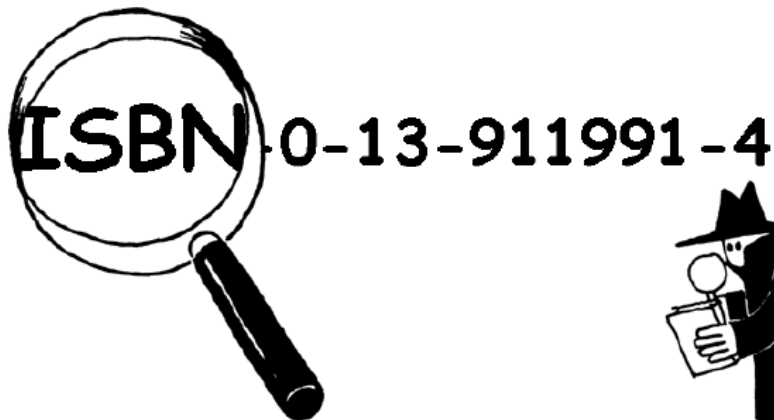


A barcode (UPC) from a box of Weet-Bix™

Another example of the use of a check digit is the bar codes on grocery items. This uses a different formula. If a bar code is misread the final digit should be different from its calculated value. When this happens the scanner beeps and the checkout operator re-scans the code.

Check that book!

Detective Blockbuster
Book Tracking Service, Inc.



We find and check ISBN checksums for a small fee.

Join our agency—look in your classroom or library for real ISBN codes.

Are their checksums correct?

Sometimes errors are made.

Some of the common errors are:

- × a digit has its value changed;
- × two adjacent digits are swapped with each other;
- × a digit is inserted in the number; and
- × a digit is removed from the number

Can you find a book with the letter X for a checksum of 10? It shouldn't be too hard to find—one in every 11 should have it.

What sort of errors might occur that wouldn't be detected?

Can you change a digit and still get the correct checksum?

What if two digits are swapped (a common typing error)?

What's it all about?

Imagine you are depositing \$10 cash into your bank account. The teller types in the amount of the deposit, and it is sent to a central computer. But suppose some interference occurs on the line while the amount is being sent, and the code for \$10 is changed to \$1,000. No problem if you are the customer, but clearly a problem for the bank!

It is important to detect errors in transmitted data. So a receiving computer needs to check that the data coming to it has not been corrupted by some sort of electrical interference on the line. Sometimes the original data can be sent again when an error has been transmitted, but there are some situations when this is not feasible, for example if a disk or tape has been corrupted by exposure to magnetic or electrical radiation, by heat or by physical damage. If data is received from a deep space probe, it would be very tedious to wait for retransmission if an error had occurred! (It takes just over half an hour to get a radio signal from Jupiter when it is at its closest to Earth!)

We need to be able to recognize when the data has been corrupted (*error detection*) and to be able to reconstruct the original data (*error correction*).

The same technique as was used in the “card flip” game is used on computers. By putting the bits into imaginary rows and columns, and adding parity bits to each row and column, we can not only detect if an error has occurred, but *where* it has occurred. The offending bit is changed back, and so we have performed error correction.

Of course computers often use more complex error control systems that are able to detect and correct multiple errors. The hard disk in a computer has a large amount of its space allocated to correcting errors so that it will work reliably even if parts of the disk fail. The systems used for this are closely related to the parity scheme.

And to finish, a joke that is better appreciated after doing this activity:

Q: What do you call this: “Pieces of nine, pieces of nine”?

A: A parrot error.

Solutions and hints

Errors that would not be detected are those where one digit increases and another decreases. Then the sum might still be the same.